SIP – 2, Fall 2013

An SIP Formulation for Production Scheduling and Application at a Gold Mine:

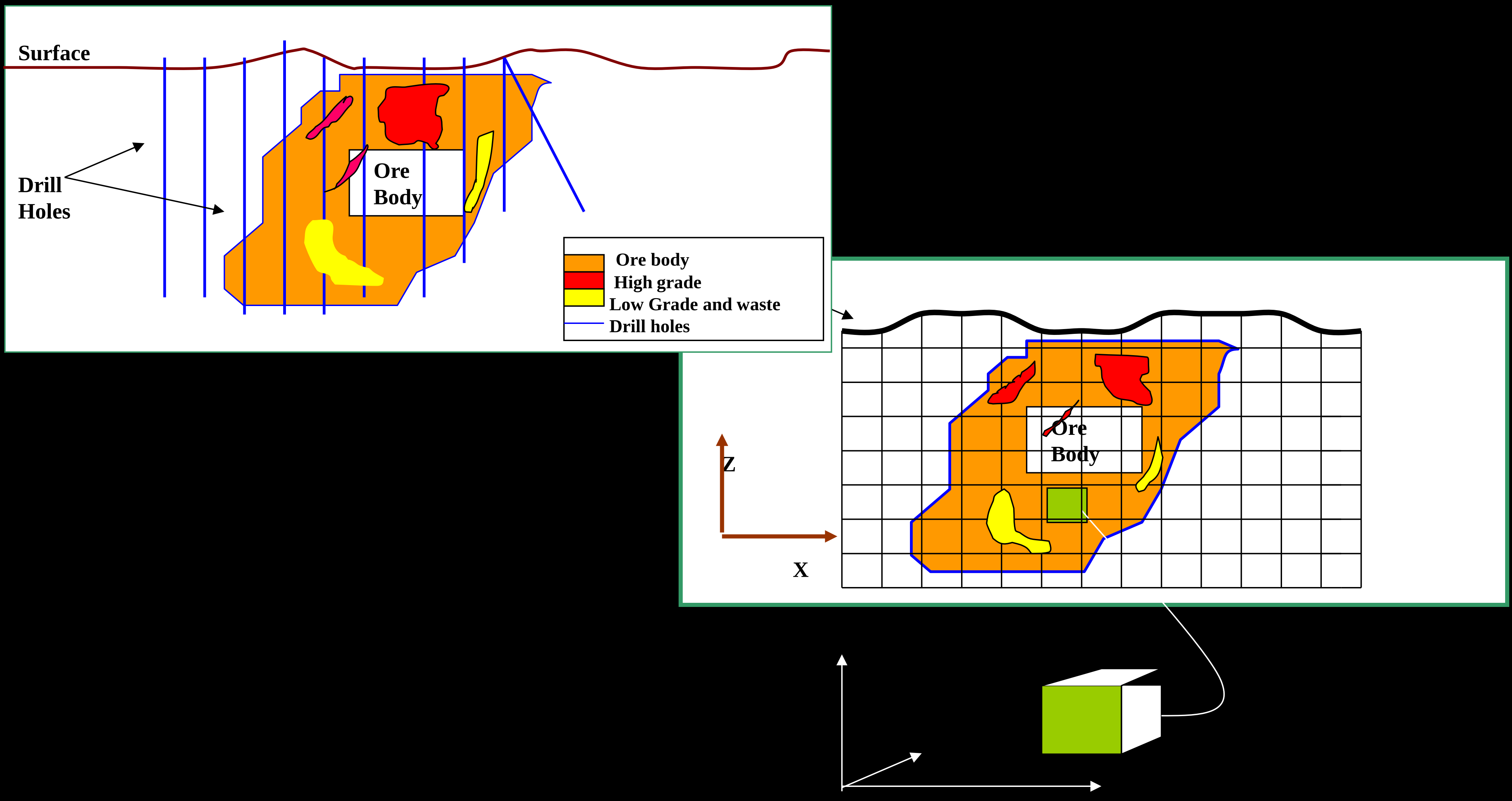
An Industry Example of the Value of Stochastic Solutions

Contents

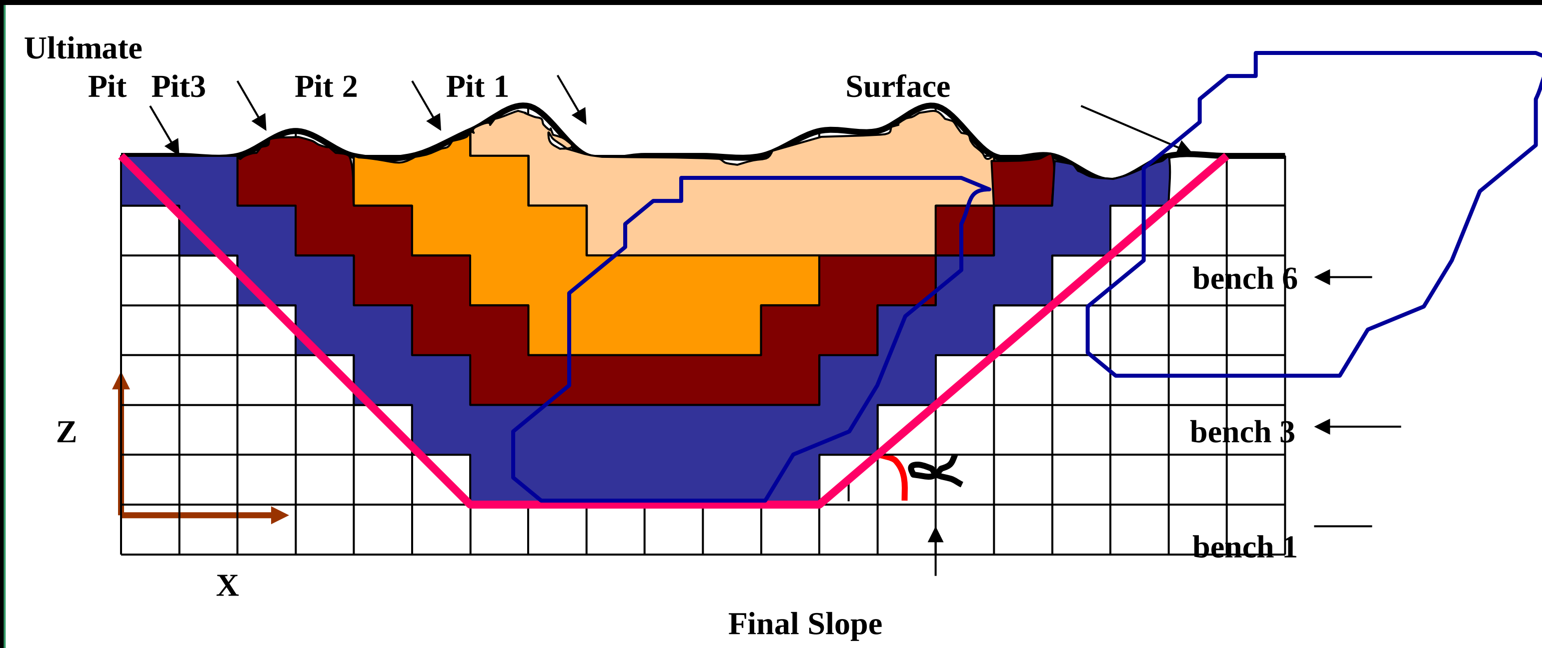
Introduction

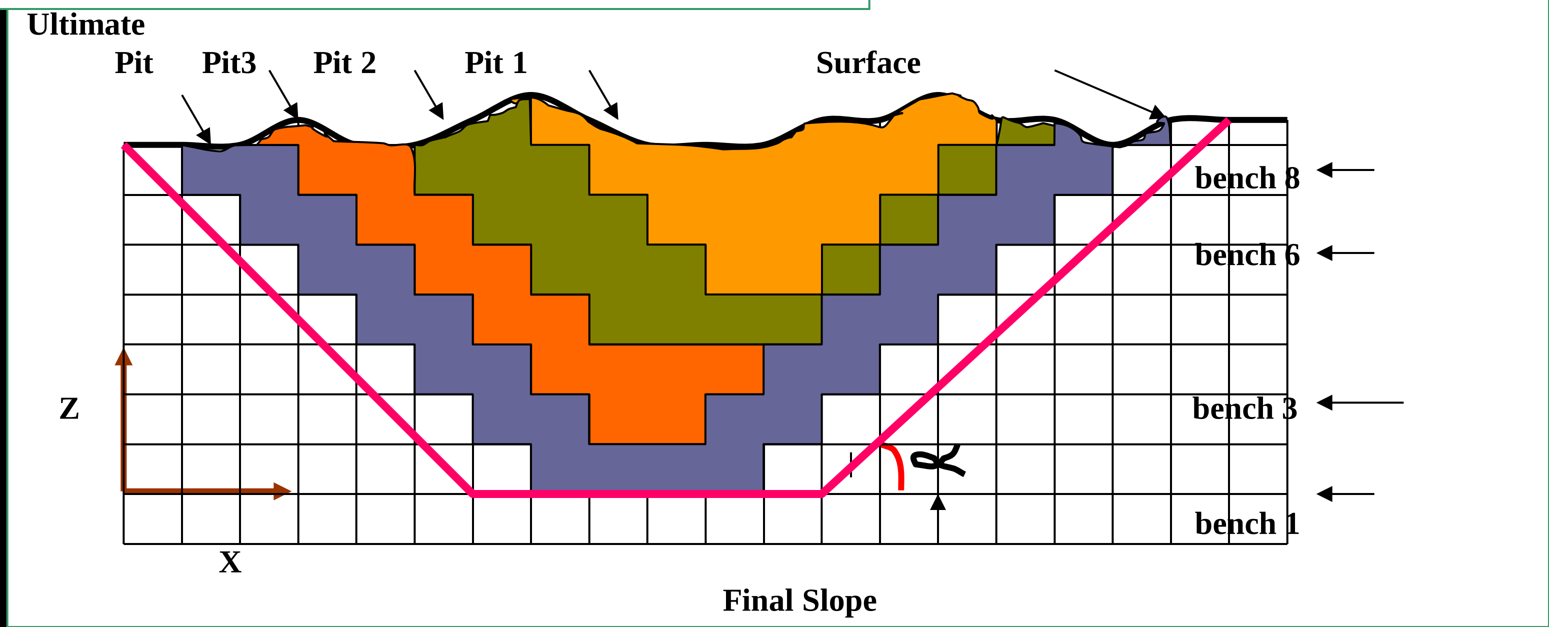
- The traditional "optimum scheduling methods" are based on mathematical models with inputs of 100% certainty.
- Uncertainty may exist from technical, environmental and market sources. Grade variability is examined in this presentation.
- A recently developed Stochastic Integer Programming (SIP) model uses multiple simulated orebody models in optimising long-term production schedules in open pit mines.
- BUT, let's provide some background

Production Scheduling - Open Pit Mine

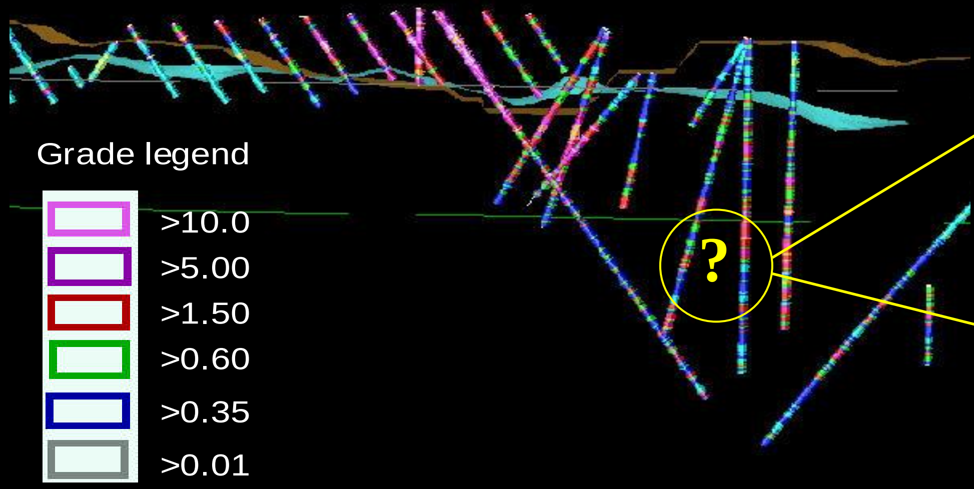


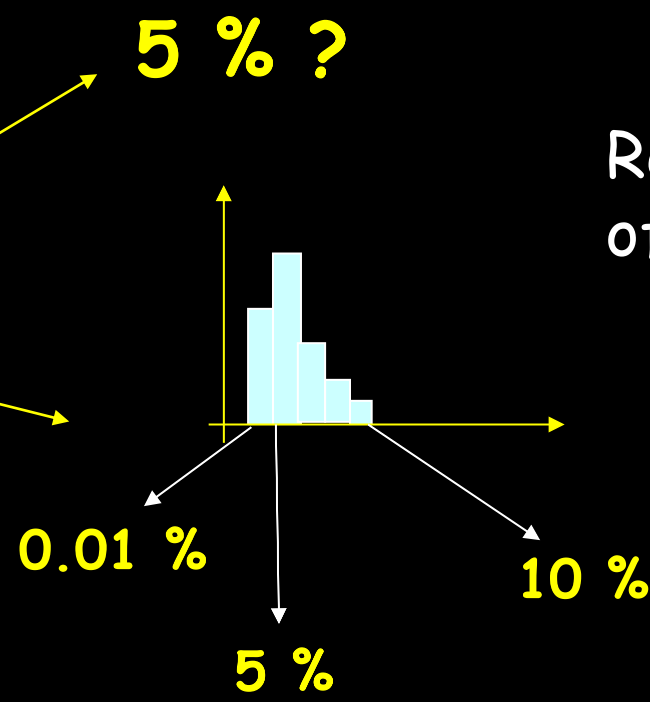
Production Scheduling - Open Pit Mine

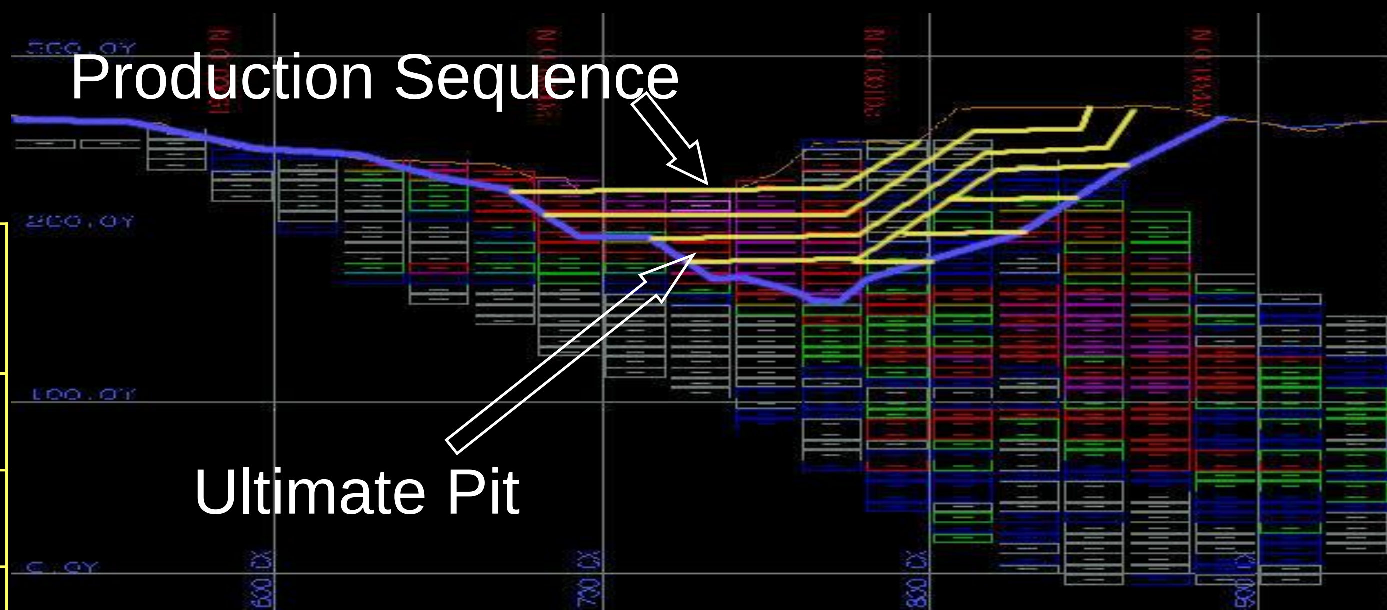




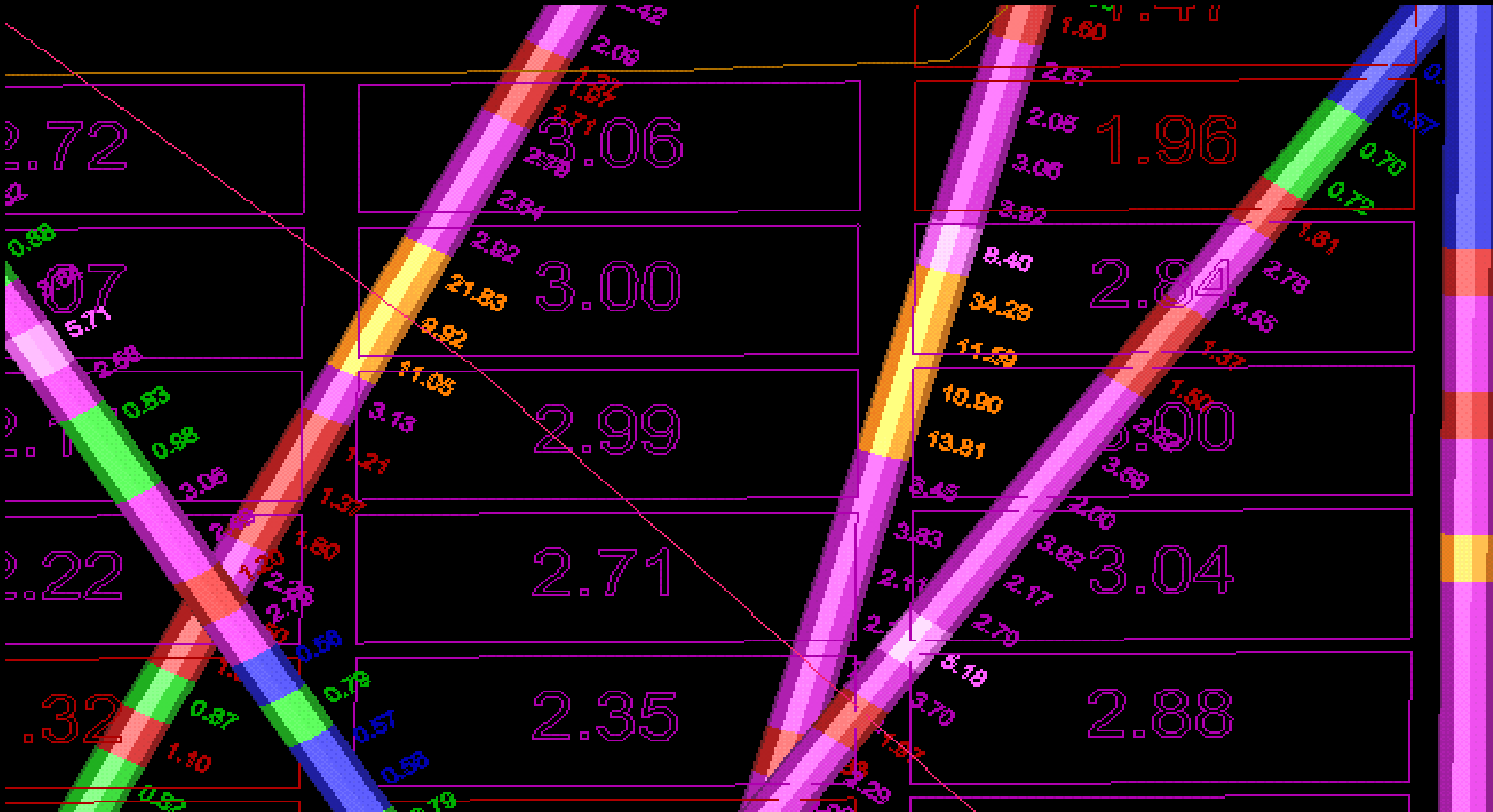
Representation of uncertainty







Period	DCF (m$)	Ore (Mt)	Waste (Mt)	Gold (Mgr)
1	14.2	1.0	0.36	1.78
2	19.0	2.0	1.4	2.9
3	22.2	3.0	2.6	4.1

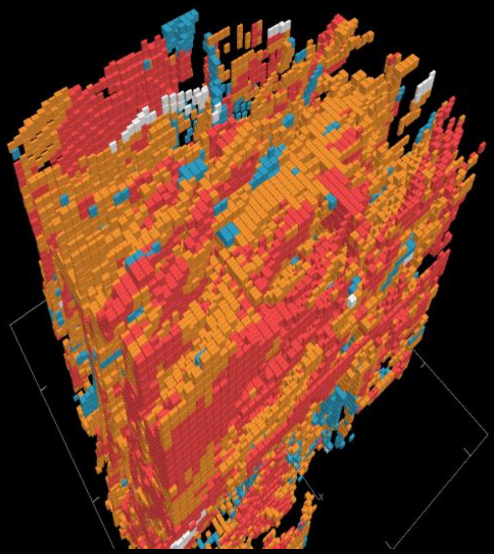


3.06
3.00
2.99
2.71
2.35
1.96
3.04
2.88
2.22
2.08
2.67
2.05
3.06
2.84
2.82
21.83
9.92
11.05
3.13
1.21
1.37
1.60
8.40
34.29
10.90
13.81
3.45
3.83
2.17
2.79
3.70
0.70
0.72
1.81
2.78
1.57
1.50
3.69
0.88
5.71
2.58
0.83
0.98
3.05
0.56
0.79
0.57
0.87
1.10

An Open Pit Gold Deposit

Lode 1502
Simulation #1

A Formula for a Block Value (Blocks Representing the Deposit) when Optimizing

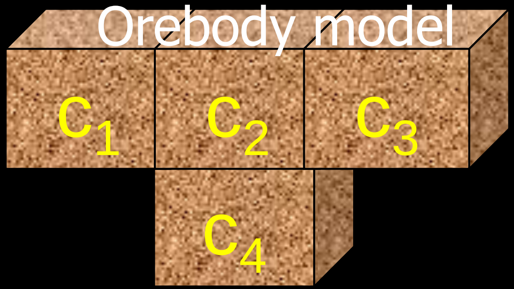


BLOCK ECONOMIC VALUE =

(METAL*RECOVERY*PRICE - ORE*COSTP)

- ROCK*COSTM

- The objective function: *The main objective of the long term production scheduling is to maximize net present value (NPV) of the mine.*



Maximize

$$\sum_{t=1}^{p} \sum_{i=1}^{n} C_i^t * X_i^t$$

where

p is the maximum number of scheduling periods

n is the total number of blocks to be scheduled

C_i^t is the NPV to be generated by mining block i in period t

X_i^t is a binary variable, equal to 1 if the block i is to be mined in period t, 0 otherwise.

Subject to the following constraints

- Grade blending constraints

  Upper bound constraints: The average grade of the material sent to the mill has to be less than or equal to a certain grade value, G_{max}, for each period, t

$$\sum_{i=1}^{n} (g_i - G_{max}) * O_i * X_i^t \leq 0$$

where

g_i is the average grade of block i

O_i is the ore tonnage in block i

- Reserve constraints

$$\sum_{t=1}^{p} X_i^t \leq 1$$

- Processing capacity constraints

Upper bound: The total tonnage of ore processed cannot be more than the processing capacity (PCmax) in any period, t

$$\sum_{i=1}^{n} (O_i * X_i^t) \leq PC_{max}$$

Lower bound: The total tonnage of ore processed cannot be less than a certain amount (PCmin) in any period, t

$$\sum_{i=1}^{n} (O_i * X_i^t) \geq PC_{min}$$

Note:

ALL DETAILS ARE IN THE PAPERS

provided for this lecture

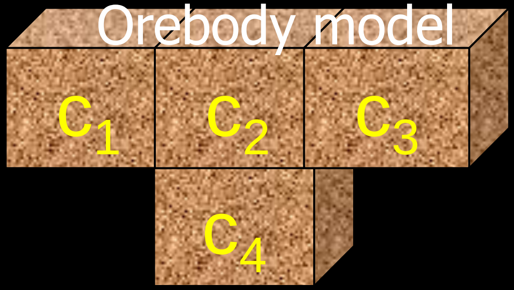
Particularly:

Ramazan and Dimitrakopoulos,

Optimization in Engineering, 2013

Models for Optimisation

Integer Programming



An objective function

Maximize $(c_1x_1^1+c_2x_2^1+\ldots.)\ldots$

c = constant

X_1^1 = binary variable

Subject to

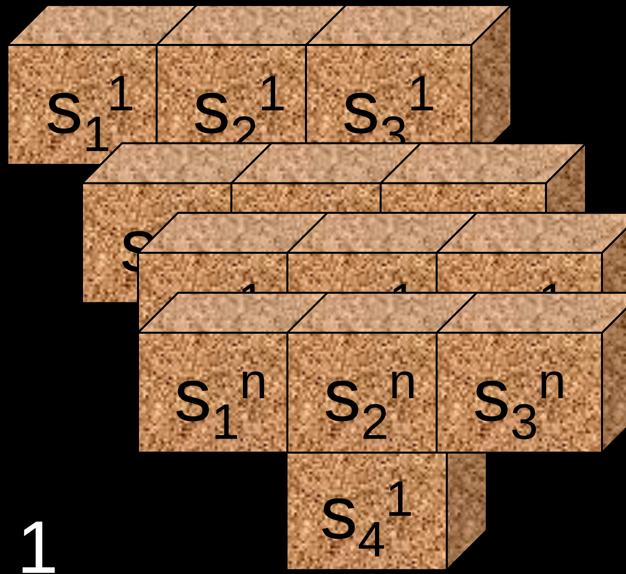
$c_1x_1^1+c_2x_2^1+\ldots. = b_1$ → Period 1

⋮

$c_1x_1^p+c_2x_2^p+\ldots. = b_p$ → Period p

Stochastic Integer Programming (SIP)

The objective function now is …..



Maximise $(s_{11}x_1^1+s_{21}x_2^1+\ldots.$
$s_{12}x_1^1+s_{22}x_2^1+\ldots.)$ …

Subject to

$s_{11}x_1^1+s_{21}x_2^1+\ldots. = b_1$ → Period 1

⋮

Simulated model 1
Simulated model 2
Simulated model r

$s_{11}x_1^p+s_{21}x_2^p+\ldots. = b_1$ → Period p

$s_{12}x_1^p+s_{22}x_2^p+\ldots. = b_1$

$s_{1r}x_1^p+s_{2r}x_2^p+\ldots. = b_1$

Stochastic Integer Programming (SIP)

- Account for uncertain inputs
- Consider simulated grade realizations in the optimization process
- Minimize the risk of not meeting production targets caused by geological variability

SIP - Production Scheduling Model

Objective function

$$\text{Max} \sum_{t=1}^{P} \Big[\sum_{i=1}^{N} E\{(NPV)_i^t\}\, b_i^t$$

$\longrightarrow$ Part 1

Mill & dump

$$- \sum_{i=1}^{U} E\{(NPV)_i^t + MC_i^t\} * s_i^t$$

$\longrightarrow$ Part 2

Stockpile input

$$+ \sum_{s=1}^{M} (SV)_s^t \, (P)\, q_s^t$$

$\longrightarrow$ Part 3

Stockpile output

$$- \sum_{s=1}^{M} (c_u^{ty} d_{su}^{ty} + c_l^{ty} d_{sl}^{ty}) \Big]$$

$\longrightarrow$ Part 4

Risk management

Note:

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Particularly:

Ramazan and Dimitrakopoulos,

Optimization in Engineering, 2013

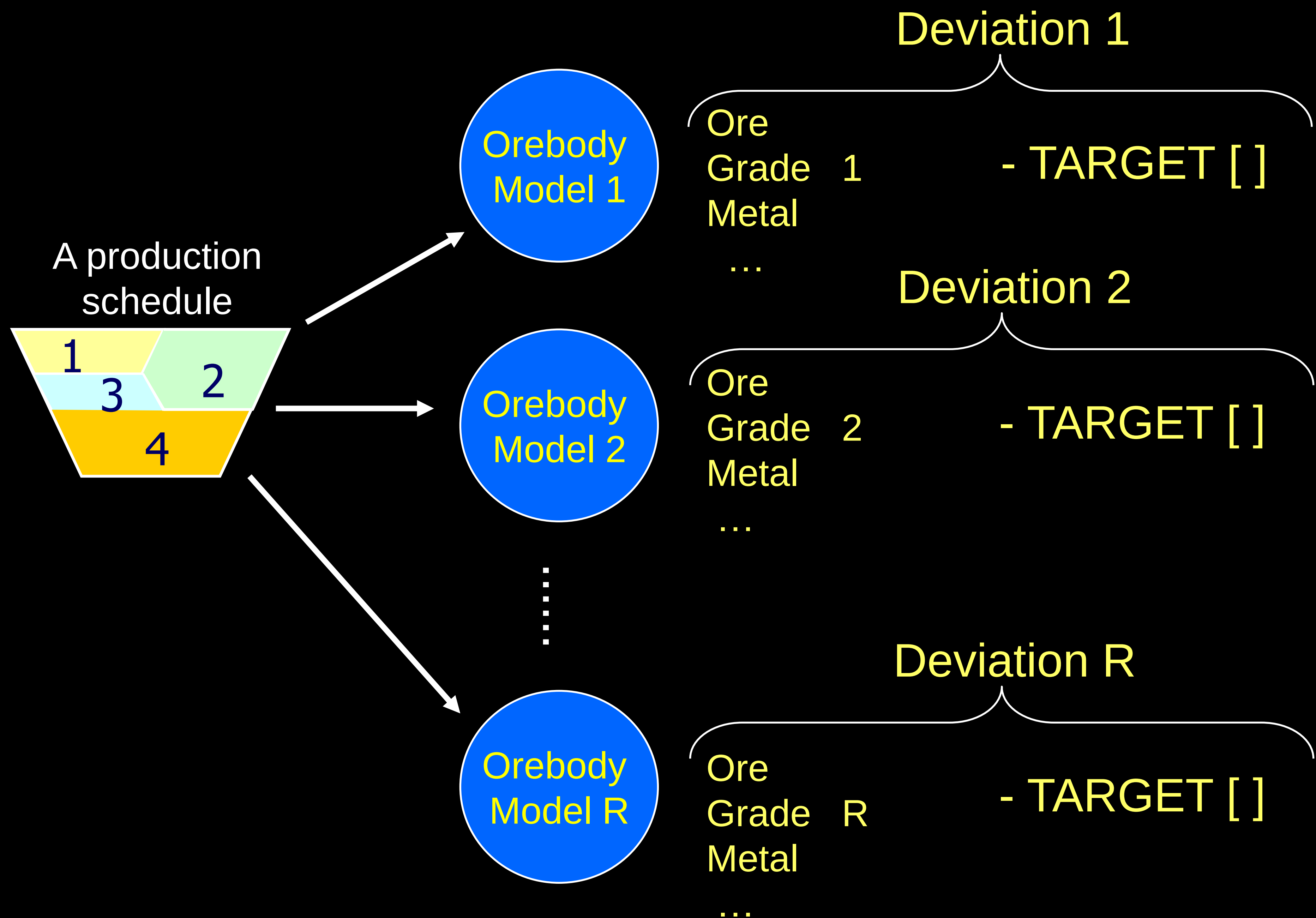
Objective Function - Deviations

$$\text{Minimise……} - \sum_{r=1}^{R} (c_u^{ty} d_{ru}^{ty} + c_l^{ty} d_{rl}^{ty})]\ldots..$$

Part 2

Deviation from production targets $c^{ty}{}_u$ and $c^{ty}{}_l$ penalized by $d^{ty}{}_{ru}$ and $d^{ty}{}_{rl}$ for each simulation r

Stochastic Integer Programming - SIP



SIP – Geological 'Discount Rate'

$$\text{Minimise}\ldots\ldots\ldots - \sum_{r=1}^{R}(c_u^{ty} d_{ru}^{ty} + c_l^{ty} d_{rl}^{ty})]\ldots\ldots$$

Risk Management

$$c_u^{ty} = \frac{c_u^{0y}}{(1+d)^t}$$

Shortage in ore production

$$c_l^{ty} = \frac{c_l^{0y}}{(1+d)^t}$$

Excess in ore production

Deviations from production targets by $d^{ty}{}_{su}$ and $d^{ty}{}_{sl}$ are penalized by $c^{ty}{}_u$ and $c^{ty}{}_l$ respectively for each simulation s

SIP – Penalties

$$\sum_{t=1}^{P}\left[\sum_{i=1}^{N}E\left\{(\mathrm{NPV})_i^t\right\}*b_i^t\right]$$

Total NPV

r = economic discount rate

$$E\left\{(\mathrm{NPV})_i^t\right\}=\frac{E\left\{\left(EV_i^0\right)\right\}}{(1+r)^t}$$

$$\sum_{t=1}^{P}\left[-\sum_{s=1}^{M}\left(c_u^{ty}d_{su}^{ty}+c_l^{ty}d_{sl}^{ty}\right)\right]$$

Risk Management

d = geological 'discount rate'

$$c_u^{ty}=\frac{c_u^{0y}}{(1+d)^t}$$

SIP – A Stochastic Definition of Ore

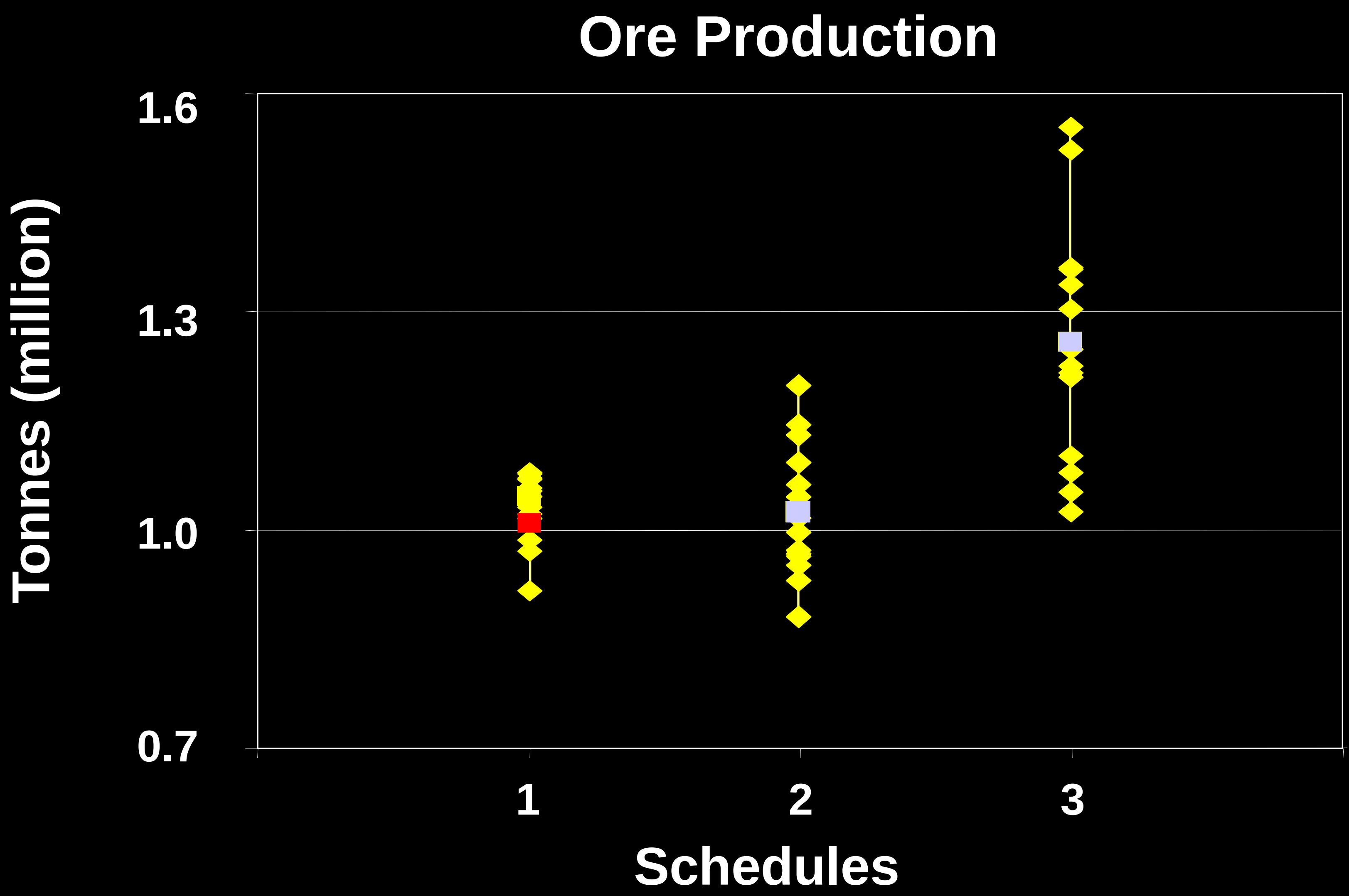
$$\mathrm{E}\{\mathrm{V_i}\} = \begin{cases} NR_i - MC_i - PC_i \text{ , if } NR_i > PC_i\text{; block i is ore} \\ -MC_i - PC_i \text{ , if } NR_i \leq PC_i\text{; block i is waste} \end{cases}$$

$$NR_i = T_i * G_i * rec * (\text{Price} - \text{Selling cost})$$

A probability cut-off (p) is also utilized to classify a block as ore

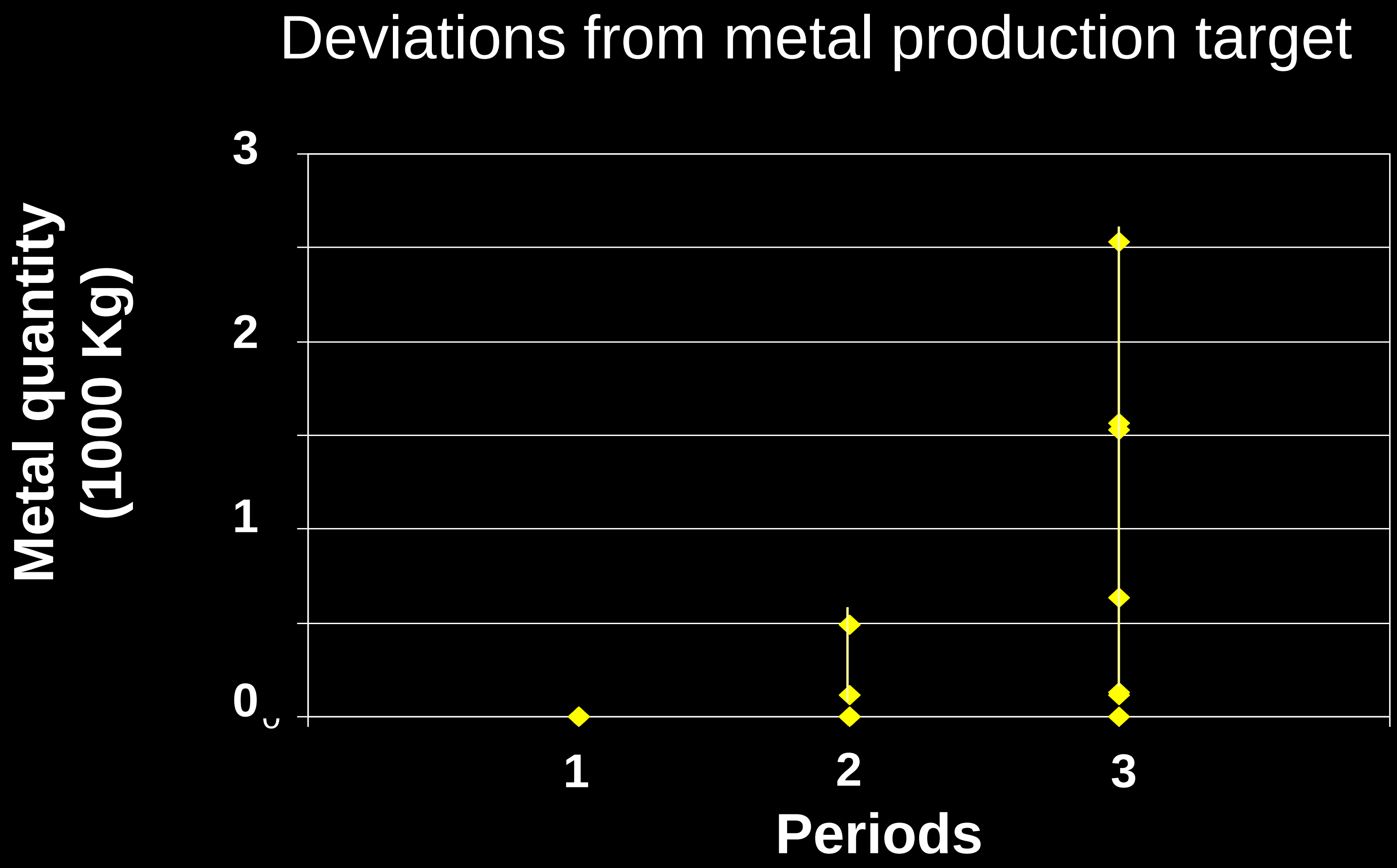
$$\text{if Prob}\{G_i \geq g_{cut-off}\} \geq p\text{, block i is ore}$$
$$\text{else, block i is waste}$$

Managing Risk within a Given Period



$$C_{l3}^{t} > (C_{l1}^{t} = C_{u1}^{t}) > (C_{l2}^{t} = C_{u2}^{t}) > C_{u3}$$

Managing Risk Between Periods



$C^t = C^{t-1} * RDF_{t-1}$ $RDF_t = 1/(1+r)^t$

RDF – risk discounting factor   r – orebody risk discount rate

Case Study on a Large Gold Mine

General information

Total blocks	**22,296**
Block dimensions (m)	**20 x 20 x 20**
Processing input capacity (PC)	**18 Mtpa**
Metal production capacity (MC)	**28,000 Kg pa**
Total mining capacity (TC)	**85 Mtpa**
Stockpile capacity (SC)	**5 Mt**
Stockpile re-handling cost	**0.6 $/t**
Discount rate	**10 %**
Mine Life	**6 yrs**

Case Study on a Large Gold Mine

The SIP specific information

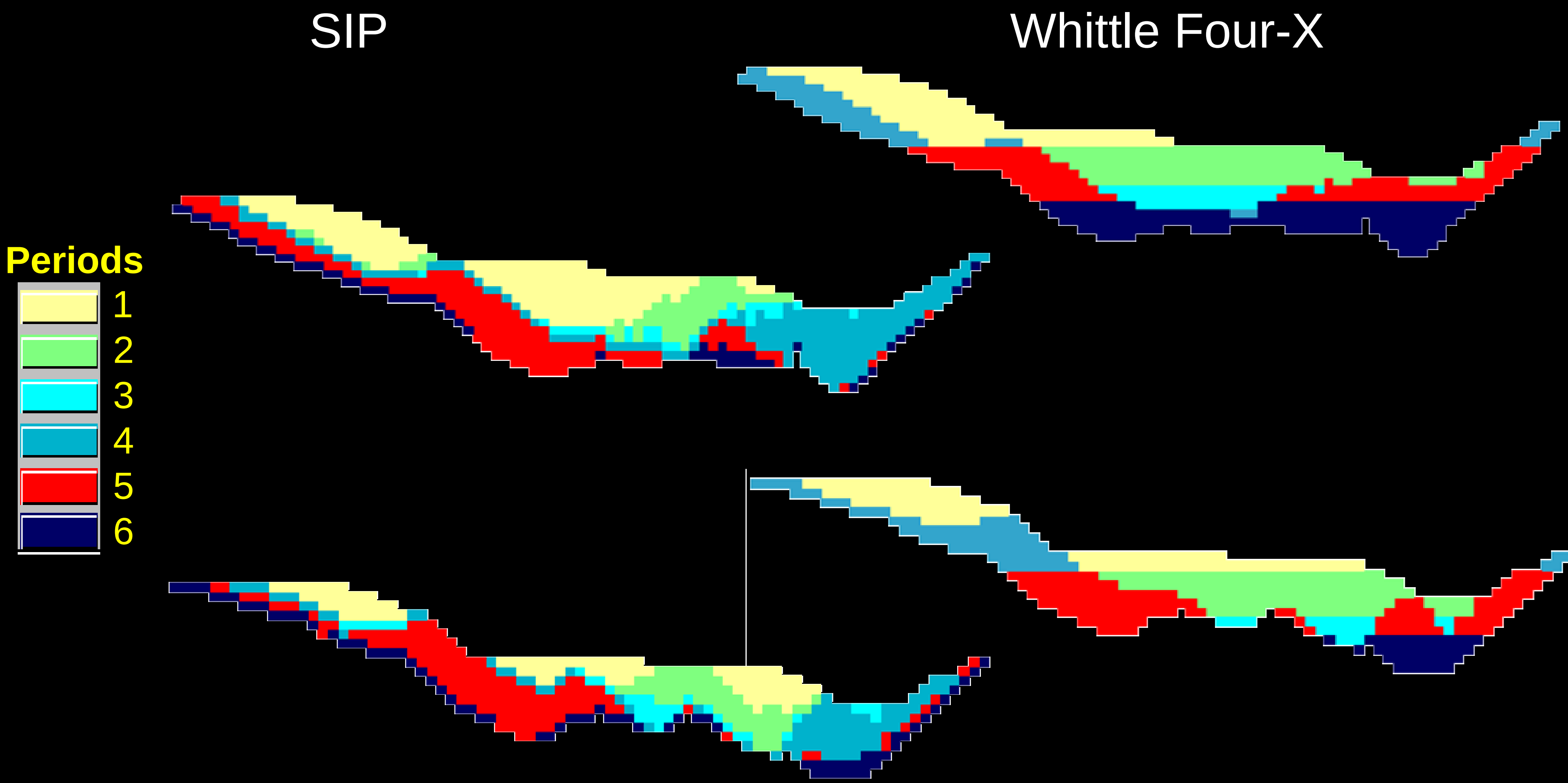
Orebody risk discounting rate	20 %
Cost of shortage in ore production	10,000 /t
Cost of excess ore production	1,000 /t
Cost of shortage in metal production	20 /gr
Cost of excess metal production	20 /gr
Number of simulated orebody models	15

The SIP Model Information

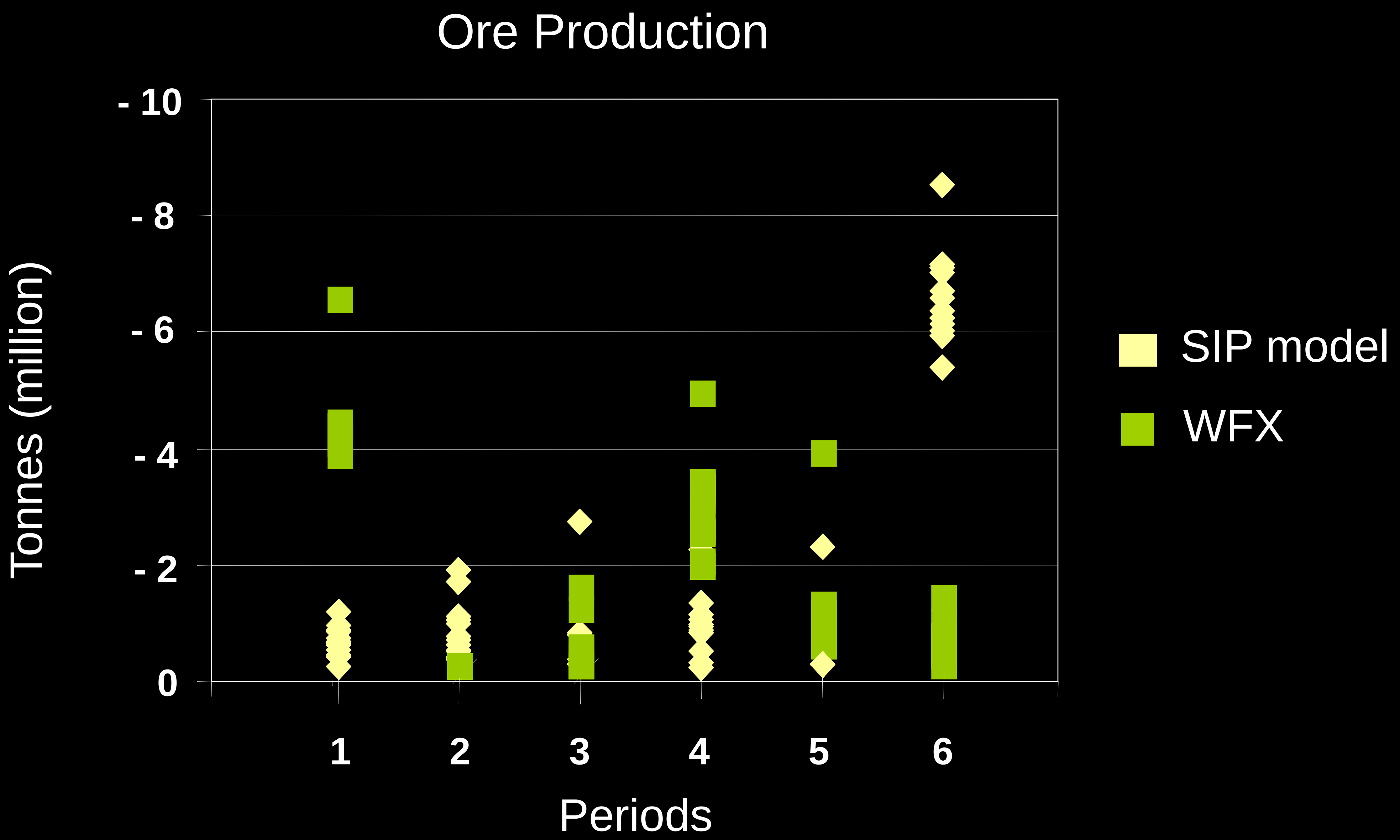
Periods	**1 - 4**	**4 - 6**
Total blocks	**11,301**	**10,995**
Constraints	**33,273**	**21,363**
Total variables	**53,301**	**37,286**
Binary:	**18,540**	**9,580**
T Time (hr:min:sec)	**<04:49:55**	**<37:15:33**

Supercomputing system used with parallel processors $\leq$ 8 in 2002

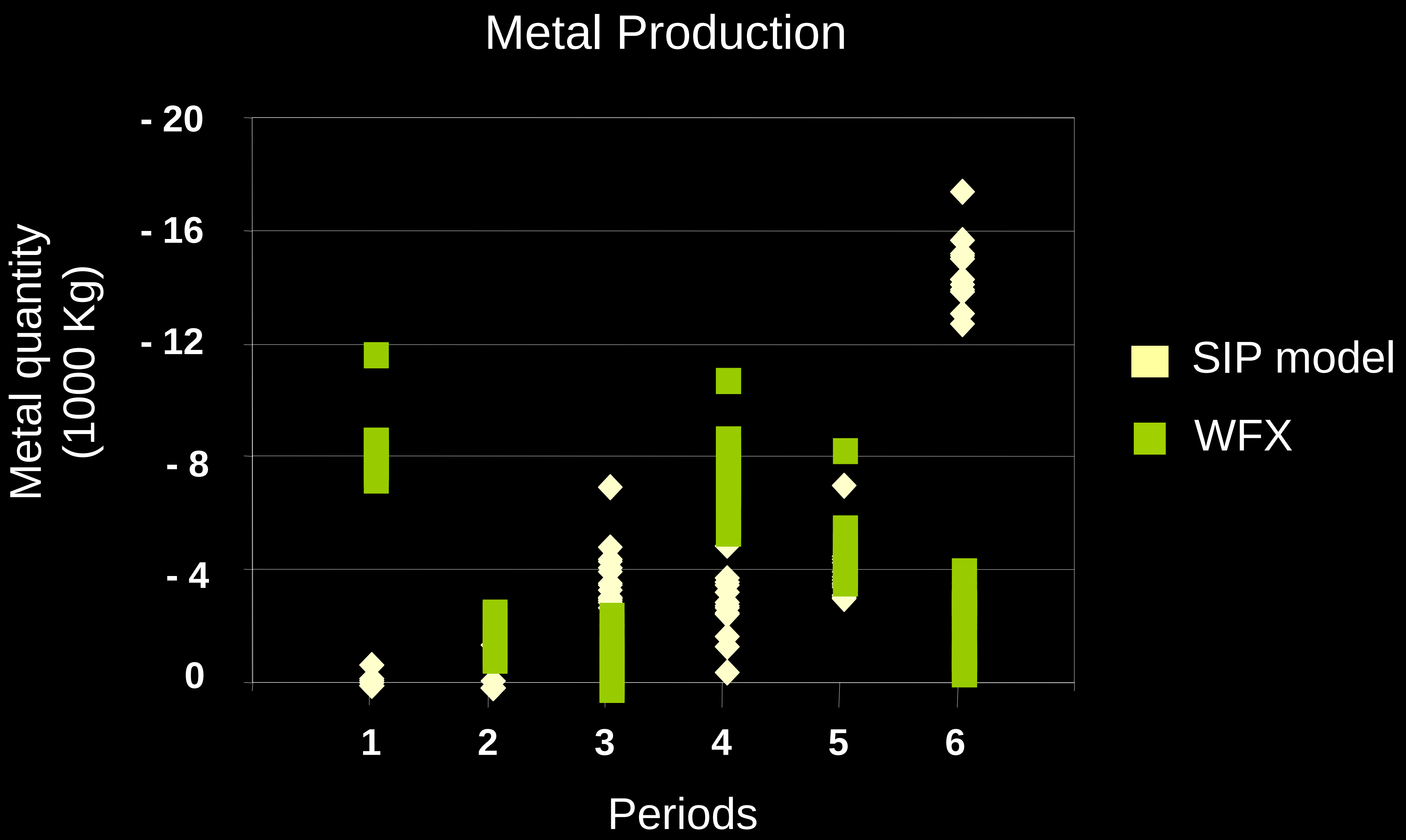
Cross-Sectional Views of the Schedules



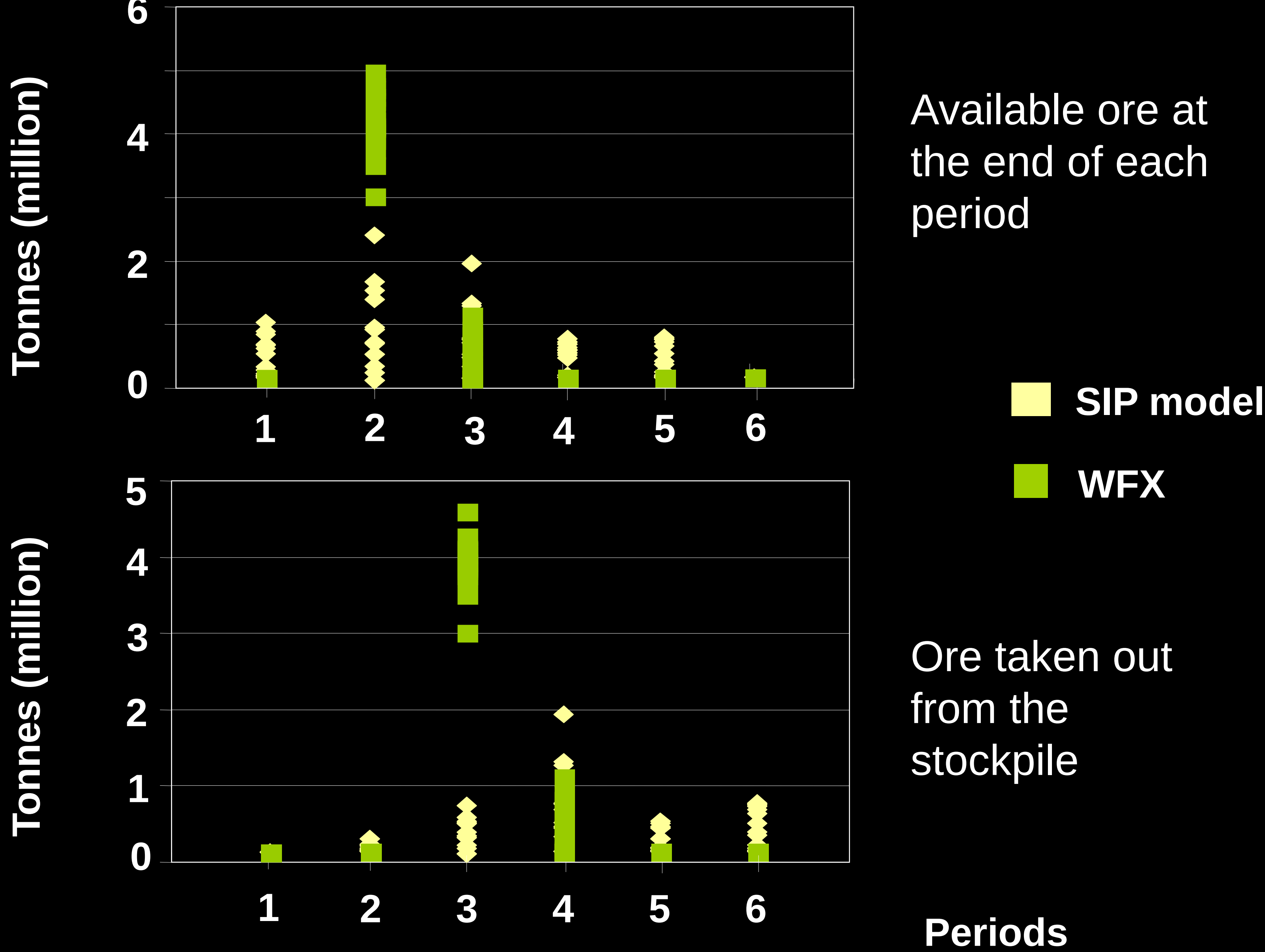
Deviations from Production Targets



Deviations from Production Targets



Stockpile's Profile

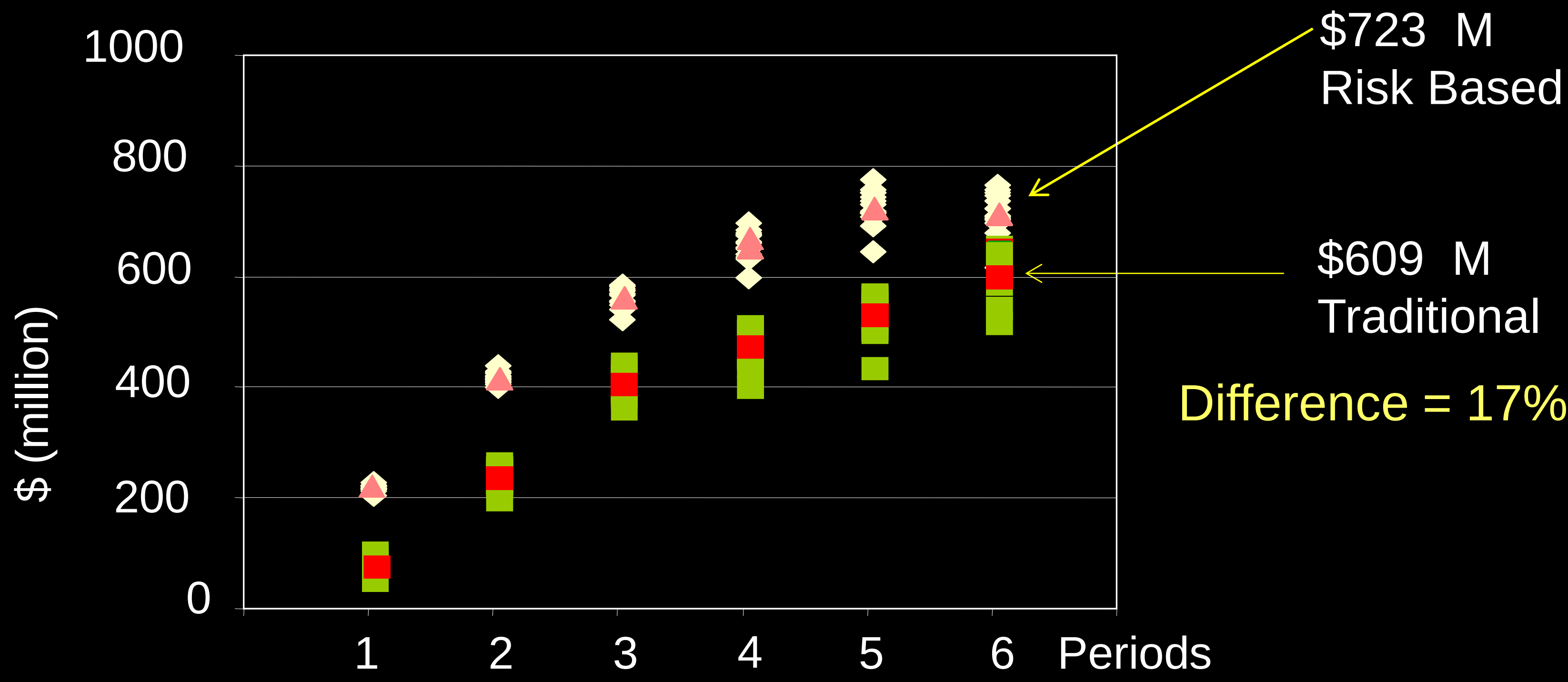


Available ore at the end of each period

Ore taken out from the stockpile

Uncertainty is Good: Traditional vs Risk-Based

Stochastic Integer Programming



Cumulative NPV values

SIP model **WFX**

Average NPV values

SIP model **WFX**

Geological Risk
Discounting= 20%

Value of SIP Solution (VSS)

ESPI = Expected Solution of Perfect Information

15 Scenarios, 15 schedules = average NPV

(a ‘theoretical NPV’ value which one to use?)

EVS = Expected Value Solution

15 Scenarios, Expected value scenario, 1 Schedule tested with 10 Scenarios = NPV

ESS = Expected Stochastic Solution

14 Scenarios, 1 Schedule tested with 14 Scenarios = NPV

Value of SIP Solution (VSS)

EVPI = Expected Value of Perfect Information

$$EVPI = ESPI - EVS$$

VSS = Value of Stochastic Programming or Solution (VSS)

$$VSS = ESS - EVS \geq 0$$

COST of IGNORING Uncertainty

Value of SIP Solution (VSS)

ESS = Expected Stochastic Solution = 723 million $

EVS = Expected Value Solution = 659 million $

VSS = Value of Stochastic Programming or Solution

VSP = ESS – EVS = 64 million $ 10%

COST of IGNORING Uncertainty

Note:

ALL DETAILS ARE IN THE PAPERS

provided for this lecture

Particularly:

Dimitrakopoulos and Ramazan,
Mining Technology, 2008

Some Key Comments

The new SIP production scheduling model:

- Uses individual realisations, thus explicitly accounts for geological risk
- Allows the risk management at three levels:
  1. manage the magnitude of risk within a period
  2. manage the variability of risk
  3. control the risk distribution between time periods
- Maximises NPV for a desired risk profile
- The SIP is efficient: Contains less binary variables than traditional MIP models

A Second Case Study

- Disseminated low-grade copper deposit
- Orebody dips mainly N180/60S
- 185 DH in a pseudo-regular grid of 50x50m^2
- Mineralized envelop defined using the drill core logs

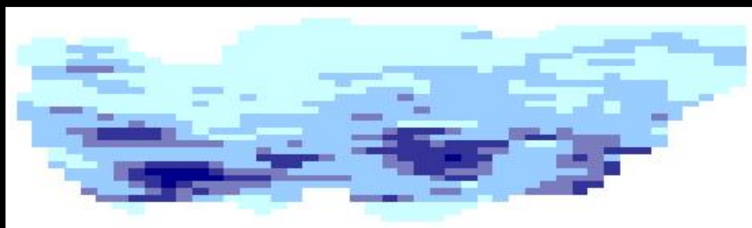
Direct block simulation

- 20 simulations, directly generated on a 20x20x10m^3 mining block size

Stochastic Simulations

Generates equally probable scenarios of the deposit

1



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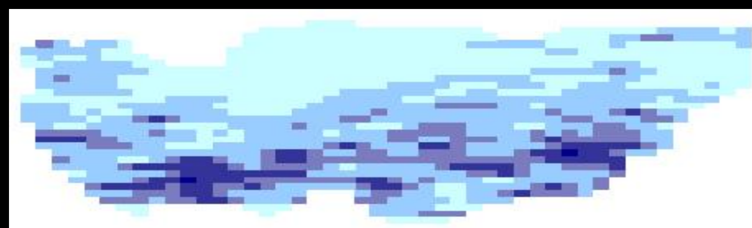

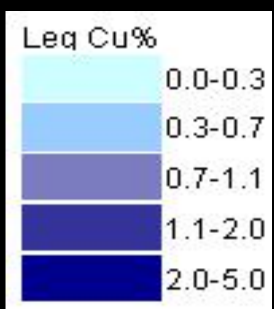

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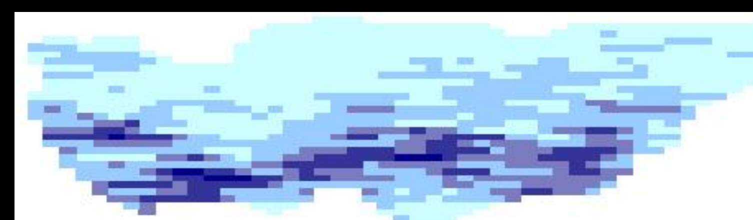
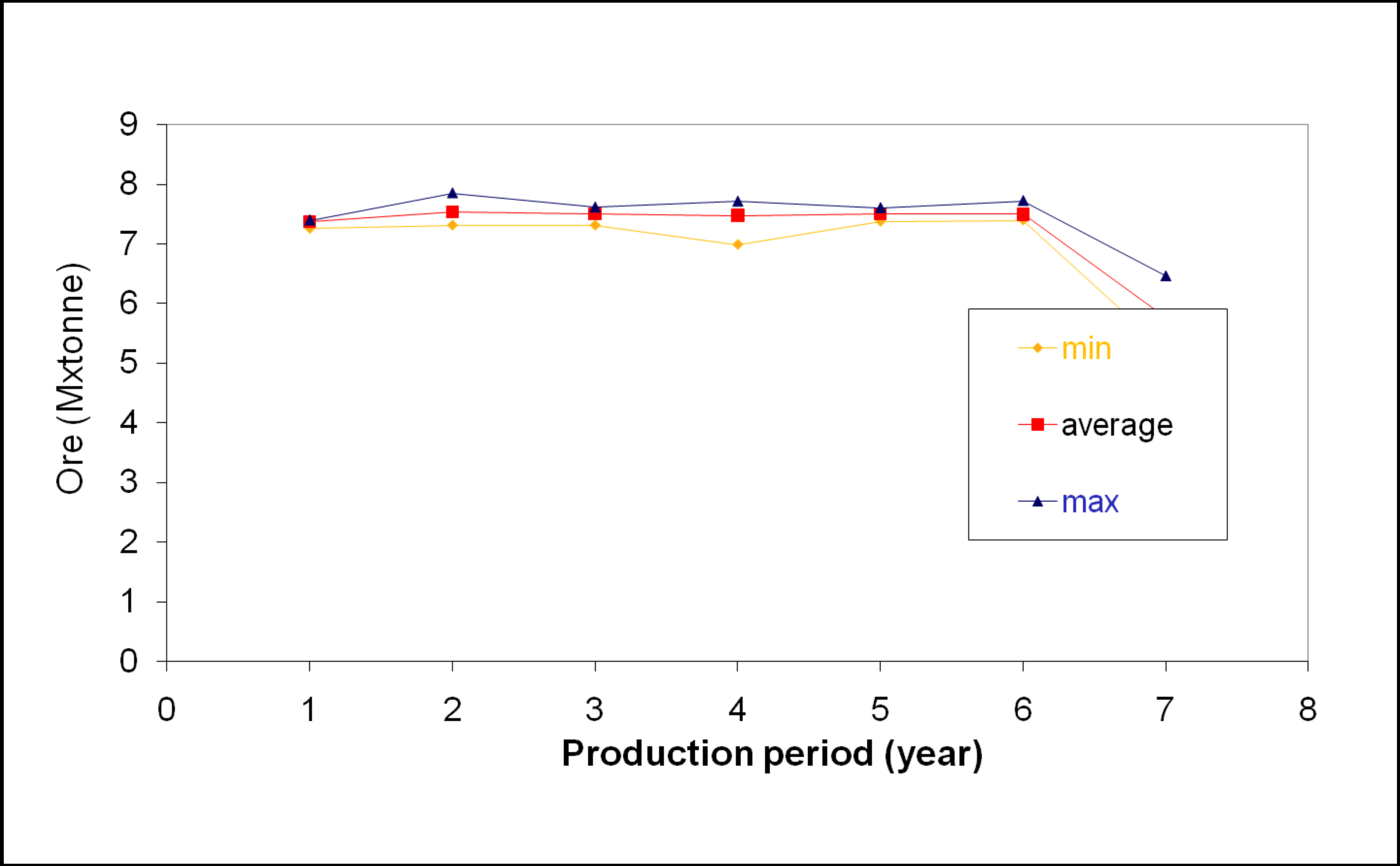


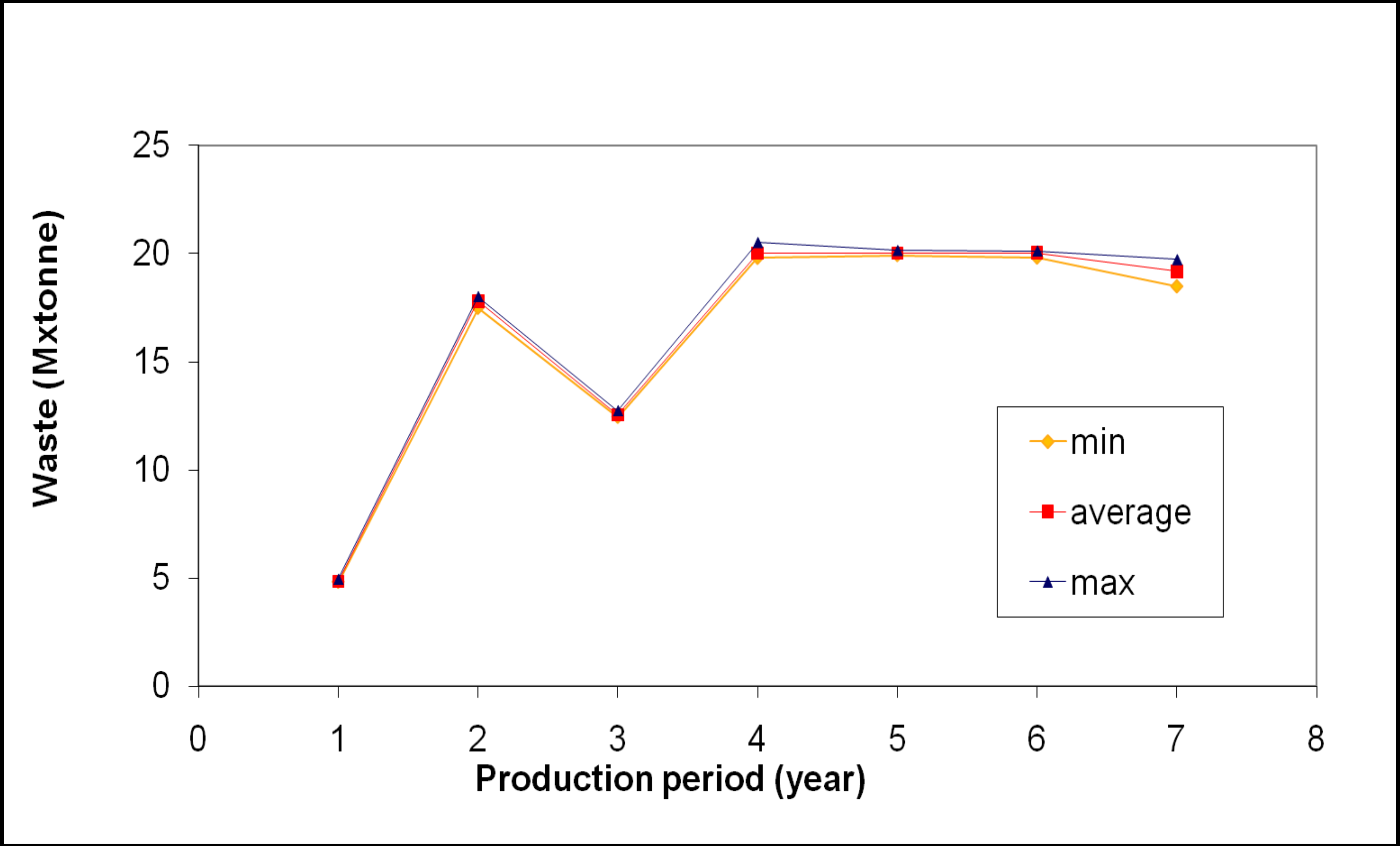
Parameters for the SIP

Total blocks	15,391
Block dimensions (m)	20 x 20 x 10
Processing input capacity (PC)	7.5 Mtpa
Total mining capacity (TC)	28 Mtpa
Economic discount rate	10 %
Cost of shortage in ore production	10,000 /t
Cost of excess ore production	1,000 /t
Cut-off	0.3% Cu
Number of simulated orebody models	20

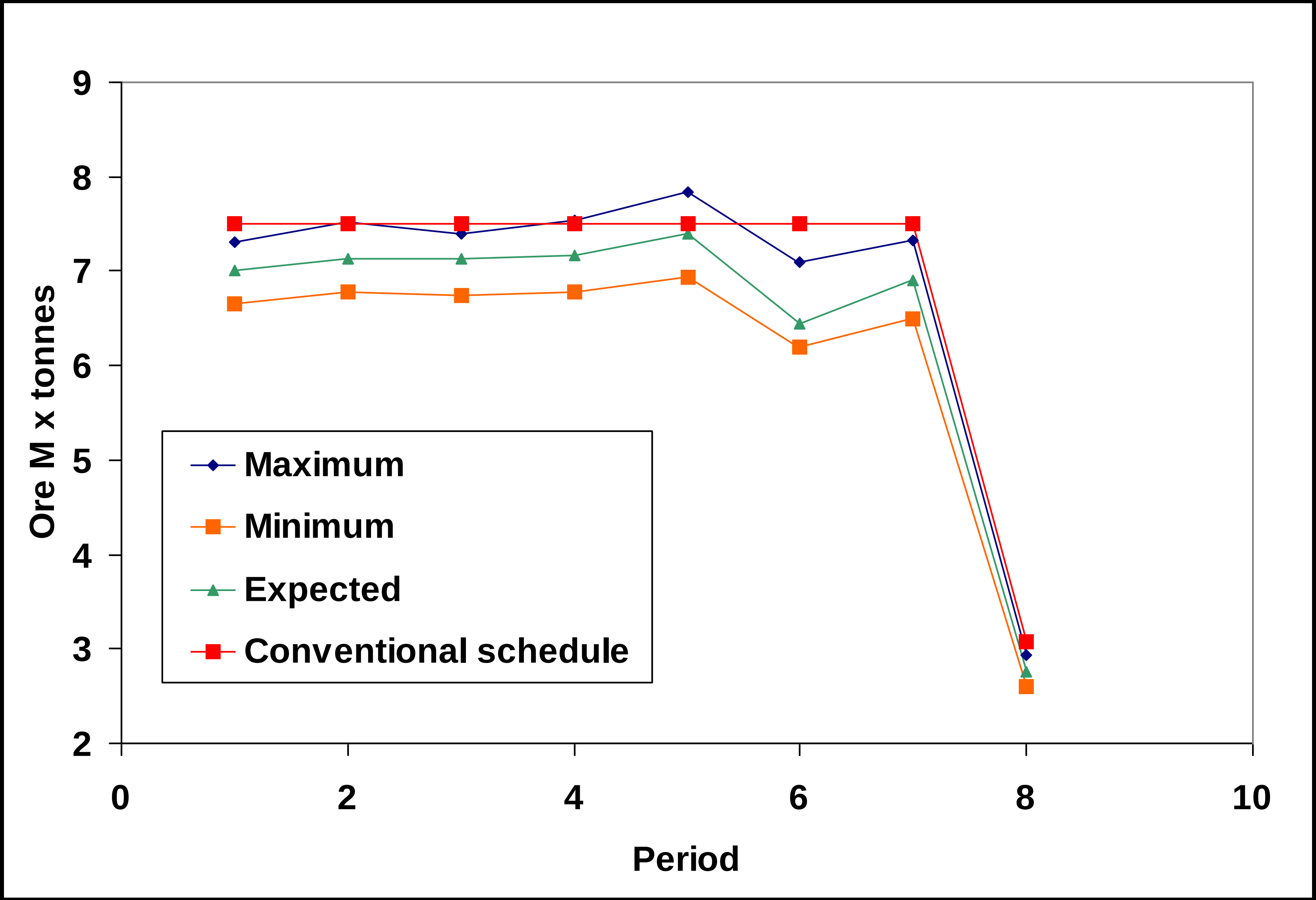
Ore Production Risk Profile



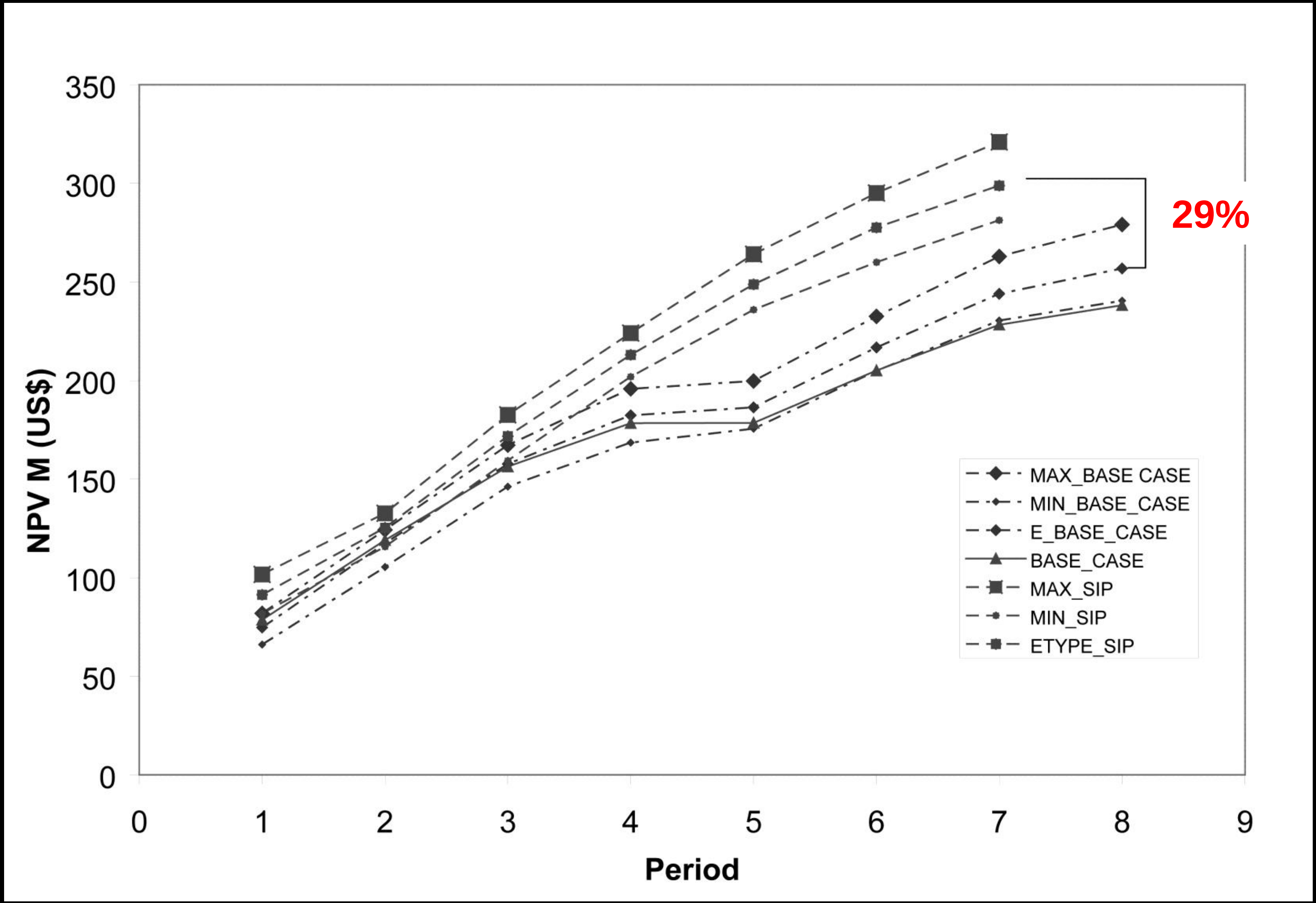
Waste Production Risk Profiles



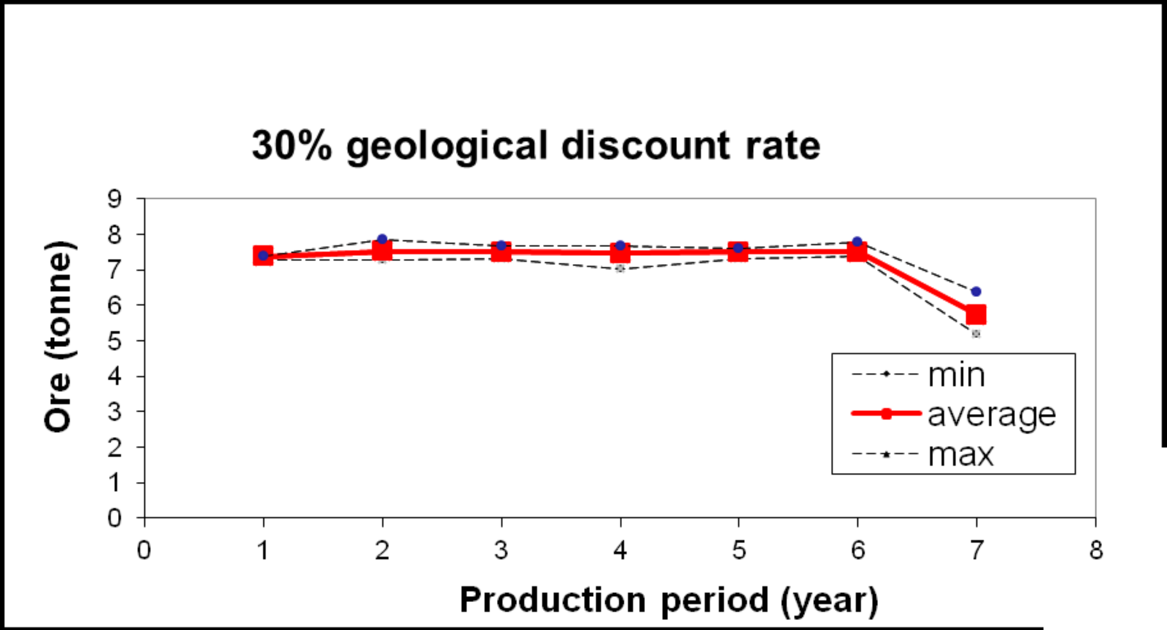
Risk Analysis - Conventional Schedule

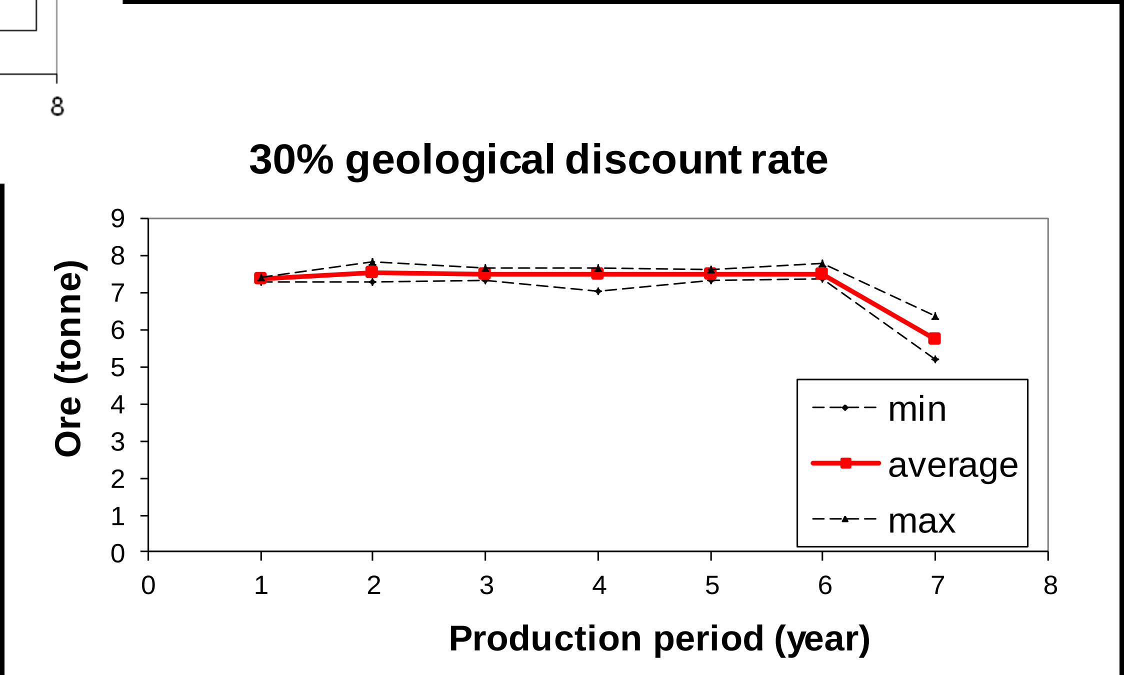


The Value the SIP Solution

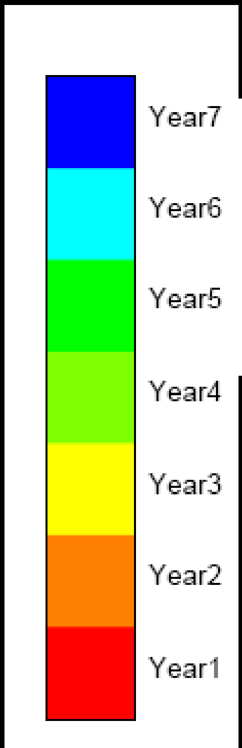


The Role of Geological Discount Rate



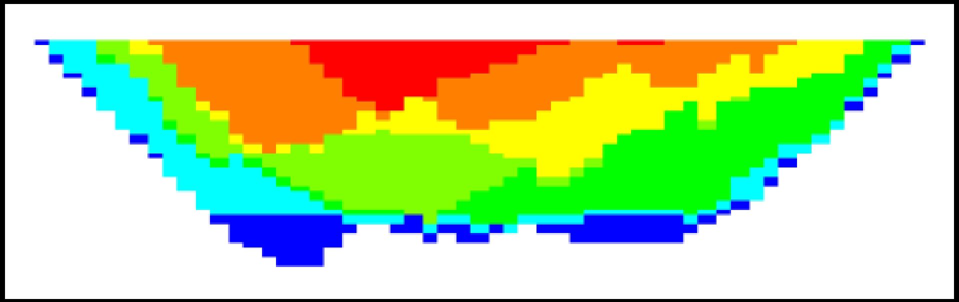


Cross-sectional Views of Schedules



20% geological discount rate

30% geological discount rate



Value of SIP Solution (VSS)

ESS = Expected Stochastic Solution = 298 million $

EVS = Expected Value Solution = 238 million $

VSS = Value of Stochastic Programming or Solution

VSS = ESS – EVS = 60 million $ or 25%

COST of IGNORING Uncertainty

Note:

The End

(of this lecture only)