Faculty of Science

FINAL EXAMINATION

COMPUTER SCIENCE COMP 360

Algorithm Design Techniques

Examiner: Prof. D. Avis Thursday, December 21, 2006, 2-5pm

Associate Examiner: Prof. L. Devroye

Instructions

Calculators are not allowed

No cell phones, books, or notes.

This exam consists of 5 questions, and has 3 pages including this cover page.

Dictionaries are not allowed

Answer in EXAM BOOK

(12 pts) 1. (Greedy) (a) What is the weighted selection problem? State three properties for such a problem to be solvable optimally by the greedy selection algorithm.

(b) Describe a problem for which the greedy selection algorithm provides an optimum solution. Give a brief justification why each of the three properties is satsified.

(c) Describe a problem for which the greedy selection algorithm fails. Explain which of the conditions fails, and give a small example illustrating this.

(12 pts) 2. (Dynamic Programming) (a) Describe an algorithm for determining whether a directed graph with weights on the edges contains a negative weight cycle. Give pseudocode for the algorithm and analyze its time complexity.

(b) Explain how to use the algorithm in part (a) to determine in polynomial time if a graph has two different negative weight cycles. The cycles may share edges. Justify your approach, and analyze its time complexity. You should call the algorithm in part (a) as few times as possible.

(12 pts) 3. (Linear Programming)

(a) Explain what is a linear program in standard form, and what is its dual. State the strong duality theorem.

(b) Show how to formulate the minimum vertex cover problem as an **integer linear program.** Write down the integer linear program for the case when the graph is a cycle of length 5.

(c) Formulate the example in part (b) as a **linear program**, and write down its dual. Solve both problems by inspection, and verify the strong duality theorem.

(d) Explain how to use the LP formulation as an approximation algorithm for vertex cover. What is the worst case bound on the ratio of the approximate solution to the optimum solution?

(12 pts) 4. (Network Flows)

(a) Give a brief description of the Ford-Fulkerson algorithm, explaining what is an augmenting path and what is the residual graph. State the max flow/min cut theorem.

(b) Give an example to show that this algorithm does not run in polynomial time. Describe briefly how to modify the algorithm to run in polynomial time using scaling.

(c) Explain how to use the Ford-Fulkerson algorithm to solve the following problem. Undergraduate students who wish to do a summer project each submit a list of names of profs for whom they would be willing to work. Each professor can accept at most two undergraduate students. The problem is to find supervisors for as many students as possible. Give an interpretation of the max flow and the minimum cut. Illustrate on a small example where not all of the students find a supervisor.

(12 pts) 5. (NP-completeness) CLIQUE: You are given an undirected graph $G = (V, E)$ and an integer k. You want to find out if there is a subset $S \subseteq V$ of k vertices such that each pair of vertices in S is joined by an edge in G.

(a) Define the class NP, and determine whether this problem is in NP.

(b) Explain what is meant by a polynomial time reduction.

(c) Reduce one of the following problems to CLIQUE.
3-SAT, SUBSET SUM, VERTEX COVER.
Give a description of the problem from the list that you have chosen, give the reduction, and prove it is correct.