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Production scheduling with uncertain supply: a new solution to the open pit mining problem

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Abstract The annual production scheduling of open pit mines determines an optimal sequence for annually extracting the mineralized material from the ground. The objective of the optimization process is usually to maximize the total Net Present Value (*NPV*) of the operation. Production scheduling is typically a Mixed Integer Programming (MIP) type problem containing uncertainty in the geologic input data and economic parameters involved. Major uncertainty affecting optimization is uncertainty in the mineralized materials (resource) available in the ground which constitutes an uncertain supply for mine production scheduling.

A new optimization model is developed herein based on two-stage Stochastic Integer Programming (SIP) to integrate uncertain supply to optimization; past optimization methods assume certainty in the supply from the mineral resource. As input, the SIP model utilizes a set of multiple, stochastically simulated scenarios of the mineralized materials in the ground. This set of multiple, equally probable scenarios describes the uncertainty in the mineral resource available in the ground, and allows the proposed model to generate a single optimum production schedule.

The method is applied for optimizing the annual production scheduling at a gold mine in Australia and benchmarked against a traditional scheduling method using the traditional single "average type" assessment of the mineral resource in the ground. In the case study presented herein, the schedule generated using the proposed SIP model resulted in approximately 10% higher *NPV* than the schedule derived from the traditional approach.

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1 Introduction

The problem of production scheduling in open pit mining is determining the parts of a mineralized deposit to be mined annually in an optimum order of sequence so as to maximize the total discounted profit. Solving the annual production scheduling problem in mining optimally is critical, because it determines the annual cash flows that can add several hundred million up to several billion dollars in magnitude, and at the same time it is based on an uncertain supply of mineralized materials for the resource available in the ground. This uncertainty is acknowledged in the related technical literature to be the major reason for not meeting production expectations (Baker and Giacomo 1998) or being the by far most dominant factor in failing mining projects (Vallee 2000). Given its substantial impact on the financial outcome of mining operations, this paper focuses on dealing with the uncertainty in metal content within a mineral deposit being mined.

Mining is an excavation activity in the earth made for the purpose of removal and sale of economically valuable minerals or materials. An orebody model containing the deposit attributes such as grades, tonnage, density, mining cost, processing cost, expected economic value etc. are used to determine the final pit limits. The final pit limits in mining can be defined as the limits of the deposit up to which it is economically feasible to mine. Lerchs and Grossmann's algorithm (LG) based on graph theory (Lerchs and Grossmann 1965) and Maxflow algorithm based on network flow concept (Johnson 1968) are the most commonly used methods in practice. Hochbaum and Chen (2000) provided a critical review and discussion of various methods in finding optimum ultimate pit limits. Gershon (1983) presented a Mixed Integer Linear Programming (MILP) model formulation that an actual mining operation is represented as a mathematical model to maximize overall *NPV* from the operation. The paper stated that the models for optimising production scheduling of open pit mines require too many binary variables and cannot be solved. There have been some publications of methods which aim to reduce the required number of binary variables in MIP formulations for production scheduling in mining. In underground mines, Topal (2003) developed a methodology that substantially reduced the number of required binary variables by defining earliest start and latest start periods of the blocks through an intelligent process for Kiruna iron ore mine located in Sweden. To reduce the variables in optimizing production scheduling in open pit mining, Ramazan (2007) developed the Fundamental Tree Algorithm (FTA) that efficiently aggregates the blocks in an optimal way based on purely linear programming without using any integer variable. There are also heuristic approaches to aggregate the mining blocks that are discussed in Ramazan (1996) and Whittle (1988).

An estimated grade or mineral content of a block within an orebody model, representing a mineral deposit, is generated as a weighted average of the surrounding samples while the actual grade is unknown; uncertainty is caused by the single estimated value of a block that, in addition, cannot represent the possible in-situ grade variations between sampled points. Alternatively, stochastic spatial simulation (Boucher and Dimitrakopoulos 2009; Scheidt and Caers 2009; Mustapha and Dimitrakopoulos 2010) is a geostatistical approach used to generate multiple orebody models that are equi-probable representations of the actual mineral deposit in the ground. However, there is not one production scheduling method that can use multiple orebody models as input to generate an optimum production schedule in the presence of supply uncertainty. Grieco and Dimitrakopoulos (2007) proposed a probabilistic MIP method to optimize annual production scheduling in underground mining. Some past efforts in open pit mining included the sequential use of stochastic orebody models in traditional optimization methods by Ravenscroft (1992) and Dowd (1997). Dimitrakopoulos and Ramazan (2004) developed a long-term probabilistic type production scheduling method and introduced the concept of Geologic Risk Discounting (GRD). However, sequential processes and the probabilistic approach are shown to be inefficient and cannot produce an optimal schedule in the presence of uncertainty. Similar are the limitations of recent experimental approaches (Dimitrakopoulos et al. 2007; Godoy and Dimitrakopoulos 2004). Golamnejad et al. (2006) propose a chance-constrained formulation to account for grade uncertainty; however, chance-constrained formulations make severe and unrealistic assumptions, such as a Gaussian distribution and independence of the metal content (grade values) of mining blocks. Boland et al. (2008) consider a multistage stochastic programming approach, which however also makes unrealistic mining assumptions to be applicable. Dimitrakopoulos (2011) provides a review of stochastic approaches used in mine scheduling. Ramazan and Dimitrakopoulos (2004b) introduced a stochastic programming optimization model in a conference which was an initial start-up of this fully developed model proposed in this paper. The model produced promising results, but it did not include the formulation for leaching, the objective function did not have the stockpiling parts and model constraints were not presented and discussed in detail except the generic grade blending constraint. The major drawback of the publication was the case study, which considered a small scale two-dimensional hypothetical data set, which doesn't guarantee the applicability of the model to a real mining operation with larger data.

In addition to the uncertainty of mineralized materials a mineral deposit can supply over time, there are other uncertainties in open pit mining that also play a significant role in defining the overall net profit expected from operations. These uncertainties include largely the fluctuating market demand for raw materials and metals affecting commodity prices, and production costs such as mining, processing, administration and so on. Technical and operational uncertainties, such as the uncertainties on the geotechnical parameters, introduce additional risk sources. The focus herein is on the uncertainty in metal content and supply as a starting point and due to its substantial impact on mining operations, as discussed above. Integrating additional sources of uncertainty in the present formulation or others remains a longer term research objective.

This paper proposes a new Stochastic Integer Programming model (SIP) that can optimize the annual production scheduling problem for open pit mines, considering uncertainty in the supply of mineralized materials extracted from the deposit. The proposed model is a novel approach in the sense that it can use multiple simulated orebody models without averaging the grade and contains parametric tools to manage the risk distribution effectively. The case study in a gold deposit in Australia shows that the SIP method has potential to improve the total *NPV* of mining projects substantially and it can be applied to large size open pit mines efficiently in terms of solution time of the large SIP model. In the following sections, the calculation of economic values is first discussed. The stochastic integer programming description and model formulation is presented after that. Subsequently, managing risk profile with the proposed method is discussed. Performance of the SIP model is analyzed and compared with a commonly used production scheduling method.

2 Economic values of mining blocks

After an orebody model with geological and economic attributes is generated, the expected economic value of a block i $E\{V_i\}$ is calculated based on the expected Net Revenue (NR_i) to be gained from selling the contained metal within block i. If PC_i is the processing cost (for all processes if multiple processing methods are used), the value of the block is:

$$E\{V_i\} = \begin{cases} NR_i - MC_i - PC_i & \text{if } NR_i > PC_i \quad \text{(block } i \text{ is an “ore block”)} \\ -MC_i & \text{if } NR_i \le PC_i \quad \text{(block } i \text{ is a “waste block”)} \end{cases}$$

$$NR_i = T_i \cdot G_i \cdot Rec \cdot (\text{Price} - \text{Selling Cost})$$

where MC_i is the cost of mining, T_i is the weight of the block; G_i is the grade within block i; Rec is the processing recovery percentage. Note that in mining, a block is classified as “waste block” if processing cost is greater than the expected revenue from selling the contained metal within the block. It is classified as “ore block” if it is profitable to process and sell the contained metal.

3 Stochastic integer programming for optimizing annual production scheduling in open pit mines

The proposed model offers a solution for the problem of which blocks should be mined in what time periods (years) without violating operational constraints (mining slope requirement, mining capacity, processing capacity, product qualities) so that the overall discounted profit from the sales of the products (metals) is maximized.

The following descriptions are adopted from Dimitrakopoulos and Ramazan (2009) where the generic two-stage recourse stochastic programming concept is explained specifically in mining terms. The two stage recourse model is a type of mathematical programming model where the fundamental mechanisms of anticipative and adaptive models are integrated within a single model. This model represents a trade-off between long term anticipatory strategies and the associated adaptive strategies. In mining terms, the trade-off could be between total expected *NPV* and associated risks in meeting production targets. Conceptually, a mathematical model is expected to result in higher *NPV* values if higher geological risks are tolerated within the model;

a conservative risk approach is expected to result in a lower *NPV* value. The recourse problem can be expressed as follows:

Find $x \in R_+^n$, such that

$$F_i(x) \le 0, \quad i = 1, \ldots, m, \quad \text{and} \tag{a1}$$

$$F_0(x) = cx - E\{Q(x, g)\} \quad \text{is maximized} \tag{a2}$$

where

$$Q(x, g) = \inf\{q(g)y | W(x, g) = h(x, g) - T\}; \tag{a3}$$

$y \in R_+^{n'}$; x is the matrix of decision variables (x_i^t) for deciding when to mine a block (if $x_i^t = 1$, mine block i in period t). If all x variables are set to 0 or 1 values that represent percentage of blocks to be mined at each period t and if this set of x values are feasible for the model constraints under (a1), the values define a production schedule. Equation (a1) is a representation of all the constraints required for mining operations ((8), (9) and (10) in the subsequent section).

In (a2), cx is the total *NPV* value to be generated from the project given a decision on when the blocks should be mined, or given a production schedule defined by x-values. $E\{Q(x, g)\}$ is the expected risk, or associated costs, of not meeting production targets under the chosen schedule. The risk in the model is defined as the deviations from the desired production targets and the unit cost multiplier matrix y as a function of the infeasibility. For a given schedule, x, and a set of grades, g, $h(x, g)$ represents the tonne, grade and quality values to be produced periodically and T is the target matrix. Therefore, $W(x, g)$ defines the risk and $q(g)y$ defines the cost of the risk for the schedule, as a function of the uncertain grade values. After the true environment is observed through a simulated orebody model, the discrepancies that may exist between $h(x, g)$ and T (for fixed x and observed $h(x, g)$ and T) are calculated using (a3) as:

$$W(x, g) = h(x, g) - T$$

The cost of risk is defined in the objective function. The model takes the corrective or recursive form to re-define the x variables and the schedule so that *NPV* and cx are maximized, while the loss, $q(g)y$, is minimized. Therefore, an optimal decision x should modify the total expected profit to be generated by carrying out the plan, i.e. the direct *NPV* (cx) as well as the costs generated by the risk defined using simulated orebody models on the schedule, ($E\{Q(x, w)\}$).

The SIP model developed herein accounts for uncertain inputs for mineral supply in a deposit. This is achieved by simultaneously considering multiple, stochastically-simulated realizations of a deposit in the optimization process, which minimizes the risk of not meeting production targets caused by uncertain supply and, in addition, allows for the management of risk by controlling the risk profile of pertinent indicators. The model considers operational requirements, a main processor plant and also leaching as a possible alternative process. The stockpiling is also considered within the stochastic concept since the amount of material to be stockpiled will be affected by the block grades in the orebody model.

3.1 Definition of symbols and terms

Subscripts and superscripts

t is a scheduling time period;
g, o and q are target parameters, or type of production target; g is for the grade targets; o is for the ore production target; q is for the metal production target;
s is a simulated orebody model;
l is for the minimum target (lower bound);
u is for the maximum target (upper bound);
i is the block identifier.

Variables to be determined

$a_{sl}^{tg}, a_{sl}^{to}, a_{sl}^{tq}, a_{su}^{tg}, a_{su}^{to}, a_{su}^{tq}$ are the dummy variables (a-parameters) used to balance the equality constraints;
b_i^t is a variable representing the fraction of block i mined in period t; if a b_i^t variable is defined as binary (0, or 1), it is 1 if block i is mined in period t and 0 otherwise;
d_{sl}^{tg}, d_{sl}^{to} and d_{sl}^{tq} are the deficient amounts (d-parameters) for the target parameters produced below a desired minimum limit;
d_{su}^{tg}, d_{su}^{to} and d_{su}^{tq} are the excess amounts (d-parameters) for the target parameters produced above a desired maximum limit;
h_s^t is the amount of material left in the stockpile at the end of period t with respect to orebody model s;
k_s^t is the amount of material (ore) in tonnes processed from the stockpile with respect to orebody model s in period t;
w_i^t is a variable representing the fraction of block i sent to the stockpile in period t.

Known constants

$C_l^{tg}, C_l^{to}, C_l^{tq}, C_u^{tg}, C_u^{to}$ and C_u^{tq} are unit costs (C-parameters) for $d_{sl}^{tg}, d_{sl}^{to}, d_{sl}^{tq}, d_{su}^{tg}, d_{su}^{to}$ and d_{su}^{tq}, respectively, in the objective function;
$C_l^{t-} = C_l^{0-}/(1 + f_l)^t$; and $C_u^{t-} = C_u^{0-}/(1 + f_u)^t$, where "$-$" after superscript t represents the type of targets g, o or q; f_l and f_u are the orebody risk discounting rates (uncertain mineral supply) used to calculate C-parameters;
$E\{(NPV)_i^t\}$ is the expected (average) net present value (NPV) to be generated if block i is mined in period t. If the discount rate is r and $E\{(EV)_i^0\}$ is the undiscounted expected economic value, it can be expressed as:

$$E\{(NPV)_i^t\} = E\{(EV)_i^0\}/(1 + r)^t.$$

G_{tar} is the targeted grade of the ore material to be processed;
G_{si} is the grade of block i in orebody model s;

GST is the average grade of the stockpiled material;
$GST_{\min}, GST_{\max}$ are minimum and maximum grades that a block is considered for stockpiling, respectively; the probability of a block to have a grade between $GST_{\min}$ and $GST_{\max}$ is the probability of that block to be considered for stockpiling;
h^0 is the amount of material already available in the stockpile;
M is the number of simulated orebody models;
MC_i^t is mining cost of block i occurred in period t and discounted to time 0;
$MCAP_{\max}$ is the total available mining capacity of the equipment;
$MCAP_{\min}$ is the minimum amount of the material (waste and ore) required to be mined in each of the periods;
N is the number of blocks within final pit limits;
O_{tar} is the targeted amounts of the ore material to be processed periodically;
O_{si} is the ore tonnage within block i in orebody model s;
P is the total number of production periods, or mine life;
Q_{tar} is the targeted amounts of the metal to be processed in a period;
Q_{si} is the quantity of the metal in block i with respect to orebody model s;
$(QST)^t$ is the percentage of the metal content at the stockpile in period;
SV^t is profit to be generated by processing a tonne of material from the stockpile in period t and discounted to time 0;
U is the number of blocks considered for stockpiling;
Y_i number of blocks overlying ore block i considered for setting the slope constraints.

3.2 Objective function

The objective function of the SIP model is constructed as the maximization of a profit function, defined as the difference between the total expected net present value and the cost of deviations from planned production targets. The objective function formulation is:

$$\underbrace{\text{Max} \sum_{t=1}^{P} \Bigg[\sum_{i=1}^{N} E\{(NPV)_i^t\} b_i^t}_{\text{Part 1}} - \underbrace{\sum_{i=1}^{U} E\{(NPV)_j^t + MC_j^t\} w_j^t}_{\text{Part 2}} + \underbrace{\sum_{s=1}^{M} (SV^t/M) k_s^t}_{\text{Part 3}}$$
$$- \underbrace{\sum_{s=1}^{M} (C_u^{to} d_{su}^{to} + C_l^{to} d_{sl}^{to} + C_u^{tg} d_{su}^{tg} + C_l^{tg} d_{sl}^{tg} + C_u^{tq} d_{su}^{tq} + C_l^{tq} d_{sl}^{tq}) \Bigg]}_{\text{Part 4}} \qquad (1)$$

The objective function in (1) has four parts: Part 1 refers to the expected total NPV to be generated if all the mined ore blocks were processed. Part 2 adjusts the economics of Part 1 for the percentage of the blocks that are sent to stockpile in a way that only mining costs are incurred in that time period. Part 3 adds the discounted profit to be generated by processing k_s^t amount of material from the stockpile. Part 4 consists of geological risk management parameters.

It is fairly straightforward to calculate the economic values for a set of market parameters and set up the model in Part 1 and Part 2. In Part 3, the amount of material processed from the stockpile depends on the simulated orebody models. At first, the model makes the decision of when to mine a block without full information being known; i.e. the information about which simulated representation (model) of the orebody is the actual deposit. However, it assumes that the decision of how much material should be taken from the stockpile for processing can be made later in time when the mining occurs. At this time of mining complete information about the blocks is known: whether there is enough ore from the mine supplied to the processing plant or not and how much more material is needed for the plant. The unit value to be gained from processing material from stockpile is divided by M to account for the probability of a simulated orebody model to represent the actual deposit.

In cases where multiple independent processes such as concentrator and leaching are used, the proposed model should be implemented according to the situation of the operation. For example, if the leaching process is relatively small portion of the material processed, Part 4 of the equations should not be implemented for that process to minimize the dilution of the *NPV* with the parameters that their deviations are not expected to have significant impact on the overall *NPV*. For these secondary processes, we suggest implementing the grade and capacity constraints as a hard constraint as in (7a) and (7b) and using average orebody model. If each of the processes have significant impact on the revenue stream, then the full objective function and the related constraints (2), (3) and (4) should be implemented for each processing types.

The objective function of the proposed model contains multiple parameters with various coefficients. Gershon (1984) has noted that one must give extra care when assigning weights to multiple parameters in quantifying relative importance of them. Therefore, in the proposed SIP model, Part 4 is used to manage the uncertainty in the supply of ore from the deposit in the optimization process. The coefficients in front of these parameters define a risk profile for the production and *NPV* produced is the optimum for the defined risk profile. It is considered that if the expected deviations from the planned amount of ore tonnage having planned grade and quality in a schedule are high in actual mining operations, it is unlikely to achieve the resultant *NPV* of the planned schedule. Therefore, the SIP model contains the minimization of the deviations together with the *NPV* maximization to generate practical and feasible schedules and achievable cash flows. Section 4 provides further discussions on how these parameters can be used to control the distribution of risks over the time periods.

3.3 Constraints

The deviation parameters in the objective function are calculated within the SIP model by the grade blending and processing plant constraints that consider each of the simulated orebody models.

Grade blending constraints for each time period t

$$\sum_{i=1}^{N}(g_{si}-G_{tar})O_{si}b_i^t-\sum_{j=1}^{U}(g_{sj}-G_{tar})O_{sj}w_j^t+(GST-G_{tarn})k_s^t+d_{sl}^{tg}-d_{su}^{tg}=0$$

$$s=1,2,\ldots,M;\; t=1,2,\ldots,P \qquad (2)$$

Processing constraints

$$\sum_{i=1}^{N} O_{si} b_i^t - \sum_{j=1}^{U} O_{sj} w_j^t + k_s^t + d_{sl}^{to} - d_{su}^{to} = O_{tar}$$
$$s = 1, 2, \ldots, M; \; t = 1, 2, \ldots, \tag{3}$$

Metal production constraints for each time period t

$$\sum_{i=1}^{N} Q_{si} b_i^t - \sum_{j=1}^{U} Q_{sj} w_i^t + (QST) k_s^t + d_{sl}^{tq} - d_{su}^{tq} = Q_{tar}$$
$$s = 1, 2, \ldots, M; \; t = 1, 2, \ldots, P \tag{4}$$

Stochastic stockpile constraints Determine the quantity of material at the stockpile at the end of first period:

$$\sum_{j=1}^{U} O_{sj} w_j^1 + h^0 - k_s^1 - h_s^1 = 0 \quad s = 1, 2, \ldots, M \tag{5a}$$

Determine the quantity of material at the stockpile at the end of period t $(t > 1)$:

$$\sum_{j=1}^{U} O_{sj} w_j^t + h_s^{t-1} - k_s^t - h_s^t = 0 \quad s = 1, 2, \ldots, M; \; t = 2, 3, \ldots, P \tag{5b}$$

Capacity of the stockpile may be limited for each time period t:

$$h_s^t \le SC \quad s = 1, 2, \ldots, M; \; t = 1, 2, \ldots, P \tag{5c}$$

The amount of material that can be taken from the stockpile cannot be more than the available amount at the stockpile at the end of previous period:

$$k_s^t - h_s^{t-1} \le 0 \quad s = 1, 2, \ldots, M; \; t = 1, 2, \ldots, P \tag{5d}$$

Linkage constraints A block must be mined before stockpiling.

$$w_i^t - b_i^t \le 0 \quad i = 1, 2, \ldots, N; \; t = 1, 2, \ldots, P \tag{6}$$

To increase the efficiency of the SIP model, the optimization is applied in two steps for large size problems. Initially, the model is formulated without including the main stochastic constraints (2), (3) and (4) that are feasible for any value of the binary (b_i^t) variables. Instead of these main stochastic constraints, the following mill input capacity and metal production constraints are first applied:

Processing capacity step 1 constraints

$$\sum_{i=1}^{N} O_{si} b_i^t - \sum_{j=1}^{U} O_{sj} w_j^t + k_s^t \ge O_{\max} F_o \quad s = 1, 2, \ldots, M; \; t = 1, 2, \ldots, P \tag{7a}$$

Metal production step 1 constraints

$$\sum_{i=1}^{N} Q_{si} b_i^t - \sum_{j=1}^{U} Q_{sj} w_i^t + (QST)^t k_s^t \geq Q_{\max} F_m$$

$$s = 1, 2, \ldots, M; \; t = 1, 2, \ldots, P \tag{7b}$$

where, F_o and F_m are fractions for ore and metal production to limit the initial results; min and max are some values around the targeted value. Presumably (7a) and (7b) hold for all s and t. For example, an initial schedule can be generated in such a way that F_o, assumed as 70% of the processing capacity, must be reached with respect to each of the simulated orebody models. Initially, a high probability such as 90% is applied. This means the capacity constraints must hold for at least 90% of the simulated orebody models. If the result is infeasible, the probability can be decreased 5 or 10% at a time. The SIP model with these hard constraints is solved faster than the SIP model with the stochastic constraints. Then, the model is formulated again without (7a), (7b), but including (2), (3) and (4). The solution generated from the initial model is used as a starting solution for the SIP model. This procedure often significantly reduces the solution time for the large SIP model.

3.3.1 Operational constraints

Slope constraints Slope constraints ensures that to mine a specific block, all the blocks overlying that specific block must be mined.

For each overlying block of the block i and each period t

$$b_i^t - \sum_{k=1}^{t} b_l^k \leq 0 \quad i = 1, 2, \ldots, N; \; t = 1, 2, \ldots, P \tag{8}$$

Reserve constraints A block has to be mined fully (100%)

$$\sum_{t=1}^{P} b_i^t = 1 \quad i = 1, 2, \ldots, N \tag{9}$$

Mining capacity constraints

$$MCap_{\min} \leq \sum_{i=1}^{N} T_i b_i^t \leq MCap_{\max} \quad t = 1, 2, \ldots, P \tag{10}$$

3.4 Binary definition

In mining, a block is usually mined completely within a period of a year during the scheduling process. To mine the entire block, the block variables are traditionally defined as binary; 1 if the block is mined in a period and 0 if not mined. However,

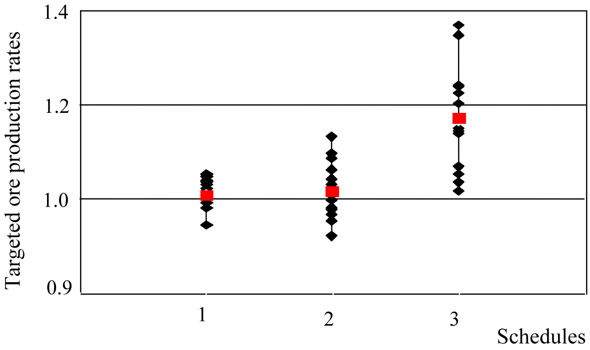

Fig. 1 Variability in the ore production within a year for 3 schedules targeting 100% production rate generated by the initially developed SIP model; *square shape* in the *middle* represents the average deviation; *diamond shape* represents the production rate for a schedule according to simulated orebody models

this logic is difficult to implement in real mining cases due to large size of block models that MIP formulations cannot solve. Therefore, only the variables representing ore blocks are defined as binary and the other variables are continuous (linear). Since waste blocks are linked to ore block mining as discussed in Ramazan and Dimitrakopoulos (2004a), waste block variables would be forced to produce binary results.

4 Managing risk profiles using the SIP model

The risk management process in the presence of uncertain mineral supply when using the proposed SIP model is demonstrated in this section using a relatively small pushback, containing 1064 blocks, from a gold deposit that can be mined within 3 years. To protect data confidentiality and without loss of generality, the production rates are given as percentages and an explanation of using the SIP model to control the risk profile is given. The probability and distribution of the geological risk can be managed between scheduling periods. More specifically, Fig. 1 shows deviations from metal production between scheduling periods for a given schedule. The aim was to mine the least risky part of the deposit early in the mine life and delay the mining of high uncertainty regions for later periods. This is done assuming that more information would become available in time and future planning would be more confident. It is very important in mining operations to meet the production targets at early years of the mine life so that the capital spent can be recovered earlier. For this reason, the geological risk discounting (GRD) is used in the SIP model. The first year production is very likely to meet the production targets. During the second year, the chance of shortage in metal production is higher than the first year, because the cost of deviating from the target in the second year is less than the cost for the first year in the objective function. The deviations are very high in the last year because of depletion of the deposit. However, if the deposit contained sufficient metal, the chance of higher deviations would still be expected as compared to the previous periods, because the cost of deviations in the last year was less than the previous years. Theoretically, the rate of variations in the magnitude of the deviations and chances of the deviations would increase as the GRD factor applied increases and would decrease as the GRD rate decreased. If a balanced-risk profile is to be produced, GRD must be set to 0.

It should be noted that there is no control of the distribution of risk profile in traditional production scheduling methods since they can only use a single orebody model

Table 1 General information on the deposit and scheduling parameters

Description	Values
Total blocks	22,296
Block dimensions (m)	$20 \times 20 \times 20$
Metal prod. capacity (1000 Kg)	30
Processing capacity (million tonnes)	15
Total mining capacity (million tonnes)	80
Stockpile capacity (million tonnes)	10
Stockpile re-handling cost ($/t)	0.3
Discount rate (%)	10
Mine life (years)	6
Number of simulated orebody models	15

as input. The risk profile on a schedule by traditional methods is random. However, one needs to be aware of the fact that the SIP model cannot control the risk distribution 100%, although it can provide substantial control as illustrated in Fig. 1. The risk distribution is also dependent on how the grade variability is spread throughout the mineralized region. Some blocks may have to be mined due to slope constraints or other operational reasons at earlier periods even if they contain high uncertainty in the estimated grades. This type of grade uncertainty can only be reduced through drilling and sampling process in the high risk areas that need to be mined during earlier periods.

In this paper, it is suggested that a series of schedules be produced with different C-parameters, which may include actual approximated cost numbers where possible based on some reasonable assumptions, and GRD factors. The resultant NPV values and risk profiles need to be analyzed using simulated orebody models and decisions must be based on the analysis considering the company's objectives.

5 Performance analysis of the sip formulation against a traditional schedule in a gold deposit

Table 1 shows the general information about the block model used for performance analysis and some of the scheduling parameters. There are 22,296 blocks within the part of the deposit optimized using the SIP model. This size of deposit can be considered as very large in terms of application of Operations Research techniques. In the schedules, total mining capacity of the shovel–truck fleet was 80 million tonnes (mt), processing plant capacity was 15 mt of ore per annum and a maximum of 5 mt stockpile capacity was available. The gold price was assumed to be Australian $560/oz. This part of the deposit would take about 6 years to mine considering the capacity constraints for the operation. The schedules were produced using 15 simulated orebody models and an e-type ("average" or "expected value" type) orebody model that was generated by averaging the grades in the simulated models.

To benchmark the proposed SIP model, two schedules were produced; one using the proposed SIP model and another one, referred to as traditional schedule (TS) in

Table 2 SIP specific information

Description	Values
Orebody risk discounting rate (GRD)	20%
Cost of shortage in ore production	1,000/tonne
Cost of excess ore production	1,000/tonne
Cost of shortage in metal production	20/tonne
Cost of excess in metal production	20/tonne

Table 3 SIP scheduling information

Description	Stage 1	Stage 2
Periods	1–4	4–6
Total blocks	11,301	10,995
Constraints	33,373	21,363
Total variables	53,301	37,286
Binary variables	18,540	9,580
Time (h:m:s)	<04:49:55	<37:15:33
Gap	0.0%	0.0%

this paper, using the e-type orebody model as input to the Whittle software package (Whittle, 1998), one of the most commonly used software packages in the mining industry. Table 1 contains the common scheduling parameters and constraints for both the SIP model and the traditional model. The SIP specific parameters for this model are shown in Table 2. The SIP schedule was produced using 20% GRD to control the risk distribution between periods. The cost of shortage and excess ore production were assigned as 1,000 per tonne. Cost of shortage or excess metal production was assigned as 20 per tonne. These numbers were used within the model to establish a priority between parameters so that *NPV* could be maximized with a risk profile, which aimed to minimize the risk during earlier years of mine production.

The deposit was scheduled by SIP model in two stages as shown in Table 3. Stage 1 contained 11,301 blocks and was scheduled over 4 periods. There were more than 33,300 constraints and 18,540 binary variables in the first stage SIP model formulation. It took about 4 hours 50 minutes to solve the first stage problem formulation. Stage 2 contained about 11,000 blocks and was scheduled over 3 periods. Note that period 4, which is the last period of the previous stage is included in the second stage to fill up the remaining capacities. The blocks used in the first stage was not enough to fill it up fully. The SIP model formulation contained over 21,300 constraints and 9,580 binary variables. Using a dual core 2 GHz machine with 1 GB RAM on a 32 bit operating system, it took about 37 hours to reach the solution for Stage 2 using CPLEX software's solver engine (ILOG ltd).

In the second stage, we removed the capacity constraints (mining and processing capacity) from the last period. The reason for the change is that (9) requires all the blocks to be mined and we know that mining all blocks over the total scheduling

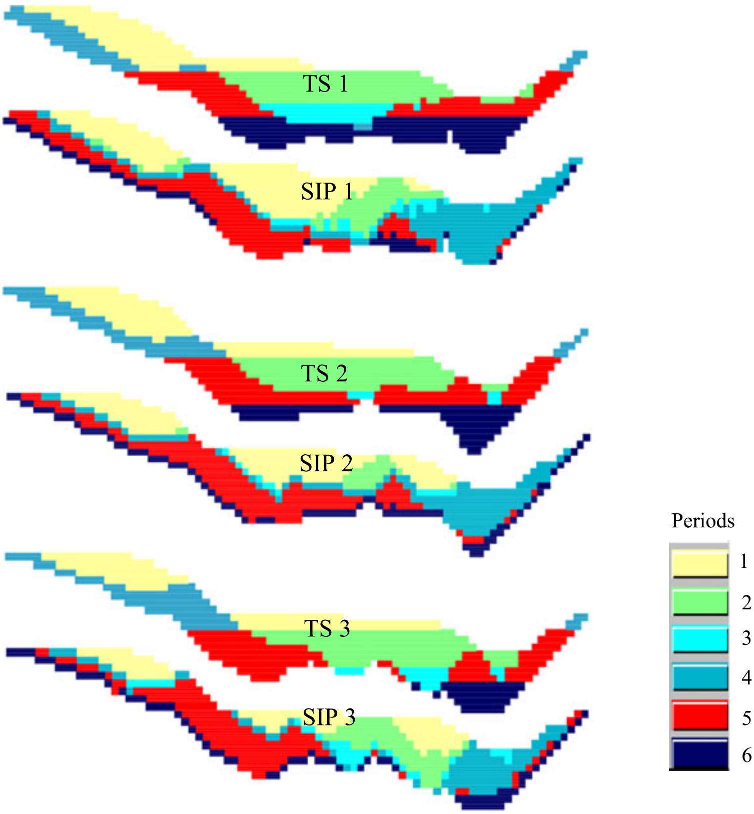

Fig. 2 Three cross-sectional views of the traditional schedule (TS) and the schedule obtained by the SIP model. *Colored regions* are the soil scheduled for mining; *uncolored parts* on *top* are the areas previously mined (air)

periods will not violate these constraints; this is because profitable blocks will be mined out in early periods with the processing plant running at full capacity. Although the material mined and processed at the last period was uneconomic according to the simulated models and SIP schedule, the traditional scheduler was not able to identify that due to the single orebody model input data, which mixes high grade ore with low grade material (smoothing effect of the estimation method used to construct them). To be comparable with the traditional schedule, all the blocks are scheduled in the SIP as well.

Another reason to schedule all the blocks is that there maybe more information until the end of mine life is reached and the new information may change the economics of this region. If it was found out that mining the end part would be uneconomic as mine gets closer to that region, the operation would stop before mining uneconomic blocks. The main purpose of SIP is still being served by delaying mining the risky parts of the mine. While in using traditional scheduling, some part of the uneconomic material would have been mined earlier and part of the higher grade ore (higher metal

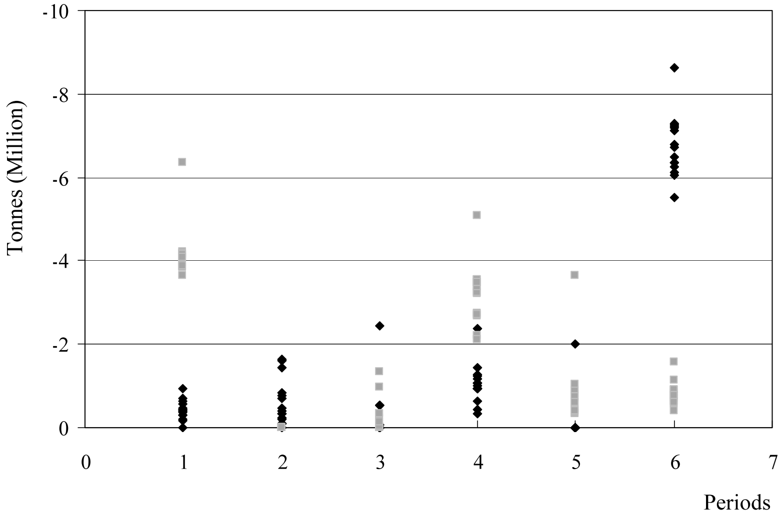

Fig. 3 Deviations from ore production target by the schedules generated by the SIP model (*dark color*) and traditional method (*light color*) using one orebody model (Whittle Four-X Milawa scheduler)

content) would have been left behind to the end making all the material appear profitable. The cost of this would not be known unless the proposed SIP model was used.

Cross-sectional views of the two schedules are shown in Fig. 2 at three locations. The figure shows that there are differences between schedules in terms of where they are physically mining each period. Both of the schedules need to be smoothed to be practically feasible for providing equipment access to all blocks to be mined within the time periods scheduled for mining.

5.1 Comparison of the traditional and SIP schedules

Annual ore and metal production for each of the schedules were calculated with respect to each of the simulated orebody models. Figure 3 shows the deviations from targeted ore production. During the first year of production, there is a deviation of about 500,000 tonnes on average with the schedule using the SIP model while the traditional model has a deviation of around 4 million tones. There is a chance that the ore production can be more than 6 million tonnes less than the planned target if TS is used as a schedule in operation. This possibility of extreme shortages in the first year of production makes the operation extremely risky in terms of achieving the company's financial objective. The maximum amount of a possible ore production shortage using the SIP model is only about 1 million tonnes in the first year. During the second year of production, there is no shortage in ore production with the TS model as shown in Fig. 3. There is a lot of excess production that is assumed to be stockpiled. The schedule by the SIP model has slightly higher expected shortages in ore production than the first year. The risk of not meeting production target increases from the first year to the end of the mine life. This was an intended trend of the schedule. In the TS model, the risk of ore production shortage is randomly distributed over

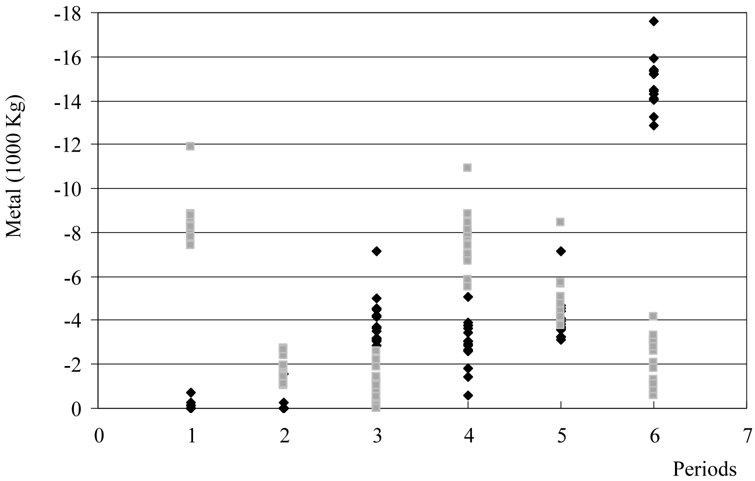

Fig. 4 Deviations from metal production target by the schedules generated by the SIP model (*dark colour*) and traditional method (*light colour*) using one orebody model (Whittle Four-X Milawa scheduler)

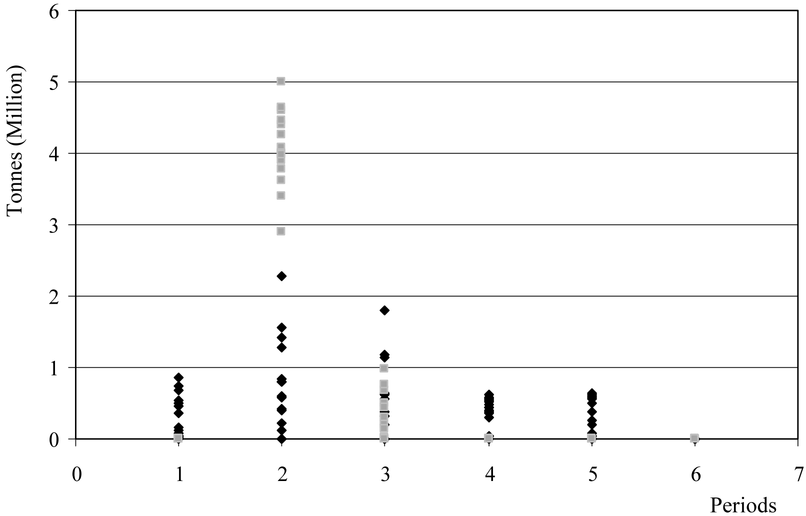

Fig. 5 Material at the stockpile from the schedules generated by the SIP model and traditional method using one orebody model (Whittle Milawa scheduler)

the production periods, as there is no provision allowing risk management in the traditional scheduling methods.

The deviations from metal production targets for the two schedules with respect to the 15 simulated orebody models are illustrated in Fig. 4. The schedule generated by the SIP model has the tendency to increase towards the later years of production. As expected, shortage in metal production in TS is also randomly distributed between periods for the same reasons as the shortage in ore production discussed above. The amount of material at the stockpile at the end of each period is given in Fig. 5. The

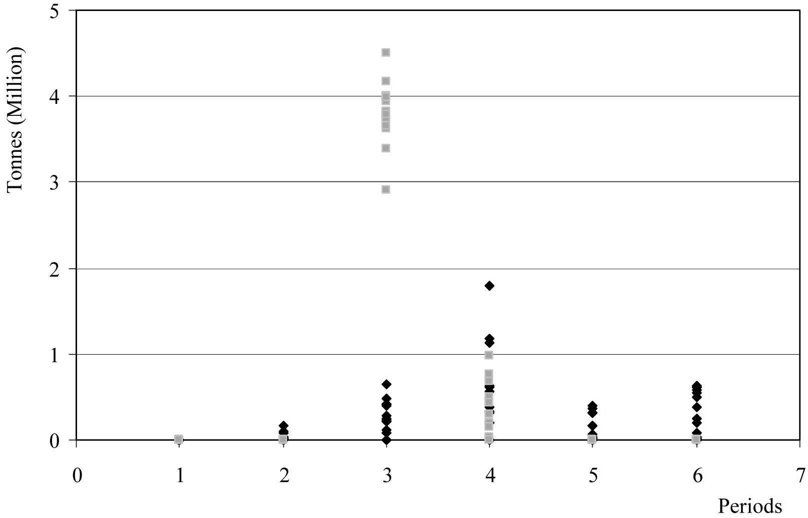

Fig. 6 Material processed from the stockpile from the schedules generated by the SIP model and traditional method using one orebody model (Whittle Milawa scheduler)

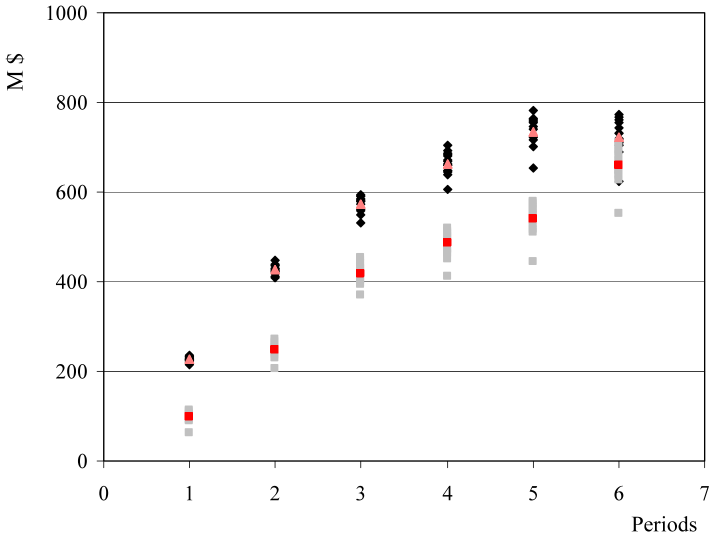

Fig. 7 Expected cumulative net present value according to the schedules generated by the SIP model (*dark colored*) and traditional method using one orebody model by Whittle Milawa scheduler (*light colored*)

figure shows that the stockpile capacity limit of 5 million tonnes is obeyed with respect to all of the simulated orebody models. The amount of material reclaimed from the stockpile for processing is given in Fig. 6. Figures 5 and 6 show that at the end of the mine life, all the stockpiling material is processed and show the capability of the SIP model to consider stockpiling as an option in the optimization process. Figure 7 illustrates the cumulative cash flows generated from the schedules according to each of the simulated orebody models. Each dot in the figure corresponds to cash flow in a year from a specific schedule for a simulated orebody model. The figure shows that during the first year, the SIP model generates over \$200 M while the traditional schedule produced only about \$100 M.

At the end of the mine life, the total *NPV* generated from the SIP schedule is \$64 M higher than the *NPV* from TS model, which is about 10%. This difference is because of economic discounting of the money since both schedules eventually produced the same amount of metal and mined out the same total volume.

6 Conclusions

In this paper, a new stochastic integer programming model is developed and applied to a gold deposit in Australia. The proposed SIP model uses multiple conditionally simulated orebody models that are equi-probable representations of the actual deposit in the ground for optimizing annual production schedules in open pit mines. This approach accounts for the uncertainty in the mineral supply from the deposit in the ground, unlike the traditional scheduling methods that are based on a single orebody model assumed to be the actual deposit in the ground being mined.

Incorporating multiple conditionally simulated orebody models without averaging the grades in a mathematical optimization model delivers a risk robust production schedule. The robustness of the schedule comes from the fact that the effect of the geological uncertainty (variability of metal content in the ground) on the periodical production targets can be managed using the model parameters.

The SIP model is found to be a powerful tool to manage the risk of not meeting production targets by controlling the magnitude and the probability of the risk within individual production periods and, in addition, controlling how the risk is distributed between production periods. The model allows the user to define the risk profile and generates the schedule with the optimum *NPV* for the resultant risk profile. The risk profile in the schedules that are generated by traditional methods is random and can lead to substantial losses, even premature closure of a mining operation, by failing to meet the planned production targets

The proposed SIP model is very efficient in that it needs no more binary variables than the traditional MIP formulations using single orebody model as input. The total number of binary variables representing the blocks for scheduling periods in the SIP formulation using many simulated orebody models is the same as in the MIP formulations using single orebody models. The only additional variables are the deviation variables and some variables to control the stockpile, which are defined as continuous. The number of constraints is larger in the SIP scheduling formulation than the traditional MIP formulations. However, possible inefficiency issues caused by this increased number of constraints can be managed as discussed in this paper.

In this study, by defining only ore block variables as binary and leaving the waste block variables as continuous, the SIP model could be applied efficiently to an Australian gold deposit and is shown to increase the overall *NPV* for about 10%. As well established in general terms (e.g. Birge and Louveaux 1997), the value generated using stochastic solutions is always greater than, or equal to, the value that can be obtained using a deterministic approach. In open pit mining applications, tests have shown that stochastic approaches produce a 10 to 25% of additional value for mining operations than the traditional scheduling methods.

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