COMMENTS ON A LOWER BOUND FOR CONVEX HULL DETERMINATION

David AVIS
McGill University, Montreal, Quebec H3C 3G1, Canada

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In [2] Van Emde Boas proves an $\Omega(n \log n)$ lower bound for the problem of finding the convex hull of a planar set of points. His computational model uses linear decision trees and (potentially) infinite precision real arithmetic. We show that, in fact, no finite algorithm exists under this model, even for determining the convex hull of a set of three points. For consider any linear decision tree for finding the convex hull of a set of three points. Now choose any three colinear points in the plane, (x_1, y_1), (x_2, y_2), (x_3, y_3), such that $p = (x_1, y_1, x_2, y_2, x_3, y_3)$ does not lie on any of the hyperplanes used in the linear decision tree. Since we are allowed unlimited precision, this is always possible. Now p must lie in the interior of a six-dimensional convex region defined by some leaf node of the linear decision tree. By perturbing p we can change the cardinality of the convex hull while remaining at this leaf node. Thus the algorithm fails.

The methods of [2] can, however, be used for a model of computation with integer arithmetic. As shown previously in [1], $O(n \log n)$ algorithms do exist under this model. The appropriate model for this problem is, in fact, the quadratic decision tree model. Yao's elegant proof of an $\Omega(n \log n)$ lower bound for this model is contained in [3]. Further information on the use of linear decision tree lower bounds for geometric problems is contained in [4].

References

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