Lectures 6 and 7: Characteristic Functions and Concentration

Part of Lecture 6 was based on a survey paper by Gabor Lugosi which can be found at: cgm.cs.mcgill.ca/$\tilde{}$breed/conc/lugosi.ps

We returned to the topic of Concentration. We first recalled Markov's Inequality, Chebyshev's inequality and Chernoff's bounding method for sums of random variables (reference: Page 3 of Lugosi's notes).

We then proved Hoeffding's inequality (pages 7-9 of the notes).

This led naturally to considering Moment Generating Functions. We saw the definition for discrete and continuous variables and noted that this function was actually the generating function of the moments of X (by Taylor's theorem).

We remarked that the moments need not be finite, so we turned to characteristic functions.

Following section 5.7 of the text, we stated (3) and its converse but did not prove them.

We stated (4) which tells us that we can often recover the moments of X from its characteristic function.

We proved (5) and (6) which tell us how to compute the characteristic function of sums and scalar multiples.

We stated (3) of Section 5.9 which states that the characteristic function determines the random variable. We also stated (5) which says informally that the characteristic function of the limit of a sequence of functions is the limit of the sequence of characteristic functions of this sequence.

We then proved the Law of Large Numbers and the Central Limit Theorem, by applying these results. This is on page 193-194 of Section 5.10.

We differed from the book's treatment in one respect here. Once we had obtained the last set off equation of the proof, we knew that $\frac{S_n - nE(X)}{\sqrt{nVAR(X)}}$ goes to the same function for every X with finite mean and finite non-zero variance. To show that the function was the normal distribution, we simply needed to prove this for some specific funxtion and we did so for the variable which is equally likely to be 1 or -1 which has mean zero and variance 1. To do so we simply plugged in Stirling's formula to approximate the probability that S_n was a particular value and took a discrete integral rather than a continuos one. We reproduce neither proof here and will not exam this line of the proof.

We note that we strayed into non-examined material in Lecture 6, discussing concentration of variables which were not sums. This material can be found in Lugosi's notes for those interested in learning more.