COMP566 Discrete Optimization I

Homework 2 Due: Tuesday, October 10 , 2006

1. (a) For problem 1 of the first assignment, write down the dual problem. Convert the dual problem into "standard form", and solve it by the two phase simplex method. Please work out which variables to pivot for each step. You may use a package to do the actual pivot step (as in problem 2 of the last assignment). Write out the final dictionary.
(b) Write out the final dictionary for this problem solved in the normal way - ie. simplex method applied to the original problem. You should be able to get this dictionary without having to do a complete simplex method run. Compare these two dictionaries and see what you can say about them. In particular, is the solution to one problem contained in the final dictionary of the other, and vice versa?
(c) Give an economic interpretation of the dual problem.

2. Read the handout for the Sep 26 lecture on solving systems of inequalities. Show how to adapt the method described there to find a solution to a system of inequalities $Ax \le b$, where only some of the variables are required to be non-negative. What is the certificate of infeasibility? Illustrate your method on the following systems:
System 1:

$$-x_1 - x_2 + x_3 \le -2$$

$$x_1 + 2x_2 - 3x_3 \le 5$$

$$2x_1 + 2x_2 - x_3 \le -6$$

$$x_1 \ge 0$$

System 2:

$$-x_1 - x_2 + x_3 \le -1$$

$$x_1 + 2x_2 - 3x_3 \le 5$$

$$4x_1 + 2x_2 - x_3 \le -2$$

$$x_1, x_3 \ge 0$$

3. Solve the LP for the bicycle problem (P. 12, 1.9) using lpsolve (or another package) for $n = 4$, $distance = 90$, $w_1 = w_2 = 13$, $b_1 = b_2 = 27$, $w_3 = w_4 = 3$, $b_3 = b_4 = 18$. Write down the dual, and give the dual optimum solution. Also verify the complementary slackness conditions. (Replace the equation by two inequalities.) Show that there is no schedule that achieves this finish time, given the bicycle changes hands only a finite number of times. Hint: First show the optimum solution is unique, then concentrate on the bicycle.