Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions

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The production scheduling of open pit mines is an intricate, complex and difficult problem to address due to its large scale and the unavailability of a truly optimal net present value (NPV) solution, as well as the uncertainty in key parameters involved. These key factors are geological and mining, financial and environmental. Geological uncertainty is a major contributor in failing to meet production targets and the financial expectations of a project especially in the early stages of a project. Stochastic integer programming (SIP) models provide a framework for optimising mine production scheduling considering uncertainty. A specific SIP formulation is shown herein that generates the optimal production schedule using equally probable simulated orebody models as input, without averaging the related grades. The optimal production schedule is then the schedule that can produce the maximum achievable discounted total value from the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed SIP model allows the management of geological risk in terms of not meeting planned targets during actual operation, unlike the traditional scheduling methods that use a single orebody model and where risk is randomly distributed between production periods while there is no control over the magnitude of the risks on the schedule. Notably, the testing of the SIP formulation in two cases, a gold and a copper deposit, shows that the expected total NPV of the schedule using the SIP approach is significantly higher (10 and 25% respectively) than the traditional schedule developed using a single estimated orebody model.

Keywords: Open pit mining, Production scheduling, Stochastic integer programming

Introduction

A main objective in mining is to maximise the total discounted economic value to be generated from an operation. Traditionally, an orebody model is generated to represent the mineral deposit using drillhole data at certain locations and estimates of the values of mining blocks between the data locations. However, a single estimated model is only a smoothed image of the actual deposit and cannot represent the natural local grade variability within deposits. The traditional methods of open pit mine planning and production scheduling using single orebody models are unable to deal with geological uncertainty caused by *in situ* grade variability. This poses substantial risks of not meeting planned production targets in terms of ore tonnes, grades and expected cash flows through actual operations.

The effect of geologic uncertainty in mine planning has been recognised in the literature. Ramazan and Dimitrakopoulos[1,2] show that conventional mixed integer programming (MIP) type optimisation methods may generate significantly different scheduling patterns from each other when geostatistically simulated and estimated orebody models are used as input. These significant differences in the scheduling results clearly indicate the need for new stochastic optimisation methods that can consider orebody uncertainty in the optimisation process. Dimitrakopoulos *et al.*[3] also show that there are substantial conceptual and economic differences between risk based frameworks and traditional approaches. Some effort is made to use stochastic orebody models sequentially in traditional optimisation methods by Ravenscroft[4] and Dowd.[5] However, sequential processes are shown to be inefficient and cannot produce an optimal schedule considering uncertainty.

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The MIP formulations for optimising production scheduling for both open pit and underground mines are usually too large to be solved without applying specific methodology to reduce the number of required binary variables and the model size. To reduce the required solution time of the mathematical optimisation methods for open pit mines, Ramazan[6] developed the fundamental tree algorithm. This algorithm reduces the number of binary variables required in formulating production scheduling optimisation as an MIP model and makes the MIP models applicable to large open pit mine optimisation processes. For underground mines, Topal[7] developed a methodology that reduced the number of required binary variables substantially by defining earliest start and latest start periods for mining blocks of the Kiruna iron ore mine, Sweden.

Dimitrakopoulos and Ramazan[8] present a new long term probabilistic type production scheduling method and introduce the concept of geological risk discounting. The method is based on MIP which solves the scheduling problem efficiently. The model is further improved and developed in Ramazan and Dimitrakopoulos[1] to consider orebody uncertainty, efficiently integrating the issue of equipment access and reduced mobility of large equipment. Although this method produces a schedule with a preferable risk profile as compared to other conventional optimisation methods, setting scheduling priorities to blocks based on the probabilities assigned to individual blocks does not always produce an optimal solution for production scheduling of large and complex deposits. This is because the blending of the blocks which have lower and higher grades than the target values (low probability blocks) is not considered effectively. Grieco and Dimitrakopoulos[9] demonstrate a new probabilistic type MIP formulation for stope designs in underground mines. Dimitrakopoulos *et al.*[10] present a new method of designing cutbacks using simulated orebody models, which considers the upside potential and downside risk of a project. Godoy and Dimitrakopoulos[11] developed a new approach for risk inclusive long term production scheduling based on simulated annealing, which yields substantial net present value (NPV) increase (28%) as compared to traditional optimisation methods. The same order of NPV increase (26%) is also shown in recent case studies[12] using a variant of the same method. Although the above studies constitute substantial developments in the field, they do not directly integrate the uncertainty related to ore grade and quality parameters in the optimisation process, and in addition, the method requires several steps in developing a final risk based schedule.

Stochastic integer programming (SIP) models are presented herein, as an alternative framework to the above developments, for generating optimum production schedules which consider ore grade and quality uncertainty. SIP is a type of mathematical programming based modelling approach that considers multiple, equally probable scenarios and generates the best result for a set of defined objectives, within the feasible solution space bounded by a set of constraints. Three alternative SIP modelling approaches are first presented, so as to discuss their applicability to mine production scheduling problems, and are followed by a method that quantifies the value of stochastic solutions. Then, one of the SIP approaches presented is revisited through a specific to mine production scheduling formulation. The formulation uses a set of simulated orebody models without averaging the grades of the simulated orebody realisations and provides a schedule having maximum total discounted economic value (NPV) with a managed risk profile. A key aspect of this formulation is that the risk of not meeting planned production targets is controlled while generating the optimum production schedule and maximising the expected NPV of the project. The new SIP formulation is applied on a part of a gold deposit and a copper deposit; results are compared with the conventional scheduling approach and finally, the value of the stochastic solution for the given two examples is assessed. Conclusions follow.

Stochastic integer programming

Stochastic integer programming (SIP) may be defined as a type of mathematical programming based modelling that can consider multiple, equally probable scenarios and generates the best result for a set of defined objectives, within the feasible solution space bounded by a set of constraints. In a SIP model, there is a set of decisions to be taken, such as when to mine a block, without full information about an event which is thus described probabilistically. Examples of an event are the unknown grades and tonnages of the orebody to be mined. The grades and tonnages are represented by multiple simulated realisations of the orebody. In the context of mathematical programming, SIP is defined in Escudero[13] as an extension of MIP with uncertainty in one or more of the related coefficients. Different approaches in SIP formulations are discussed by Birge and Louveaux.[14] The existing developments in the technical literature are not, however, directly applicable to mining problems. In stochastic programming, there are mainly three main models: anticipative models, adaptive models, and anticipation and adaptation (recourse) models.[15] A fourth approach, the so called chance constraints, is unsuitable for the scheduling problem due to severely unrealistic assumptions, such as normality of grade distributions in mining blocks.

Anticipative models

Assuming that $w_i(x,m)$ is a function where x is the decision of which blocks to mine in what production period and m is the annual metal production, which is a function of grade (g). Since the grades are uncertain within the mining blocks, there may be r representations, or simulated realisations, of the actual grades with equal probability of each one occurring. Then, i is the ith realisation of the grades, i.e. i=1, 2, …, r. In stochastic programming for mining, i is termed the 'here and now situation' when a decision must be made on the mining periods of the blocks without a priori knowledge of the grades. The problem consists of finding $x \in X \subset R^n$ (X is a given subset of R^n , i.e. real numbers) in the expression

$$\text{Maximise } w_i(x, m) \tag{1}$$

Subject to:

$$w_i(x,m) \leqslant 0 \tag{2}$$

$$\nabla x \leqslant 0 \tag{3}$$

Equation (2) is a generic representation of all the operational constraints such as annual ore tonnes, grade and quality requirements. Equation (3) represents the physical restrictions such as slope constraints in production scheduling of open pit mines. Some $x \in X$ may satisfy the constraints $w_i(x,m) \leqslant 0$, with a certain level of reliability

$$\mathrm{Prob}\{m|w_i(x,m)\cdot \leq 0,\ i=1,2,\ \ldots,\ m\} \geqslant \alpha \tag{4}$$

where $\alpha \in (0,1)$ or the average $E\{w_i(x,m)\} \leqslant 0$. Equation (4) signifies that the probability that the simulated realisations satisfy the model constraints must be greater than a certain reliability level α.

Suppose that the objective function is of type $E\{w_0(x,m)\}$, in the simple case, then violating the constraint $w_i(x,m) \leqslant 0$ generates a cost, $c_i w_i(x,m)$ where $c_i \geqslant 0$, proportional to the amount by which the constraint is violated. This leads to an objective function

$$\mathrm{f}_0(x,m) = w_0(x,m) + \sum_{i=1}^{r} q_i(\max[0, c_i w_i(x,m)]) \tag{5}$$

Adaptive models

Adaptive models cannot directly represent mine production scheduling problems due to their nature, especially considering the presence of grade uncertainty which is the main focus of the new SIP model proposed in this paper. It is difficult to interpret the adaptive models in terms of the mine production scheduling problem, because block grades will not be known in time, or by waiting, as required for the purpose of optimisation. Therefore, a brief description of the model is provided here as in the general operations research literature such as Birge and Louveaux,[14] Ermoliev and Wets,[15] and Heyman and Sobel.[16]

Adaptive models consider the situation where an observation is allowed before making a decision on x. This corresponds to the idea of optimisation in a learning environment. Typically, observations will only give partial information on the environment g. Suppose that B is a collection of sets that contain all the relevant information that could become available after making an observation. The decision x must be determined on the basis of the information available in B, i.e. it must be a function of g whose values are B dependant or 'B measurable'. Now the problem is to find a B measurable function

$$g \rightarrow x(g)$$

which satisfies $x(g) \in X$ for all g, and $E\{\mathrm{f}_i(x(\cdot),\cdot)\backslash B\}(g) \leqslant 0$, i=1, 2, …, m, such that

$$z = E\{\mathrm{f}_0(x(g),g)\} \tag{6}$$

is minimised, where $E\{\cdot\backslash B\}$ denotes the conditional expectation given B.

For the case where g becomes completely known, then the optimal $g \rightarrow x(g)$ is obtained by solving for all w, the optimisation problem

Find $x \in X \subset R^n$such that $\mathrm{f}_i(x,g) \leqslant 0$, i=1, 2, …, m and $z_w = \mathrm{f}_0(x,g)$ is minimised.

Anticipation and adaptation: recourse models

The two stage recourse model is a type of mathematical model where the fundamental mechanisms of anticipative and adaptive models are integrated within a single model. This model represents a trade-off between long term anticipatory strategies and the associated adaptive strategies. In mining terms, the trade-off could be between total expected NPV and associated risks in meeting production targets. Conceptually, a mathematical model is expected to result in higher NPV values if higher geological risks are tolerated within the model; a conservative risk approach is expected to result in a lower NPV value. The recourse problem can be expressed as follows:

Find $x \in R^n_+$, such that

$$\mathrm{F}_i(x) \leqslant 0,\ i=1,2,\ \ldots,\ m \tag{7}$$

and

$$\mathrm{F}_0(x) = cx - E\{Q(x,g)\} \tag{8}$$

is maximised, where

$$Q(x,g) = \inf\{q(g)y | W(x,g) = h(x,g) - T\cdot\} \tag{9}$$

where $y \in R^{n'}_+$ and x is the matrix of decision variables (x_i^t) for deciding when to mine a block (if $x_i^t = 1$, mine block i in period t). If all x variables are set to 0 or 1, representing the percentage of blocks to be mined at each period t, and if this set of x values are feasible for the model constraints under equation (7), the values define a production schedule. Equation (7) is a representation of all the constraints required for the mining operations.[17–19] In equation (8), cx is the total NPV value to be generated from the project given a decision on when the blocks should be mined, or given a production schedule defined by x values. $E\{Q(x,g)\}$ is the expected risk, or associated costs, of not meeting production targets under the chosen schedule. The risk in the model is defined as the deviations from the desired production targets and the unit cost multiplier matrix y as a function of the infeasibility. For a given schedule x and a set of grades g, $h(x,g)$ represents the tonne, grade and quality values to be produced periodically and T is the target matrix. Therefore, $W(x,g)$ defines the risk and $q(g)y$ defines the cost of the risk for the schedule, as a function of the uncertain grade values.

After the true environment is observed through a simulated orebody model, the discrepancies that may exist between $h(x,g)$ and T (for fixed x and observed $h(x,g)$ and T) are calculated using equation (9) as

$$W(x,g) = h(x,g) - T$$

The cost of risk is defined in the objective function. The model takes the corrective or recursive form to redefine the x variables and the schedule so that NPV and cx are maximised, while the loss $q(g)y$ is minimised. Therefore, an optimal decision x should modify the total expected profit to be generated by carrying out the plan, i.e. the direct NPV (cx) as well as the costs generated by the risk defined using simulated orebody models on the schedule $E\{Q(x,w)\}$.

Proposed SIP model for long term production scheduling of open pit mines

A SIP model for optimising long term production scheduling in open pit mines is developed with an objective function that maximises the total NPV of the project under a managed risk profile. The general form of the objective function is expressed as

$$\mathrm{Max} \sum_{t=1}^{p} \left[\sum_{i=1}^{n} E\{(\mathrm{NPV})_i^t\} b_i^t - \sum_{s=1}^{m} (c_u^{to} d_{su}^{to} + c_l^{to} d_{sl}^{to} + c_u^{tg} d_{su}^{tg} + c_l^{tg} d_{sl}^{tg}) \right] \quad (10)$$

where p is the total production period, n is number of blocks, and b_i^t is the decision variable for when to mine block i (if mined in period t, b_i^t is 1 and otherwise b_i^t is 0). The c variables are the unit costs of deviation (represented by the d variables) from production targets for grades and ore tonnes. The subscripts u and l correspond to the deviations and costs from excess production (upper bound) and shortage in production (lower bound) respectively, while s is the simulated orebody model number, and g and o are grade and ore production targets.

The NPV of a block is calculated for all the simulated orebody models and averaged. The cost parameters are discounted by time using the geological risk discount factor developed by Dimitrakopoulos and Ramazan.[8] The geological risk discount rate (GRD) allows the management of risk to be distributed between periods. If a very high GRD is used, the lowest risk areas in terms of meeting production targets will be mined earlier and the most risky parts will be left for later periods. If a very small GRD or a GRD of zero is used, the risk will be distributed at a more balanced rate among production periods depending on the distribution of uncertainty within the mineralised deposit.

The 'c' variables in the objective function (equation (10)) are used to define a risk profile for the production, and NPV produced is the optimum for the defined risk profile. It is considered that if the expected deviations from the planned amount of ore tonnage having planned grade and quality in a schedule are high in actual mining operations, it is unlikely to achieve the resultant NPV of the planned schedule. Therefore, the SIP model contains the minimisation of the deviations together with the NPV maximisation to generate practical and feasible schedules and achievable cash flows. Note that the developed SIP model for mine production scheduling is very similar to the anticipation and adaptation/recourse model given in equation (8). In addition, note that the same operational constraints for production scheduling of open pit mines given in Ramazan[17] and Ramazan and Dimitrakopoulos[2] can be used in the SIP model. Stochastic constraints used to calculate the deviations from production targets are given in Ramazan and Dimitrakopoulos.[19]

Value of stochastic programming

In mining, the production scheduling problem is a stochastic problem because uncertainty exists on the grades used as input. The random variables, grades, are replaced by their expected values to simplify the stochastic production scheduling problem to a deterministic one. Recently, there have been some approaches presented in the literature for solving several deterministic scheduling problems each corresponding to one particular scenario, and then either to combine these different solutions, or choose the best solution among the solutions obtained, by heuristic methods such as presented in Dimitrakopoulos *et al.*[8] It is not known how different the solutions from these heuristic methods are from the true optimal stochastic solution. The answer to this theoretical question is provided by two concepts: the expected value of perfect information and the value of the stochastic solution.[10]

Simulated orebody models are equally probable representations of the actual deposit. Each simulated orebody model corresponds to one particular scenario for variables such as grade and quality. Assume that there are 10 simulated orebody models and each orebody model has exactly 10% chance of being the same as the actual deposit. For the sake of discussion in this section, the probability indicates that the deposit is fully represented by the 10 orebody models. Assume that there is a true optimisation method for annual production scheduling of mines given an orebody model. Then, each of the simulated orebody realisations could be used to generate 10 optimised production schedules. If the authors knew which simulated orebody model was the true representation of the deposit, the schedule generated using that orebody model would be used to calculate the total discounted economic value of the project (NPV). This NPV value obtained from the schedule using the perfect information, the simulated realisation that is known to be the exact representation of the deposit, would be the highest NPV that can be generated for the project. For example, assume that the actual orebody model grades were known to be exactly the same as simulation no. 3. The NPV value calculated using the schedule based on simulation 3 would be higher than the NPV calculated using the schedule based on simulation no. 1, because the grades used in calculating the NPV is still based on the actual deposit grades (simulation 3) although schedule is based on simulation 1.

Since it is not known which simulated realisation is the exact representation of the deposit under study, the NPVs obtained from the 10 optimised schedules can be averaged to determine the maximum expected value of the project. This expected NPV is termed the 'expected solution of perfect information (ESPI)'.

The simulated orebody models may be averaged and this average model may be used as input in the optimisation method. After generating a schedule using this average orebody model, 10 different NPVs would be calculated for this schedule, since the deposit is one of the 10 orebody models, resulting in 10 different metal and ore production values for each production period. The average of the 10 NPVs is called the 'expected value solution (EVS)'. A different and more efficient way of scheduling is to use the stochastic programming method given in equation (10). The stochastic programming method uses the 10 simulated orebody models as input and generates one schedule that is optimum, given the orebody models available and the operational constraints considered. From the schedule obtained through stochastic optimisation, 10 different NPVs can be calculated for each of the 10 simulated realisations. The average value of the 10 NPVs is called the 'expected stochastic solution (ESS)'.

The expected value of perfect information (EVPI) is the difference between the ESPI and the EVS. The EVPI can be expressed as follows

$$\mathrm{EVPI} = \mathrm{ESPI} - \mathrm{EVS} \quad (11)$$

The EVPI is the maximum economic value that a decision maker would be willing to pay for the perfect, complete and accurate information about the deposit. The EVPI concept is first developed in the context of decision analysis. Further discussions can be found in Raiffa and Schlaifer.[20]

The value of stochastic programming (VSP) is the difference between the value of the stochastic solution and the expected value solution. It can be expressed as

$$\mathrm{VSP} = \mathrm{ESS} - \mathrm{EVS} \tag{12}$$

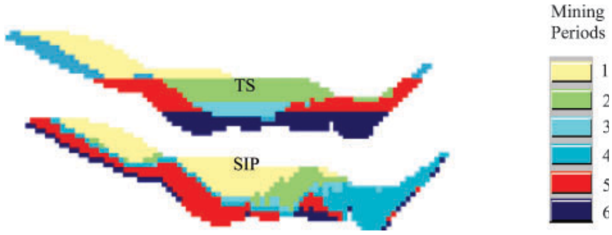
The VSP represents the cost of ignoring uncertainty in making a decision and is always positive if the optimal solution to the problem depends on the uncertain variable.[21] It is well known and rigorously proven property of SIP models in the field of operations research[13–16,20,21] that the VSP is always non-negative (greater than, or equal to 0); this is also intuitive in mine production scheduling, considering that the schedule using the proposed SIP model takes into account all possible outcomes (simulated orebody models) in meeting production targets. If only a single orebody model is used in optimisation, the NPV may appear to be the highest for that specific orebody model. However, during the actual operation when the estimated grades of the blocks turns out to be different than the actual block grades, the actual produced ore tonnes and grades can vary from the planned tonnes and grades. Owing to these variations, the actual produced NPV can be much smaller than the NPV value calculated from the optimised schedule using a single orebody model. The SIP considers these possible variations through the available simulated models, and as a result, quantifies and manages the risk of not meeting production targets properly. In the traditional optimisation approaches, the process is at the mercy of some 'unknown factor' in terms of distribution of grade variability and uncertainty, while the SIP model provides an operation with a powerful risk management tool.

Note that the VSP can only be 0 for some extreme cases where the optimal solution is not sensitive to the uncertain variables. This means that regardless of the scenario used as input to the mathematical programming model, the result would be exactly the same. It is a highly unlikely case to occur in mine production scheduling that the schedule does not depend on the grades and quality parameters involved. Although there is no general rule for the magnitude of the VSP, VSP is expected to increase with increasing variance in the variables related to the optimisation. Some examples of large VSP and EVPI are illustrated in Louveaux and Smeers[22] and Birge.[23]

Case studies

Gold deposit

The proposed SIP model is applied to a part of a gold deposit and contains 22 296 blocks. The optimisation of production scheduling is performed in two stages: in the first stage on a volume containing 11 301 blocks and in the second stage for the remaining blocks. As can be seen in Table 1, it took ~5 h for the first stage model to be solved with a dual processor (2 GHz) PC, while the second stage took over 37 h. It should be noted that traditional optimisation methods taking only one orebody model as input would be expected to require similar solution times if efficient MIP formulations are applied, as in Ramazan and Dimitrakopoulos.[18] Although multiple simulated orebody models are used in the SIP formulations, the number of binary variables is not higher than that in the traditional MIP formulations. In this case study, 14 simulated orebody models are used. The schedule using single average, or estimated, orebody model is expected to result in approximately \$659M NPV, which is the EVS value in this case study. The schedule obtained using the stochastic programming model is expected to generate about \$723M ESS value. Therefore, the VSP is \$64M (\$723M–\$659M) or a contribution of ~9·7% additional NPV to the project compared with the expected NPV from the traditional schedule. Figure 1 shows a cross-section of the two schedules: one obtained using the SIP model and the other generated by a traditional method (traditional schedule or TS) using a single estimated orebody model. Both schedules need to be smoothed further to be practical in operation; the effects of smoothing the schedules are discussed at the end of the second case study.



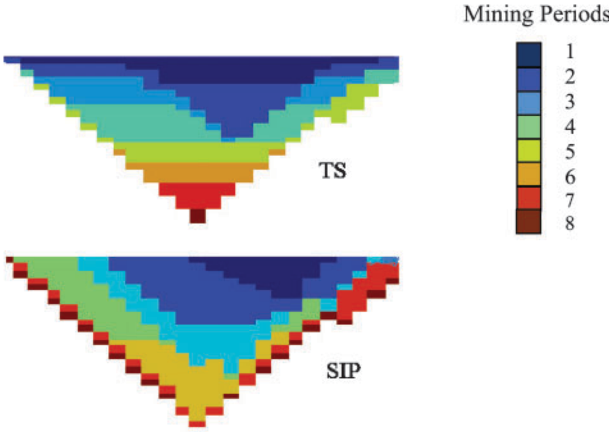
1 Cross-sectional views of SIP (bottom) and traditional schedule (TS, top) at gold deposit

Copper deposit

The second example of production scheduling with the proposed SIP model is an application at a copper deposit composed of 14 480 blocks. The scheduling considers ore capacity of 7·5 Mt per year, maximum mining capacity is 28 Mt, and the scheduler decides the optimal waste removal strategy. The SIP run considers 20 simulated orebody models that are available and tests show that, as in earlier studies,[24,25] the results are not sensitive to the use of more simulated orebody models. The conventional schedule using a single estimated orebody model forecasts an expected NPV at about \$238M, which is the EVS value in this second case study. The SIP schedule is expected to generate about \$298M ESS value. Similarly to before, the VSP is \$60M or a contribution of ~25% additional NPV to the project, compared with the expected NPV from the conventionally generated schedule.

Table 1 Information of SIP run for gold deposit in case study

Description	Stage 1	Stage 2
Total number of blocks	11 301	10 995
Constraints	33 273	21 363
Total number of variables	53 301	37 286
Number of binary variables	18 540	9580
Solution time (hour:min)	04:50	37:15
Production periods (years)	1, 2, 3 and 4	4, 5 and 6



2 Cross-sectional views of the SIP (bottom) and traditional schedule (TS, top) for copper deposit

Figure 2 shows a cross-section of the two schedules from the copper deposit: one obtained using the SIP model (bottom) and the other generated by a traditional method (top) using a single estimated orebody model. Both schedules shown are the raw outputs and need to be smoothed to become practical. It is important to note that:

(i) the results in the second case study are similar in % improvement compared with other stochastic approaches reviewed in the introduction[11,12]

(ii) although the schedules compared in the studies herein are not smoothed out, other existing SIP applications[24,25,26] show that the effect of generating smooth and practical schedules has marginal impact on the forecasted performance of the related schedules, thus the order of improvements in SIP schedules reported here remain.

Conclusions

Stochastic integer programming models offer a framework to address uncertainty in key inputs of mine production schedules, including geological (grade) uncertainty. A new SIP formulation was shown to generate the optimal production schedule using as input a set of equally probable simulated orebody models and without averaging the related grades of mining blocks. This set of simulated scenarios descries geological (metal) uncertainty and its use allows the management of geological risk in terms of not meeting planned production targets. This differs from the traditional scheduling methods that use a single model of the deposit, thus leading to not considering risk and entailing the random risk distribution in different production periods and without any control over the magnitude of the risk. As it was shown in the case studies above, the proposed scheduling approach considers multiple simulated orebody models without increasing the required number of binary variables and thus computational complexity. The SIP approach can be used to minimise the risk of not meeting production targets as a function of ore, metal and grade blending.

The value of the stochastic solution presented in the applications is significant: in the first case study of a gold deposit, it is \$64M or 10% higher NPV; in the second example with the copper deposit, the value of the stochastic solution is \$60M or ~25% higher NPV. Again, this difference is largely based on the ability of the stochastic optimiser to quantify and manage the risk of not meeting production targets. The simulated realisations of the orebody provide a range of possible values of metal content in the blocks being considered along with its neighbours, which then allows the optimisation process to assess and utilise the 'upside potential' separately from the 'downside risk'. This can only be carried out if simulated orebodies are jointly considered.

References

1. S. Ramazan and R. Dimitrakopoulos: 'Probabilistic practical optimization of production scheduling for multi-element deposits', *CIM Mag.*, CIM website, April 2006, 12 p.
2. S. Ramazan and R. Dimitrakopoulos: *Int. J. Surf. Min.*, 2004, **18**, 85–98.
3. R. Dimitrakopoulos, C. Farrelly and M. Godoy: *Trans. Inst. Min. Metall. Sect. A, Min. Ind.*, 2002, **111A**, 82–89.
4. P. J. Ravenscroft: *Trans. Inst. Min. Metall. Sect. A, Min. Ind.*, **101A**, 104–108.
5. P. A. Dowd: *Trans. Inst. Min. Metall. Sect. A, Min. Ind.*, **106A**, 9–18.
6. S. Ramazan: *Eur. J. Oper. Res.*, 2007, **177**, (2), 1153–1166.
7. E. Topal: 'Advanced underground mine scheduling using mixed integer programming', PhD thesis, Colorado School of Mines, Golden, CO, USA, 2003.
8. R. Dimitrakopoulos and S. Ramazan: *SME Trans.*, 2004, **316**, 106–112.
9. N. Grieco and R. Dimitrakopoulos: *Trans. Inst. Min. Metall. Sect. A, Min. Technol.*, 2007, **116A**, (2), 49–57.
10. R. Dimitrakopoulos, L. Martinez and S. Ramazan: *J. Min. Sci.*, 2007, **43**, 73–82.
11. M. C. Godoy and R. Dimitrakopoulos: *SME Trans.*, 2004, **316**, 43–50.
12. A. Leite and R. Dimitrakopoulos: *Trans. Inst. Min. Metall. Sect. A, Min. Technol.*, 2007, **116A**, (3), 109–118.
13. L. F. Escudero: *Ann. Oper. Res.*, 1993, **43**, 311–335.
14. J. R. Birge and F. Louveaux: 'Introduction to stochastic programming', Springer series in operations research; 1997, New York, Springer-Verlag.
15. Y. Ermoliev and R. Wets (eds.): 'Numerical techniques for stochastic optimization'; 1988, Berlin, Springer-Verlag.
16. D. P. Heyman and M. J. Sobel: 'Stochastic models in operations research', Vol. II, 'Stochastic optimization'; 1984, New York, McGraw-Hill.
17. S. Ramazan: 'Open pit mine scheduling based on fundamental tree algorithm', PhD thesis, Colorado School of Mines, Golden, CO, USA, 2001.
18. S. Ramazan and R. Dimitrakopoulos: *SME Trans.*, 2004, **316**, 73–78.
19. S. Ramazan and R. Dimitrakopoulos: in 'Orebody modelling and strategic mine planning: uncertainty and risk management models', Spectrum series no. 14, 2nd edn, 385–392; 2007, Carlton South, Vic., Australasian Institute of Mining and Metallurgy.
20. H. Raiffa and R. Schlaifer: 'Applied statistical decision theory'; 1961, Boston, MA, Harvard University Press.
21. A. Madansky: *Manage. Sci.*, 1960, **6**, 197–204.
22. F. V. Louveaux and Y. Smeers: 'Stochastic optimization for the introduction of new energy technology', *Stochastics*, 1997.
23. J. R. Birge: in 'Numerical techniques for stochastic optimization', (ed. Y. Ergoliev and R. Welts), 481–488; 1988, Berlin, Sringer-Verlag.
24. A. Jewbali: 'Modelling geological uncertainty for stochastic short-term production scheduling in open pit mines', PhD thesis, University of Queensland, Brisbane, Qld, Australia, 2007.
25. J. Benndorf and R. Dimitrakopoulos: 'Stochastic long-term production scheduling of iron ore deposits: integrating joint multi-element geological uncertainty', COSMO Laboratory Research report no. 2, McGill University, Montreal, Que., Canada, 2008.
26. M. Menabde, G. Froyland, P. Stone and G. Yeates: in 'Orebody modelling and strategic mine planning: uncertainty and risk management models', Spectrum series no. 14, 2nd edn, 379–384; 2007, Carlton South, Vic., Australasian Institute of Mining and Metallurgy.