Here is a note on **degenerate** games.

The **support** of a mixed strategy, e.g. x of player I, is the set of pure strategies that have positive probability under x : support of x = { i | $x_i > 0$}.

The *best response condition* says: any i in the support of x must be a pure best response to the mixed strategy y if x is to be a best response against y . An *equilibrium* (x,y) is given if x is a best response to y and y is a best response to x.

One way to *find equilibria* is to consider two **supports** of x and y (these are finite sets of pure strategies) of **equal size** k, say, that is, both x and y mix between the same number k of strategies. If k=1, then x and y are both pure strategies. If k=2, then x and y both mix exactly two pure strategies, and so on.

Why the same support size for both x and y? As an example, let k=2 and the supposed equilibrium support are strategies (rows) 1 and 2 for player I, i.e. x_1 and x_2 can be positive, and strategies (columns) 2 and 3 for player II, so that y_2 and y_3 can be positive. In order to have an equilibrium, rows 1 and 2 must have equal payoff (and in addition, that payoff must not be higher in any other row). That payoff is the expected payoff against y, denoted $(Ay)_1$ and $(Ay)_2$ if A is the matrix of payoffs for player I. But in that expectation, only **two variables** can be used, namely y_2 and y_3, and they have to fulfill **two linear equations**, namely $(Ay)_1 = (Ay)_2$ with $y = (0,y_2,y_3,0,\ldots,0)$ and $y_2 + y_3 = 1$ since they are probabilities. So two unknowns, two equations, that normally gives a unique solution.

If we had **unequal supports**, e.g. player I mixing 3 pure strategies (rows 1,2, and 4, say) and player II mixing only 2, say still columns 2 and 3, then there would have to be an extra equation to be fulfilled (for that extra expected payoff in the extra row 4 of player I, i.e. the equation $(Ay)_1 = (Ay)_4$ where the other equation $(Ay)_2 = (Ay)_4$ is implied by the already given equation $(Ay)_1 = (Ay)_2$) and the two variables y_2 and y_3 simply don't suffice for that, **except by accident**.

Now, such an accident happens exactly in the case of degeneracy. In our example, assume we equate $(Ay)_1 = (Ay)_2$ and these expected payoffs in rows 1 and 2 are not only equal but not exceeded in any other row. Normally, we have for row 4 either $(Ay)_4 < (Ay)_1$ (meaning row 4 is not a best response to y) or $(Ay)_4 > (Ay)_1$ (meaning rows 1 and 2 are not best responses against y after all) but one could also have the case (accidentally, when solving $(Ay)_1 = (Ay)_2$ using only y_2 and y_3) that $(Ay)_4 = (Ay)_1$. This would mean that there is a mixed strategy y of support size 2 that has 3 pure best responses (here rows 1, 2, and 4). The trouble with that is that you now can use three variables on the side of player I, x_1, x_2, and x_4, since they are all best responses against y, to fulfill the equilibrium condition that the two columns 2 and 3 used by player II have equal expected payoff against x (one equation), and the equation $x_1 + x_2 + x_4 = 1$, so we have 3 unknowns subject to 2 equations, and that typically means an **underdetermined** system with multiple solutions, and resulting complications.

So here is the **definition of a nondegenerate game**: A bimatrix game is called *nondegenerate* if to any mixed strategy z (of player I or player II), the *number of pure best responses against z is never larger than the size of the support of z*.

For a nondegenerate game, the above complications cannot occur. In contrast, a degenerate game has, for example, a pure strategy (support size 1) with 2 or more best pure responses, or a mixed strategy with support size 2 that has 3 or more pure best responses, in general some mixed strategy that has support size k but that has more than k best responses. In our small games, degeneracy only occurs with k=1 or k=2.