Lecture 5: The Long Range Behaviour of Symmetric Random Walks

In this lecture we present an approach to analyzing random walks which does not give results which are as exact as those we obtained using symmetry and difference equations. However the approach is more robust in that it requires only very weak assumptions about the variables.

We will show using these techniques:

(A) for any random walk in Z_1 with mean zero, the expected number of returns to zero is infinite (recall that a walk is symmetric if the step distribution x satisfies $Pr(X = v) = Pr(X = -v)$).

It follows that the probability we return to zero is one.

We then used this to show that

(B) the claim on page 161 that $T < \infty$ with probability 1 is true.

Thereby completing the proof of 5.2(24) began in the last lecture.

Proof of (A): We derived as an immediate corollary of the law of large numbers that for any $\epsilon > 0$ there is an N such that we expect S_n to lie between $-\epsilon N$ and ϵN for $\frac{N}{2}$ values of n between N and $2N$. It follows that there is some k within this range such that the expected number of times we visit k in the first $2N$ steps is at least $\frac{1}{4\epsilon}$. Since we expect to visit 0 more often than k by homogeneity (once we visit k for the first time it is symmetric with zero), it follows that for every $\epsilon > 0$ the expected number of times we visit 0 is greater than $\frac{1}{4\epsilon}$. It follows that we expect to visit 0 infinitely often and that the probability we return to it is 1.

Proof of (B): The text's proof of this result is on pages 199-200. We considered a similar but different proof. I now prefer the texts.