Assignment 2

Due at start of class on Thursday February 3rd.

1.
a) how many binomial trees are used when forming a binomial heap with 743 elements?
b) how many comparisions are needed to add a key to a binomial tree with 743 elements?
c) how many comparisions are needed to merge a binomial tree with 1221 elements with a binomial tree with 829 elements?

2) Consider a heap H on a 7 element set $X = \{x_1, ..., x_7\}$.
a) For how many of the 7! permutations of X is the heap order given by H valid?
b) Show that any decision tree in the comparision model which constructs a heap on 7 elements has at least 42 leaves.
b) Show that any decision tree in the comparision model which constructs a heap on 7 elements has at least 43 leaves.

3. a) Describe an adversarial strategy which proves that 8 comparisions are necessary to build a heap with 8 elements in the decision-tree comparision model.
b) Show that this bound is tight (i.e there is an algorithm in the model which builds a heap on 8 elements using 8 comparisions).

4. Show that given a heap on 2^k elements, we can construct a binomial tree on the same set withoutmaking any extra comparisions. Try to make your answer precise.

5. Suppose a recursive algorithm always splits a problem with n elements into two subproblems such that if the number of elements in these subproblems are n_1 and n_2 then

(a) $n_1 < 2n/3, n_2 < 2n/3$, and

(b) $n_1 + n_2 \leq n + \sqrt{n}$.

We want to determine an upper bound on the number of subproblems that the procedure considers when solving a problem on n elements. Show that such a procedure considers $O(n)$ subproblems when handling a problem of size n.

6. Describe an adversarial strategy which shows that it requires at least $5n/4 - 5$ comparisions to find the $n/4$th largest element from a set of n keys in the comparision-decision tree model.