1. A manufacturer of animal feed wants to produce a feed mix of minimum cost. The feed mix contains three active ingredients and a filler to provide bulk. One kg of feed mix must contain a minimum quantity of each of four nutrients as below:

Nutrient	A	B	C	D
gram	90	50	20	3

The ingredients have the following nutrient values and cost

Nutrient	A	B	C	D	Cost/kg
Ingredient 1 (gram/kg)	100	80	40	10	40
Ingredient 2 (gram/kg)	200	150	20	-	60
Ingredient 3 (gram/kg)	20	100	20	1	50

Formulate this as an LP, and find the minimum cost solution. You may use a package such as lp_solve, Maple etc.

2. Solve the following LP by hand using the two phase simplex method and Dantzig's rule. (Choose entering variable with largest coefficient. If there is a tie for leaving variable, choose the variable with smallest index.)

$$\max 6x_1 + 7x_2 + 2x_3$$

$$-x_1 - x_2 + x_3 \leq -2$$

$$x_1 + 2x_2 - 3x_3 \leq 5$$

$$2x_1 + 2x_2 - x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0$$

For each pivot, list the entering and leaving variable. You need not write out each dictionary, but give the final dictionary in phase 1, and the final dictionary in phase 2. You may use maple or some other system to solve the systems of equations.

3. Consider an LP problem in standard form where $b_i = 0, i = 1, \ldots, m$. Show that it is impossible that there are two dictionaries D and D' for this system with the following properties.

(1) In D the variable x_{n+m} is cobasic, and in the objective row all coefficients are non-positive except for the coefficient of x_{n+m}, which is positive.

(2) In D' the variable x_{n+m} is basic. There is a cobasic variable x_j such that it has positive coefficient in the objective row, and all the coefficients in this column are non-negative, except for the coefficient in the row for x_{n+m} which is negative. (Ie. if we pivot x_j into the basis, we must pivot out x_{n+m}).