

8. The residents of the underground city of Zion defend themselves through a combination of kung fu, heavy artillery, and efficient algorithms. Recently they have become interested in automated methods that can help fend off attacks by swarms of robots.

Here's what one of these robot attacks looks like.

- A swarm of robots arrives over the course of n seconds; in the i^{th} second, x_i robots arrive. Based on remote sensing data, you know this sequence x_1, x_2, \dots, x_n in advance.
- You have at your disposal an *electromagnetic pulse* (EMP), which can destroy some of the robots as they arrive; the EMP's power depends on how long it's been allowed to charge up. To make this precise, there is a function $f(\cdot)$ so that if j seconds have passed since the EMP was last used, then it is capable of destroying up to $f(j)$ robots.
- So specifically, if it is used in the k^{th} second, and it has been j seconds since it was previously used, then it will destroy $\min(x_k, f(j))$ robots. (After this use, it will be completely drained.)
- We will also assume that the EMP starts off completely drained, so if it is used for the first time in the j^{th} second, then it is capable of destroying up to $f(j)$ robots.

The problem. Given the data on robot arrivals x_1, x_2, \dots, x_n , and given the recharging function $f(\cdot)$, choose the points in time at which you're going to activate the EMP so as to destroy as many robots as possible.

Example. Suppose $n = 4$, and the values of x_i and $f(i)$ are given by the following table.

i	1	2	3	4
x_i	1	10	10	1
$f(i)$	1	2	4	8

The best solution would be to activate the EMP in the 3rd and the 4th seconds. In the 3rd second, the EMP has gotten to charge for 3 seconds, and so it destroys $\min(10, 4) = 4$ robots; In the 4th second, the EMP has only gotten to charge for 1 second since its last use, and it destroys $\min(1, 1) = 1$ robot. This is a total of 5.

- (a) Show that the following algorithm does not correctly solve this problem, by giving an instance on which it does not return the correct answer.

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Schedule-EMP( $x_1, \dots, x_n$ )
  Let  $j$  be the smallest number for which  $f(j) \geq x_n$ 
  (If no such  $j$  exists then set  $j = n$ )
  Activate the EMP in the  $n^{\text{th}}$  second
  If  $n - j \geq 1$  then
    Continue recursively on the input  $x_1, \dots, x_{n-j}$ 
    (i.e., invoke Schedule-EMP( $x_1, \dots, x_{n-j}$ ))

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In your example, say what the correct answer is and also what the algorithm above finds.

- (b) Give an efficient algorithm that takes the data on robot arrivals x_1, x_2, \dots, x_n , and the recharging function $f(\cdot)$, and returns the maximum number of robots that can be destroyed by a sequence of EMP activations.