2) Matrix Chain Multiplication

This question is worth 10 points out of 25. Here is the deduction scheme:
    1 point if you didn't give the answer, i.e. the best/worst parenthesizations
    (0.5 point for each of a and b)
    2 points if you didn't give any illustration of the algorithm, at least m[i,j] for the
    different values of i and j (0.5 point for each of a and b)
    2 points if you didn't give the s table or some indication that you understand how to obtain
     the best/worst parenthesization (1 point for each of a and b)
    1 point if you didn't explain the modification to the algorithm in part b
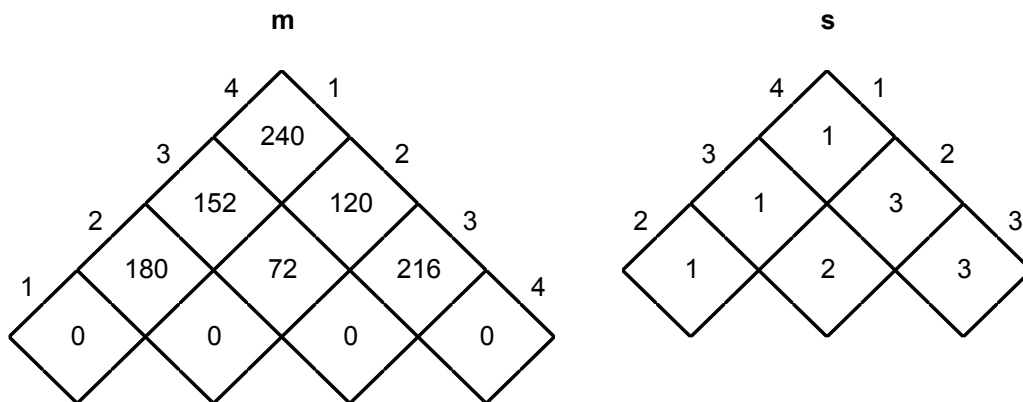   Up to 3 or 4 points if you didn't modify the algorithm correctly,
    or proposed a different algorithm altogether
    0.5 or 1 point for other errors, depending on importance

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p0    p1    p2    p3    p4
10     2     9     4     6
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(a)

l	i	j	k	q				m[i, j]	s[i, j]
1	1	2	1	m[1,1]+m[2,2]+p0*p1*p2	=	0+0+180	= 180	180	1
2	2	3	2	m[2,2]+m[3,3]+p1*p2*p3	=	0+0+72	= 72	72	2
	3	4	3	m[3,3]+m[4,4]+p2*p3*p4	=	0+0+216	= 216	216	3
3	1	3	1	m[1,1]+m[2,3]+p0*p1*p3	=	0+72+80	= 152	152	1
			2	m[1,2]+m[3,3]+p0*p2*p3	=	180+0+360	= 540		
	2	4	2	m[2,2]+m[3,4]+p1*p2*p4	=	0+216+108	= 324	120	3
			3	m[2,3]+m[4,4]+p1*p3*p4	=	72+0+48	= 120		
4	1	4	1	m[1,1]+m[2,4]+p0*p1*p4	=	0+120+120	= 240	240	1
			2	m[1,2]+m[3,4]+p0*p2*p4	=	180+216+540	= 936		
			3	m[1,3]+m[4,4]+p0*p3*p4	=	152+0+240	= 392		

    MCM (1, 4)
        X = MCM (1, 1) = A1
        Y = MCM (2, 4)
            X = MCM (2, 3)
                X = MCM (2, 2) = A2
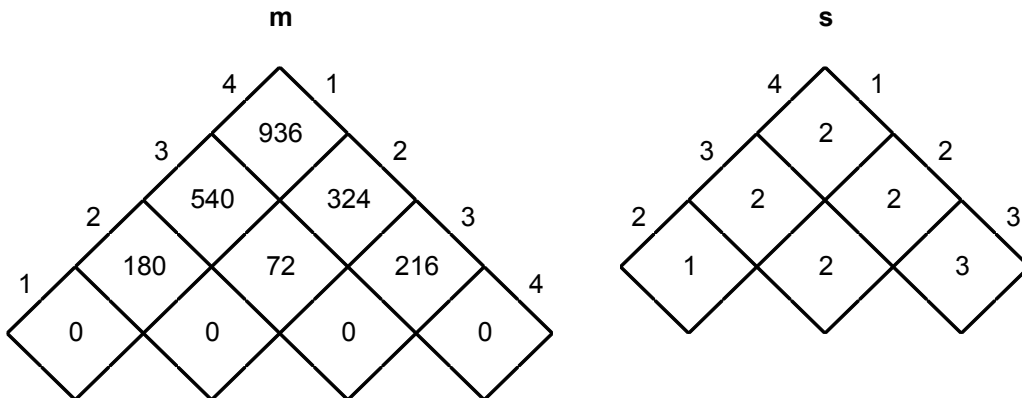   Y = MCM (3, 3) = A3
            Y = MCM (4, 4) = A4

    The best parenthesization is: (A1 * ((A2 * A3) * A4))

(b)   We use the same algorithm, except we change it in two places:

Line	Change	To
7	m[i, j] = ∞	m[i, j] = 0 (or any non positive value)
10	if q < m[i, j]	if q > m[i, j]

m

s

l	i	j	k	q				m[i, j]	s[i, j]
	1	2	1	m[1,1]+m[2,2]+p0*p1*p2	=	0+0+180	= 180	180	1
2	2	3	2	m[2,2]+m[3,3]+p1*p2*p3	=	0+0+72	= 72	72	2
	3	4	3	m[3,3]+m[4,4]+p2*p3*p4	=	0+0+216	= 216	216	3
3	1	3	1	m[1,1]+m[2,3]+p0*p1*p3	=	0+72+80	= 152	540	2
			2	m[1,2]+m[3,3]+p0*p2*p3	=	180+0+360	= 540		
	2	4	2	m[2,2]+m[3,4]+p1*p2*p4	=	0+216+108	= 324	324	2
			3	m[2,3]+m[4,4]+p1*p3*p4	=	72+0+48	= 120		
4	1	4	1	m[1,1]+m[2,4]+p0*p1*p4	=	0+324+120	= 444	936	2
			2	m[1,2]+m[3,4]+p0*p2*p4	=	180+216+540	= 936		
			3	m[1,3]+m[4,4]+p0*p3*p4	=	540+0+240	= 780		

MCM (1, 4)
  X = MCM (1, 2)
    X = MCM (1, 1) = A1
    Y = MCM (2, 2) = A2
  Y = MCM (3, 4)
    X = MCM (3, 3) = A3
    Y = MCM (4, 4) = A4

The worst parenthesization is: ((A1 * A2) * (A3 * A4))