Limit Distributions in Finite Chains

We noted that in an irreducible chain the mean number of visits to j before we return to i is a geometric sum which must be finite. Thus in a finite irreducible chain, the mean time between visits to any state must be finite and all the states are positive persistent.

So, finite irreducible aperiodic chains have limit distributions.

Wer noted that if our states are the vertices of a graph, and we are equally likely to leave on each edge out of a vertex, then provided the graph has an odd cycle it is aperiodic and the unique limit distribution is given by:

$$x(i) = \frac{|\{j|ij \in E\}|}{2|E|}.$$

We said a distribution x satisfied the detailed balance condition if for all i, j with $p_{ij} > 0$ we had $x(i)p_{ij} = x(j)p_{ji}$ and noted that this implied it was the unique limit distribution.

We discussed modofying a chain by adding loops (details to be added).