Algorithm Design Techniques - COMP 360B

October 20, 2005 MIDTERM EXAM 11:35 - 12:55

No calculators, cell phones, books, notes or cheating.

(10 pts) 1. (a) State three properties for selection problems to be solvable by greedy selection.

(b) For any $n \geq 4$, let S be the set of edges of the complete graph K_n. A subset of edges is admissible if it is a cycle or a subset of a cycle. Which of the three properties of greedy selection are satisfied in this case? Justify your answer.
(A cycle in K_n is a set of edges $u_1u_2, u_2u_3, \cdots, u_ku_1$, where $u_1, u_2, \cdots, u_k$ are distinct vertices.)

(c) Assign positive edge weights to the complete graph K_n, $n \geq 4$. Does the the greedy selection algorithm always find a maximum weight cycle? Give a proof or counterexample.

(10 pts) 2. You are given a set of n positive integers, $a_1, a_2, \cdots, a_n$. Write in pseudocode an algorithm that runs in $O(n)$ time to determine if any subset of the integers sums to exactly 1000. If there is such a subset, your algorithm should output it.

(10 pts) 3. Formulate the following problem as a network flow problem. There are n students seeking jobs from m different companies. For $i = 1, \ldots, m$, company i has up to k_i jobs available. For $j = 1, \ldots, n$, student j selects t_j companies for which she is willing to work. Your job is to find an algorithm that finds a job for as many students as possible. (One student one job.) Give a small example and interpret the meaning of a flow in your network in terms of the original problem. Can you find any interpretation of a cut?

(10 pts) 4. In the network below, each edge is marked with a pair f, c where f is the flow on the edge and c is the capacity.

(a) Verify that the flow is feasible.

(b) Draw the residual graph, and find an augmenting path. Repeat until the flow is maximum.

(c) Give a minimum weight cut to certify the flow is maximum.