

COMP566

Homework 1

Due: Tues September 21, 2004

1. Design a version of the simplex method using dictionaries, that works directly with problems written in the standard form:

$$\min c^T x, \quad Ax \geq b, \quad x \geq 0.$$

Note: Do not simply transform this to the standard form used in class (and in the text). Prove that for your algorithm the objective function can not **increase** at any iteration.

2. Work through the simplex method using Dantzig's rule (maximum positive cost coefficient, ties broken by minimum index) on the following example:

$$\max 5x_1 + 3x_2 + 4x_3$$

$$2x_1 + x_2 + x_3 \leq 20$$

$$3x_1 - x_2 + 2x_3 \leq 30$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

You may use maple or some other package to solve systems of equations. Show the dictionary for each iteration.

3. Suppose you have a problem in standard form (m constraints and n variables) and a proposed optimum solution x^* for which exactly m variables are non-zero. Explain how you would adapt the dictionary method to test if this is an optimum solution, and if not, to find an optimum solution. Illustrate by considering the proposed solution $x^* = (0, 10/3, 50/3)$ for the LP in exercise 2. What difficulties would you encounter (if any) for the cases when the proposed solution has either less than or more than m non-zero variables.

4. There are 500 acres of farmland used to grow corn, wheat, soybeans, and oats. On average each acre of yields 110, 35, 32 and 55 bushels of the respective crop. No more than 120 acres of soybeans can be planted, and at least 10,000 bushels of corn are required. The total acreage of wheat should equal or exceed the total combined acreage of oats and soybeans. A bushel of each crop earns respectively 36, 90, 82 and 98 cents. Formulate the problem of maximizing earnings as an LP in standard form. Solve the problem using `lp_solve`. Suppose that the farmland is divided into 50 pieces of 10 acres each. Each piece can be planted with at most one crop. Formulate and solve this problem using `lp_solve`.

Policy on Late Assignments: -10% per day, including weekends. Assignments are due in class. Hand in late assignments to Bohdan Kaluzny, MC232.