Question 1: This question was very well done in general. I accepted any problem of the following form:

	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	Supply
Resource 1	a1,1	a1,2									b1
Resource 2	a2,1										b2
Resource 3				…	…						b3
Resource 4											b4
Resource 5										a5,10	b5
Profit	p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	

Objective function:

Maximize: $p1 * x1 + p2 * x2 + \ldots + p10 * x10$

Subject to:

$a1,1 * x1 + a1,2 * x2 + \ldots + a1,10 * x10 \leq b1$

…

$a5,1 * x1 + a5,2 * x2 + \ldots + a5,10 * x10 \leq b5$

$x1, x2, \ldots, x10 \geq 0$

where the a's and b's are any real numbers, and the x's represent the number of units of each product to produce.

The question is out of 9 points:

(a) (3 points) I deducted 0.5 point if you didn't include the non negativity constraints (so 1 point in total if you made this mistake for both the primal and dual). Also I deducted 1 point if you had some constraints with strict inequalities (those are disallowed in LP). I also made small deductions if you provided no interpretation of your problem whatsoever.

(b) (3 points) As long as you showed me that you had been able to use lp_solve to solve your problem, you got full marks here. If your optimal values for the primal and dual were not equal and you didn't include integrality constraints, then your solution had to be wrong.

(c) (3 points) The usual technique was to obtain the upper bound by multiplying each primal constraint by the corresponding y values given by the dual solution. I also accepted Strong Duality as a proof of optimality. As for an interpretation to the dual variables (not required), yi can be seen as the value that the corresponding resource i has to us. With each extra unit of resource i, the profit increases by yi units. Therefore yi can be thought of as being the maximum amount that we should be willing to pay to acquire an extra unit of resource i.

Question 2: This question is out of 9 points (5 for the formulation and 4 for the solution).

There were 2 common mistakes.

Mistake 1: Some people could be left unassigned.

For example, consider the following matrix of preferences (cij in row i, column j)

	Person 1	Person 2	Person 3	Person 4
Person 1		10		1
Person 2	10		1	
Person 3		1		
Person 4	1			

Then many students defined xij for all i<j, and defined their set of inequalities as follows:
$x12 + x14 \leq 1$
$x12 + x23 \leq 1$
$x23 \leq 1$
$x14 \leq 1$
i.e. for every person, the sum of the variables involving that person should not exceed 1.

To maximize cij*xij, the solution to this would be to put x12=1 and all other variables to 0. But this leaves person 3 and person 4 unassigned! Or at least, no variable tells us to put them together. If we generalize this idea to a problem with more than 4 people, we could get a situation where some even number of people are not assigned, and we have an incomplete solution. We need to use equalities instead of inequalities to make sure we'll get a complete matching.

Mistake 2: Formulation may not maximize total preference

Consider these preferences:

	Person 1	Person 2	Person 3	Person 4
Person 1		10	9	1
Person 2	1			1
Person 3	9			
Person 4			1	

Some students defined xij for all possible (i,j) pairs, but then formulated constraints that disallowed xij and xji to both be 1. For example:
$x12 + x13 + x14 + x21 + x31 \leq 1$
$x21 + x24 + x12 \leq 1$
$x31 + x13 \leq 1$
$x43 + x14 + x24 \leq 1$

Then you may not get an optimal solution. For the preferences above, setting x12 = x43 = 1 (thus forcing x21 = x34 = 0) would maximize cij*xij to a value of 11. But observe that matching person 1 with person 3, and person 2 with person 4, and letting

$x13=x31=x24=x42=1$, would be a better solution (satisfaction=$c13 + c31 + c24 + c42 = 19$).

Of course you could get lucky and get the optimal solution even if you make these two mistakes, but that doesn't mean the formulation is not flawed. I usually gave full marks for the solution (4 points) if you were able to get a valid (and in most cases, probably optimal) one using lp_solve, but I deducted formulation points if you made any of the two mistakes explained above (1 point each). I usually deducted 1 point if you made some modification to the problem (for example if you assumed a symmetric preference matrix, which is a bit of an easier problem). I also deducted 0.5 or 1 for other errors, for example with the bounds or with integrality.

Here is a formulation that works:
Variables: xij = 1 if i is matched with person j, and 0 if not (defined for $1 \le i < j \le n$)
Known: cij = satisfaction that person i has of being matched with person j ($1 \le cij \le 10$; defined for $1 \le i, j \le n$)

Objective function:

$$\text{Maximize} \sum_{i=1}^{15} \sum_{j=i+1}^{16} (cij + cji) * xij$$

Subject to:

$0 \le xij \le 1$ for $1 \le i < j \le n$

xij integer for $1 \le i < j \le n$

$$\sum_{j=1}^{i-1} xji + \sum_{j=i+1}^{n} xij = 1 \qquad \text{for all } 1 \le i \le 16$$

(i.e. for each i, sum over all variables where i appears must be exactly 1)

Note that this will allow matchings where two people could be matched together even though neither of them included the other person in their preference list. It just assumes a satisfaction of 0 in these cases. This was one acceptable interpretation of the problem. Notice that for this interpretation, the problem can never be infeasible.

Another valid interpretation is that we only allow matchings where at least one of two people matched together is willing to room with the other. In this case, the formulation is the same but we restrict everything to the appropriate subset of variables (and we accept the fact that we might get infeasibility of the problem).

Question 3: This question is out of 6 points.

(a) (3 points)
Here you have to provide a feasible solution x, and a direction z such that $z \geq 0$, $Az \leq 0$, $c^T z > 0$. One possible certificate is x=(0,0,0) and z=(0,1,2).

Some people forgot to provide a feasible solution x, I deducted 1 point for that. Other deductions depend on whether your direction z was ok, and on the general demonstration of the correctness of your certificate.

One mistake that some people made is to prove that the dual is infeasible. This doesn't prove that the primal is unbounded, because both the primal and dual can be infeasible simultaneously. For example this is the case for the following problem:
Max $2*x1 – x2$ subject to $x1-x2 \leq 1$, $-x1+x2 \leq -2$, $x1 \geq 0$, $x2 \geq 0$

(b) (3 points)
Here you have to find some y=(y1,y2,y3) such that $y \geq 0$, $A^T y \geq 0$, $b^T y < 0$.

Check that (2,1,1) satisfies those conditions. It can be found using the b-rule algorithm (see paper by D. Avis and B. Kaluzny). I will not run it here because almost everybody got this.