How Good is the Simplex Algorithm?

Victor Klee and George Minty

February 1970

Abstract

By constructing long 'increasing' paths on appropriate convex polytopes, it is shown that the simplex algorithm for linear programs (at least with its most commonly used pivot rule) is not a 'good algorithm' in the sense of J. Edmonds. That is, the number of pivots or iterations that may be required is not majorized by any polynomial function of the two parameters that specify the size of the program.

Almost all string graphs are intersection graphs of plane convex sets

János Pach[1]

Ecole Polytechnique Fédérale de Lausanne and Rényi Institute, Hungarian Academy of Sciences
[Lausanne, Switzerland and Budapest, Hungary]
pach@cims.nyu.edu

Bruce Reed

School of Computer Science, McGill University, Laboratoire I3S CNRS, and Professor Visitante Especial, IMPA
[Montreal, Canada and Rio de Janeiro, Brazil]
breed@cs.mcgill.ca

Yelena Yuditsky

School of Computer Science, McGill University
[Montreal, Canada]
yuditskyL@gmail.com

Abstract

A *string graph* is the intersection graph of a family of continuous arcs in the plane. The intersection graph of a family of plane convex sets is a string graph, but not all string graphs can be obtained in this way. We prove the following structure theorem conjectured by Janson and Uzzell: The vertex set of *almost all* string graphs on n vertices can be partitioned into *five* cliques such that some pair of them is not connected by any edge ($n \to \infty$). We also show that every graph with the above property is an intersection graph of plane convex sets. As a corollary, we obtain that *almost all* string graphs on n vertices are intersection graphs of plane convex sets.

2012 ACM Subject Classification G.2.2 Graph Theory

Keywords and phrases String graph, intersection graph, plane convex set

Digital Object Identifier 10.4230/LIPIcs.SoCG.2018.68

1 Overview

The *intersection graph* of a collection C of sets is a graphs whose vertex set is C and in which two sets in C are connected by an edge if and only if they have nonempty intersection. A *curve* is a subset of the plane which is homeomorphic to the interval $[0, 1]$. The intersection graph of a finite collection of curves ("strings") is called a *string graph.*

Ever since Benzer [Be59] introduced the notion in 1959, to explore the topology of genetic structures, string graphs have been intensively studied both for practical applications and theoretical interest. In 1966, studying electrical networks realizable by printed circuits, Sinden [Si66] considered the same constructs at Bell Labs. He proved that not every graph is a string graph, and raised the question whether the recognition of string graphs is decidable. The affirmative answer was given by Schaefer and Štefankovič [ScSt04] 38 years later. The difficulty of the problem is illustrated by an elegant construction of Kratochvíl and Matoušek [KrMa91], according to which there exists a string graph on n vertices such that no matter

[1] [Supported by Swiss National Science Foundation Grants 200021-165977 and 200020-162884.]

34th International Symposium on Computational Geometry (SoCG 2018).
Editors: Bettina Speckmann and Csaba D. Tóth; Article No. 68; pp. 68:1–68:20
LIPICS Leibniz International Proceedings in Informatics
Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

JOURNAL OF COMBINATORIAL THEORY, Series B 35, 39–61 (1983)

Graph Minors. I. Excluding a Forest

NEIL ROBERTSON AND P. D. SEYMOUR*

Department of Mathematics, Ohio State University, 231 W. 18th Ave, Columbus, Ohio 43210

Communicated by the Editors

Received May 3, 1982

The path-width of a graph is the minimum value of k such that the graph can be obtained from a sequence of graphs $G_1,..., G_r$ each of which has at most $k+1$ vertices, by identifying some vertices of G_i pairwise with some of G_{i+1} $(1 \leqslant i < r)$. For every forest H it is proved that there is a number k such that every graph with no minor isomorphic to H has path-width $\leqslant k$. This, together with results of other papers, yields a "good" algorithm to test for the presence of any fixed forest as a minor, and implies that if P is any property of graphs such that some forest does not have property P, then the set of minor-minimal graphs without property P is finite.

1. INTRODUCTION

Let G be a graph. (All graphs in this paper are finite, and may have loops or multiple edges unless we state otherwise.) A sequence $X_1,..., X_r$ of subsets of $V(G)$ (the vertex set of G) is a *path-decomposition* of G if the following conditions are satisfied.

(W1) *For every edge e of G, some X_i $(1 \leqslant i \leqslant r)$ contains both ends of e.*

(W2) *For $1 \leqslant i \leqslant i' \leqslant i'' \leqslant r$, $X_i \cap X_{i''} \subseteq X_{i'}$.*

The *path-width* of G is the minimum value of $k \geqslant 0$ such that G has a path-decomposition $X_1,..., X_r$ with $|X_i| \leqslant k+1$ $(1 \leqslant i \leqslant r)$.

H is a *minor* of G if H can be obtained from G by deleting some vertices and/or edges, and/or contracting some edges. The main theorem of this paper is the following:

(1.1). *For every forest H there is an integer w such that every graph with no minor isomorphic to H has path-width $\leqslant w$.*

* Partially supported by National Science Foundation Grant MCS 8103440.

0095-8956/83 $3.00

Journal of Combinatorial Theory, Series B 99 (2009) 583–616

Contents lists available at ScienceDirect

Journal of Combinatorial Theory, Series B

www.elsevier.com/locate/jctb

Graph minors. XXI. Graphs with unique linkages

Neil Robertson [a,1], Paul Seymour [b,2]

[a] Ohio State University, Columbus, OH 43210, USA
[b] Princeton University, Princeton, NJ 08544, USA

ARTICLE INFO

Article history:
Received 3 January 1995
Available online 10 February 2009

Keywords:
Graph minors
Disjoint paths
Intertwining conjecture
Tree-width

ABSTRACT

A *linkage* L in a graph G is a subgraph each component of which is a path, and it is *vital* if $V(L) = V(G)$ and there is no other linkage in G joining the same pairs of vertices. We show that, if G has a vital linkage with p components, then G has tree-width bounded above by a function of p. This is the major step in the proof of the unproved lemma from Graph Minors XIII, and it has a number of other applications, including a constructive proof of the intertwining conjecture.

1. Introduction

A *linkage* in a graph G is a subgraph every component of which is a path. (All graphs in this paper are finite and undirected, and may have loops or parallel edges. *Paths* have at least one vertex, and have no “repeated” vertices or edges.) A vertex of G is a *terminal* of a linkage L in G if $v \in V(L)$ and v has degree $\leqslant 1$ in L. The *pattern* of a linkage L is the partition of its terminals in which two terminals are in the same block if and only if they belong to the same component of L. A linkage is a p-*linkage* if it has $\leqslant p$ terminals, where $p \geqslant 0$ is an integer. A linkage L in G is *vital* if $V(L) = V(G)$, and no linkage in G different from L has the same pattern as L.

A *tree-decomposition* of a graph G is a pair (T, W), where T is a tree and $W = (W_t\colon\ t \in V(T))$ is a family of subgraphs of G, satisfying

1. $\bigcup(W_t\colon\ t \in V(T)) = G$, and
2. if $t, t', t'' \in V(T)$ and t' lies on the path of T between t and t'', then $W_t \cap W_{t''} \subseteq W_{t'}$.

E-mail address: p.seymour@math.princeton.edu (P. Seymour).

[1] This research was supported by NSF grant DMS 8903132.
[2] This work was carried out while the author was employed at Telcordia Technologies in Morristown, NJ.

0095-8956/$ – see front matter © 2008 Published by Elsevier Inc.
doi:10.1016/j.jctb.2008.08.003

NOTES ON GROWING A TREE IN A GRAPH[ʘ]

Luc Devroye,[☿] *Vida Dujmović,*[♀] *Alan Frieze,*[♁] *Abbas Mehrabian,*[♂] *Pat Morin,*[♃] *and Bruce Reed*[♄]

Abstract. We study the height of a spanning tree T of a graph G obtained by starting with a single vertex of G and repeatedly selecting, uniformly at random, an edge of G with exactly one endpoint in T and adding this edge to T.

[ʘ] This research was partly supported by NSERC and by NSF grants DMS1661063 and DMS1362785. Some of this work was carried out while Vida Dujmović, Pat Morin, and Bruce Reed were visiting IMPA in Rio de Janeiro. They thank the Institute for its hospitality.

[☿] School of Computer Science, McGill University, Montréal, Canada

[♀] Department of Computer Science and Electrical Engineering, University of Ottawa, Ottawa, Canada

[♁] Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, USA. Research supported in part by NSF Grant DMS1362785

[♂] Department of Computer Science, University of British Columbia, Vancouver, Canada. This author was supported by an NSERC Postdoctoral Fellowship and a Simons-Berkeley Research Fellowship. Part of this work was done while he was visiting the Simons Institute for the Theory of Computing at UC Berkeley.

[♃] School of Computer Science, Carleton University, Ottawa, Canada

[♄] CNRS, Projet COATI, I3S (CNRS and UNS) UMR7271 and INRIA, Sophia Antipolis, France. Instituto Nacional de Matemática Pura e Aplicada (IMPA), Brasil. Visiting Research Professor, ERATO Kawarabayashi Large Graph Project, Japan.