Musical Rhythm and Computational Mathematics

Godfried T. Toussaint

School of Computer Science, McGill University, 3480 University St., Montreal, Canada
[godfried@cs.mcgill.ca]

2000 Mathematics Subject Classification. 68R01, 52C99, 52C10, 90B80, 51K99

In this lecture musical rhythm is represented symbolically, which renders it eminently suitable for the analysis from the discrete-mathematical and computational points of view. Several methods used to represent rhythm timelines (ostinatos), as well as measure rhythmic similarity are discussed. New measures of rhythm similarity are compared in terms of musical relevance and their computational efficiency. Analysing the histograms of inter-onset duration intervals in rhythms yields interesting mathematical and computational problems, and suggests methods for the automatic generation of new rhythms that have specific properties. For example, a large class of noteworthy rhythms used in World music may be generated by the *Euclidean* algorithm originally proposed by Euclid for computing the greatest common divisor of two given integers. Viewing rhythms as sequences (or strings) of symbols, suggests using tools from bioinformatics to perform phylogenetic analyses of families of rhythms. Such tools allow us to "reconstruct" "ancestral" rhythms, which may be used in automatic music composition systems, or to test theories of the evolution of musical rhythms. These applications will be illustrated with examples from African, Latin-American, and Flamenco music. Most of the material relevant to this talk may be found scattered in: [1], [2] [3], [4], [5], [6], and [7].

[1] Toussaint, G. T., The geometry of musical rhythm. In *Proceedings of the Japan Conference on Discrete and Computational Geometry*, (eds. Jin Akiyama, Mikio Kano, and Xuehou Tan). LNCS 3742, Springer-Verlag, Berlin-Heidelberg, 2005, 198–212.

[2] Toussaint, G. T., A comparison of rhythmic similarity measures. In *Proceedings of the 5th International Conference on Music Information Retrieval*, Universitat Pompeu Fabra, Barcelona, Spain, October 10-14, 2004, 242–245.

[3] Toussaint, G. T., The Euclidean algorithm generates traditional musical rhythms. In *Proceedings of BRIDGES: Mathematical Connections in Art, Music and Science*, Banff, Canada, July 31 - August 3, 2005, 47–56.

[4] Toussaint, G. T., A mathematical analysis of African, Brazilian, and Cuban *clave* rhythms. In *Proceedings of BRIDGES: Mathematical Connections in Art, Music and Science*, Towson University, Towson, MD, USA, July 27-29, 2002, 47–56.

[5] Toussaint, G. T., Classification and phylogenetic analysis of African ternary rhythm timelines. In *Proceedings of BRIDGES: Mathematical Connections in Art, Music and Science*, Granada, Spain, July 23-27, 2003, 47–56.

[6] Toussaint, G. T., Mathematical features for recognizing preference in Sub-Saharan African traditional rhythm timelines. In *Proceedings of 3rd International Conference on Advances in Pattern Recognition*, Bath, England, August 22-25, 2005, 18–27.

[7] Diaz-Bañez, M., Farigu, G., Gomez, F., Rappaport, D., and Toussaint, G. T., El compas flamenco: A phylogenetic analysis. In *Proceedings of BRIDGES: Mathematical Connections in Art, Music and Science*, Winfield, Kansas, USA, July 30 to August 1, 2004, 61–70.