Faculty of Science

FINAL EXAMINATION

COMPUTER SCIENCE COMP 360

Algorithm Design Techniques

Examiner: Prof. D. Avis Thursday, December 8, 2005, 2-5pm

Instructions

Calculators are not allowed

Additional paper to be supplied for rough work. This will not be graded.

No cell phones, books, notes or cheating.

This exam consists of 5 questions.

Dictionaries are not allowed

(10 pts) 1. (a) What is the weighted selection problem? State three properties for such a problem to be solvable optimally by the greedy selection algorithm.

(b) You are given an unweighted graph and asked to find a **matching** of maximum size in the graph. Show that the greedy selection algorithm may fail to do this by giving a counterexample, and by showing which property, or properties, in (a) fail.

(c) Describe a heuristic based on matchings for the vertex cover problem in an unweighted graph. State and prove a bound on the size of the vertex cover produced by the heuristic divided by the size of the minimum vertex cover.

(10 pts) 2. (a) Describe the KNAPSACK problem, and formulate it as an optimization problem.

(b) Show how to solve it by dynamic programming by giving a suitable pseudocode.

(c) State the time complexity of your algorithm. Explain why it is not polynomial in the size of the input. Give some (non-trivial) condition on the input which makes the time complexity polynomial in the input size.

(10 pts) 3. (a) Explain what is a linear program in standard form, and what is its dual.

(b) Describe the three possible outcomes for a linear programming problem.

(c) Describe a way of verifying each of the outcomes in (b), by means of suitable certificates.

(d) For the system of inequalities below, either find a solution or prove that none exists.

$$\begin{gathered} x_1 - x_2 + x_3 \leq -4 \\ -2x_1 + x_2 - x_3 \leq 6 \\ x_1 + x_2 + x_3 \leq 1 \\ x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{gathered}$$

(10 pts) 4. You are given n positive integers $a_1, a_2, \cdots, a_n$ and an integer k. You want to find out if the integers can be partitioned into two subsets S and T such that

$$\sum_{i \in S} a_i = k + \sum_{j \in T} a_j.$$

(a) Formulate this as an integer linear programming problem.

(b) Define the class NP, and determine whether this problem is in NP.

(c) Reduce one of the following problems to it:
3-SAT, SUBSET SUM, VERTEX COVER.
Describe in detail the problem from the list that you have chosen, give the reduction, and prove it is correct.

(10 pts) 5. (a) You are given an integer k and a directed graph with two specified vertices: a source s and a sink t. Explain how you could use a network flow algorithm to determine if there are k paths from source to sink, such that no edge of the graph appears in more than one of the paths. (Such a set of paths are called edge-disjoint.)

(b) What is the maximum number of iterations that the Ford-Fulkerson algorithm could take to solve this problem?

(c) Suppose that for a given input, there is no set of k edge-disjoint paths. How could you easily convince someone, who knows nothing about network flows, of this?

(d) For the example below, find the maximum set of edge-disjoint paths, and give a proof of optimality, along the lines you described in your answers to parts (a) and (c).