

How Good is the Simplex Algorithm?

Victor Klee and George Minty

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Abstract

By constructing long 'increasing' paths on appropriate convex polytopes, it is shown that the simplex algorithm for linear programs (at least with its most commonly used pivot rule) is not a 'good algorithm' in the sense of J. Edmonds. That is, the number of pivots or iterations that may be required is not majorized by any polynomial function of the two parameters that specify the size of the program.

Almost all string graphs are intersection graphs of plane convex sets

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Abstract

A *string graph* is the intersection graph of a family of continuous arcs in the plane. The intersection graph of a family of plane convex sets is a string graph, but not all string graphs can be obtained in this way. We prove the following structure theorem conjectured by Janson and Uzzell: The vertex set of *almost all* string graphs on n vertices can be partitioned into *five* cliques such that some pair of them is not connected by any edge ($n \rightarrow \infty$). We also show that every graph with the above property is an intersection graph of plane convex sets. As a corollary, we obtain that *almost all* string graphs on n vertices are intersection graphs of plane convex sets.

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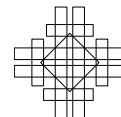
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1 Overview

The *intersection graph* of a collection C of sets is a graph whose vertex set is C and in which two sets in C are connected by an edge if and only if they have nonempty intersection. A *curve* is a subset of the plane which is homeomorphic to the interval $[0, 1]$. The intersection graph of a finite collection of curves (“strings”) is called a *string graph*.

Ever since Benzer [Be59] introduced the notion in 1959, to explore the topology of genetic structures, string graphs have been intensively studied both for practical applications and theoretical interest. In 1966, studying electrical networks realizable by printed circuits, Sinden [Si66] considered the same constructs at Bell Labs. He proved that not every graph is a string graph, and raised the question whether the recognition of string graphs is decidable. The affirmative answer was given by Schaefer and Štefankovič [ScSt04] 38 years later. The difficulty of the problem is illustrated by an elegant construction of Kratochvíl and Matoušek [KrMa91], according to which there exists a string graph on n vertices such that no matter

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Graph Minors. I. Excluding a Forest

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The path-width of a graph is the minimum value of k such that the graph can be obtained from a sequence of graphs G_1, \dots, G_r , each of which has at most $k + 1$ vertices, by identifying some vertices of G_i pairwise with some of G_{i+1} ($1 \leq i < r$). For every forest H it is proved that there is a number k such that every graph with no minor isomorphic to H has path-width $\leq k$. This, together with results of other papers, yields a "good" algorithm to test for the presence of any fixed forest as a minor, and implies that if P is any property of graphs such that some forest does not have property P , then the set of minor-minimal graphs without property P is finite.

1. INTRODUCTION

Let G be a graph. (All graphs in this paper are finite, and may have loops or multiple edges unless we state otherwise.) A sequence X_1, \dots, X_r of subsets of $V(G)$ (the vertex set of G) is a *path-decomposition* of G if the following conditions are satisfied.

(W1) For every edge e of G , some X_i ($1 \leq i \leq r$) contains both ends of e .

(W2) For $1 \leq i \leq i' \leq i'' \leq r$, $X_i \cap X_{i''} \subseteq X_{i'}$.

The *path-width* of G is the minimum value of $k \geq 0$ such that G has a path-decomposition X_1, \dots, X_r with $|X_i| \leq k + 1$ ($1 \leq i \leq r$).

H is a *minor* of G if H can be obtained from G by deleting some vertices and/or edges, and/or contracting some edges. The main theorem of this paper is the following:

(1.1). For every forest H there is an integer w such that every graph with no minor isomorphic to H has path-width $\leq w$.

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Graph minors. XXI. Graphs with unique linkages

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A linkage L in a graph G is a subgraph each component of which is a path, and it is *vital* if $V(L) = V(G)$ and there is no other linkage in G joining the same pairs of vertices. We show that, if G has a vital linkage with p components, then G has tree-width bounded above by a function of p . This is the major step in the proof of the unproved lemma from Graph Minors XIII, and it has a number of other applications, including a constructive proof of the intertwining conjecture.

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1. Introduction

A *linkage* in a graph G is a subgraph every component of which is a path. (All graphs in this paper are finite and undirected, and may have loops or parallel edges. *Paths* have at least one vertex, and have no “repeated” vertices or edges.) A vertex of G is a *terminal* of a linkage L in G if $v \in V(L)$ and v has degree ≤ 1 in L . The *pattern* of a linkage L is the partition of its terminals in which two terminals are in the same block if and only if they belong to the same component of L . A linkage is a p -*linkage* if it has $\leq p$ terminals, where $p \geq 0$ is an integer. A linkage L in G is *vital* if $V(L) = V(G)$, and no linkage in G different from L has the same pattern as L .

A *tree-decomposition* of a graph G is a pair (T, W) , where T is a tree and $W = (W_t: t \in V(T))$ is a family of subgraphs of G , satisfying

- $\bigcup (W_t: t \in V(T)) = G$, and
- if $t, t', t'' \in V(T)$ and t' lies on the path of T between t and t'' , then $W_t \cap W_{t''} \subseteq W_{t'}$.

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² This work was carried out while the author was employed at Telcordia Technologies in Morristown, NJ.

NOTES ON GROWING A TREE IN A GRAPH^⓪

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ABSTRACT. We study the height of a spanning tree T of a graph G obtained by starting with a single vertex of G and repeatedly selecting, uniformly at random, an edge of G with exactly one endpoint in T and adding this edge to T .

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