Algorithm Design Techniques - COMP 360B

October 19, 2006 MIDTERM EXAM 11:35 - 12:55

No calculators, cell phones, books, notes or cheating.

(10 pts) 1. (a) Define what is meant by a weighted selection problem. State three properties for such a problem to be solvable by greedy selection.

(b) You have a weighted selection problem that can be solved optimally by greedy selection. Suppose you have an optimum solution for a given instance of your problem. Will this still be an optimum solution if the weight of each item in the given instance is increased by one? Give a proof or counterexample.

(c) Suppose you have a directed graph with weights on the edges, and have found a shortest path from some vertex s to another vertex t. Will this still be a shortest path if each edge weight in the graph is increased by one? Give a proof or counterexample.

(10 pts) 2. You have N dollars to spend on game software. There are n games available with prices in dollars c_j, $j = 1, \ldots, n$ (assume these are positive integers). Each game has a rating r_j, which is also a positive integer. Your goal is to buy games with total price at most N dollars, so as to maximize the sum of the ratings of the games bought. Give a dynamic programming solution to this problem in pseudocode, and justify its correctness. Derive its running time. Is it polynomial in the input size?

(10 pts) 3. Supersushi has n take out shops in Montreal that are supplied from k production locations labelled $L_1, L_2, \cdots, L_k$. Each production location can supply t boxes of sushi. For $j = 1, \ldots, n$, shop j orders r_j boxes of sushi, and can be supplied from a subset $R_j \subseteq \{L_1, L_2, \cdots, L_k\}$ of nearby locations. (The stuff has to be fresh.) Formulate the problem of supplying the shops as a network flow problem. If all the orders cannot be filled, you supply as much sushi as possible.

(b) Let $n = 4$, $k = 3$, $t = 3$, $r_1 = 3$, $r_2 = 2$, $r_3 = 1$, $r_4 = 2$ and

$$R_1 = \{L_1, L_3\}, R_2 = \{L_1\}, R_3 = \{L_2, L_3\}, R_4 = \{L_1, L_3\}.$$

Draw the corresponding network, and find the maximum flow. For each iteration show the residual graph. On completion show how to find the minimum cut. Can all the orders be satisfied? Justify your answer.