Discrete Optimization-II COMP 567
Homework 1 Due: Wed, January 27 in class

1. Show how to model the following constraints as standard linear constraints in an ILP. In all cases the variables x and y are allowed to take only the values zero and one.
(i) $x^2 + y^2 \leq 1$.
(ii) $xy > 0$.
(iii) $(ax + by)^2 \leq c$, where a, b, c are given integer constants.

2. Consider a production problem containing the objective function:

$$z = \max 20x_1 + 30x_2 + 40x_3$$

and many resource constraints, one of which reads

$$7x_1 + 2x_2 + 4x_3 \leq 100 = b.$$

Suppose the objective function z is measured in dollars, and the right hand side constant b is measured in kg. The manager tells you that in fact the above constraint can be relaxed at a certain cost.
(a) Suppose that b must be an integer, and can be as big as 150. There is a penalty of 5 dollars for each kg by which b excedes 100kg. Show how to model this.
(b) Same as part (a), except there is also a fixed cost of 20 dollars if b excedes 100kg. Show how to model this.

3. (i)S-foods makes four types of cookies each of which is made in an integer number "batches". For each batch of cookie type $j = 1, 2, 3, 4$, the profit is c_j, the ammount of flour used is f_j and the ammount of butter used is b_j. There are f kg of flour and b kg of butter available. The company wants to maximize profit, but there are additional production constraints:
(a) To make cookie type 1 there is a fixed cost of h_1.
(b) Cookie type 2 must be made in an **even** number of batches.
(c) If either of cookies of type 3 or 4 are made, their combined total production must be at least 10 batches.
(d) At most 50 batches of cookies (all types combined) can be made.
Formulate this as an integer programming problem.

(ii) Choose suitable values of the constants, and solve the problem using cplex or lp_solve. Try to find constants so that all of the constraints (a) to (d) are tight (ie. inequalities satisfied as equations). Instructions for using cplex and lp_solve are given on the News web page.
Please work individually on this part of the question.

4. (a) Let $X = (1,1), (1,2), (1,3), (2,2), (2,3)$. Is the following a formulation for X?

$$x_1 \leq 3, \quad x_2 \geq 1, \quad x_1 + x_2 \geq 2$$

Give a proof or counterexample.
(b) Give an ideal formulation for the set in part (a).