COMP566 assignment: Bimatrix games

October 19, 2007

1. Construct a 2×2 bimatrix game with *exactly* two Nash equilibria.

2. Prove that the set of Nash equilibria of a bimatrix game (A, B) does not change if a constant is added to every entry of some column of A (or some row of B).

3. (a) A nondegenerate bimatrix game has an odd number of Nash equilibria. Show that a nondegenerate *symmetric* bimatrix game has at least one *symmetric Nash equilibrium* $(x, y) = (z, z)$.

   (b) Give a family of symmetric (nondegenerate) $n \times n$ bimatrix games where every game has *exactly* $2^n - 1$ Nash equilibria and all are symmetric. For every n, what is the length of the shortest Lemke–Howson path?

4. Consider the following symmetric 3×3 game (A, B) with

$$A = B^\top = \begin{pmatrix} 0 & 3 & 0 \\ 2 & 2 & 0 \\ 3 & 0 & 1 \end{pmatrix} .$$

   Show that the Lemke–Howson algorithm cannot be used to enumerate all Nash equilibria of the game. In other words, show that the game has a Nash equilibrium (x, y) such that there is no sequence of missing labels $r_1, r_2, \ldots, r_k$ so that (x, y) is connected to the artificial equilibrium by the concatenation of r_i-almost-completely-labeled paths for $i = 1, 2, \ldots, k$.