COMP566
Homework 1 Due: Thurs September 20, 2007

1. Three part-time workers are available to do 5 jobs in a given work week. The hours required by each worker to perform each job is given in the table below. Hourly wage rates for workers 1, 2 and 3 are 7, 8, and 9 dollars per hour. A worker can be assigned to more than one job, but only one worker can be assigned to each job. A worker can work at most 40 hours per week. Formulate an integer linear program to assign the workers to do the jobs at minimum cost. Solve the integer program by CPLEX, and also the linear programming relaxation (drop integer condition on the variables). Can you give a direct method to solve the LP relaxation without using the simplex method?

Worker	Job				
	1	2	3	4	5
1	30	18	22	24	23
2	20	19	21	17	16
3	18	18	25	26	19

2. Show that the following example, due to Komei Fukuda, can be made to cycle through a series of six dictionaries. You need not follow any specific pivoting rule, but the pivots must be chosen in accordance with the simplex method. (Hint: start by letting x_1 enter the basis and x_5 leave. Choose the next pivot so that the coefficient matrix obtained looks identical to the original dictionary, even though the basis is different) Show all dictionaries in the cycle.

$$\max x_1 - 2x_2 + x_3$$

$$2x_1 - x_2 + x_3 \leq 0$$

$$3x_1 + x_2 + x_3 \leq 0$$

$$-5x_1 + 3x_2 - 2x_3 \leq 0$$

$$x_1, x_2, x_3 \geq 0$$

You may use maple or some other system to solve the systems of equations.

3. Consider an LP problem in standard form where $b_i = 0, i = 1, \ldots, m$. Show that it is impossible that there are two dictionaries D and D' for this system with the following properties.

(1) In D the variable x_{n+m} is cobasic, and in the objective row all coefficients are non-positive except for the coefficient of x_{n+m}, which is positive.

(2) In D' the variable x_{n+m} is basic. There is a cobasic variable x_j such that it has positive coefficient in the objective row, and all the coefficients in this column are non-negative, except for the coefficient in the row for x_{n+m} which is negative. (Ie. if we pivot x_j into the basis, we must pivot

out x_{n+m}).

Policy on Late Assignments: -10% per day, including weekends. Assignments are due in class. Hand in late assignments to Conor Meagher, MC232.