How to Draw a Graph, Revisited

Peter Eades
University of Sydney

This talk

1. Review
    a) Graphs
    b) Planar graphs

2. How to draw a planar graph?
    a) Before Tutte: 1920s – 1950s
    b) Tutte: 1960s
    c) After Tutte: 1970s – 1990s
    d) Recent work: since 2000

1. Review
   (b) graphs

A *graph* consists of

- Nodes, and
- Binary relationships called “edges” between the nodes

Example: a “Linked-In” style social network

Nodes:

- Alice, Andrea, Annie, Amelia, Bob, Brian, Bernard, Boyle

Edges

- Bob is connected to Alice
- Bob is connected to Andrea
- Bob is connected to Amelia
- Brian is connected to Alice
- Brian is connected to Andrea
- Brian is connected to Amelia
- Boyle is connected to Alice
- Boyle is connected to Andrea
- Boyle is connected to Annie
- Bernard is connected to Alice
- Bernard is connected to Andrea
- Bernard is connected to Annie

Drawings of graphs

A *graph* consists of

- Nodes, and
- Binary relationships called “edges” between the nodes

A *graph drawing* is a picture of a graph

- That is, a graph drawing a mapping that assigns a location for each node, and a curve to each edge.

- That is, if G=(V,E) is a graph with node set V and edge set E, then a *drawing* p(G) consists of two mappings:

$p_V: V \rightarrow R^2$

$p_E: E \rightarrow C^2$

where R^2 is the plane and C^2 is the set of open Jordan curves in R^2

A social network

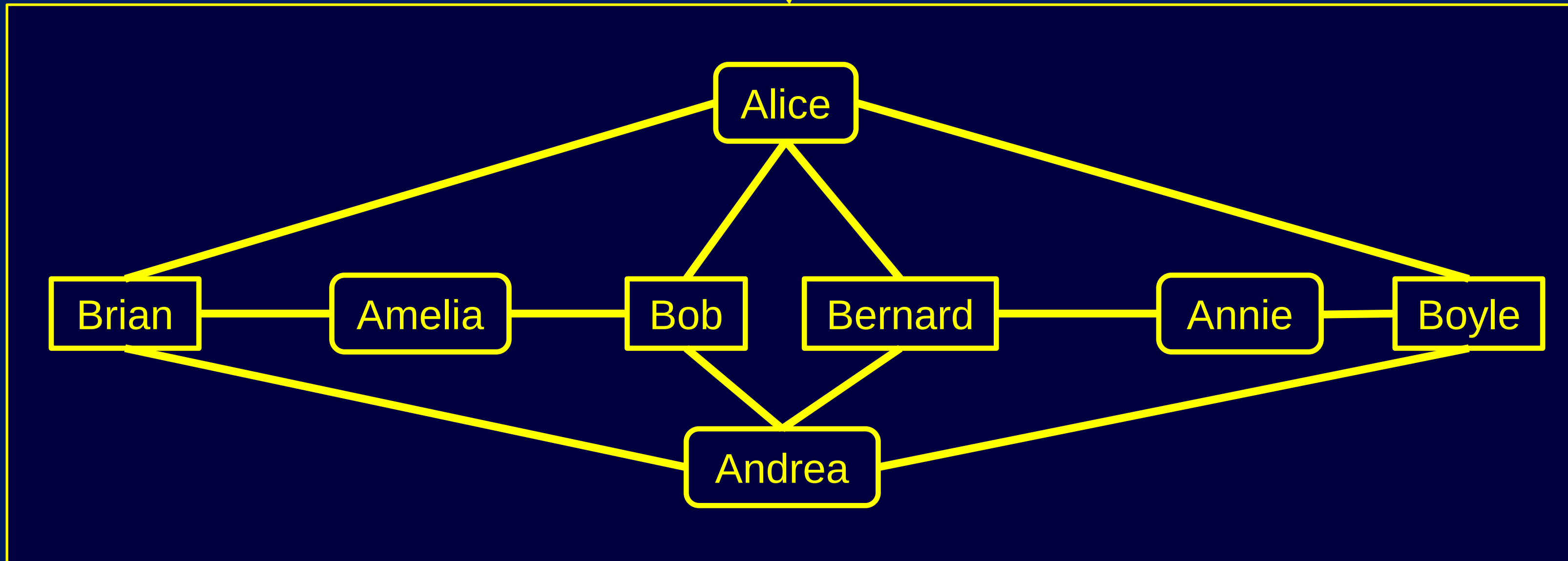
Nodes:

- Bob, Brian, Bernard, Boyle, Alice, Andrea, Annie, Amelia

Edges

- Bob is connected to Alice
- Bob is connected to Andrea
- Bob is connected to Amelia
- Brian is connected to Alice
- Brian is connected to Andrea
- Brian is connected to Amelia
- Boyle is connected to Alice
- Boyle is connected to Andrea
- Boyle is connected to Annie
- Bernard is connected to Alice
- Bernard is connected to Andrea
- Bernard is connected to Annie

A drawing of the social network



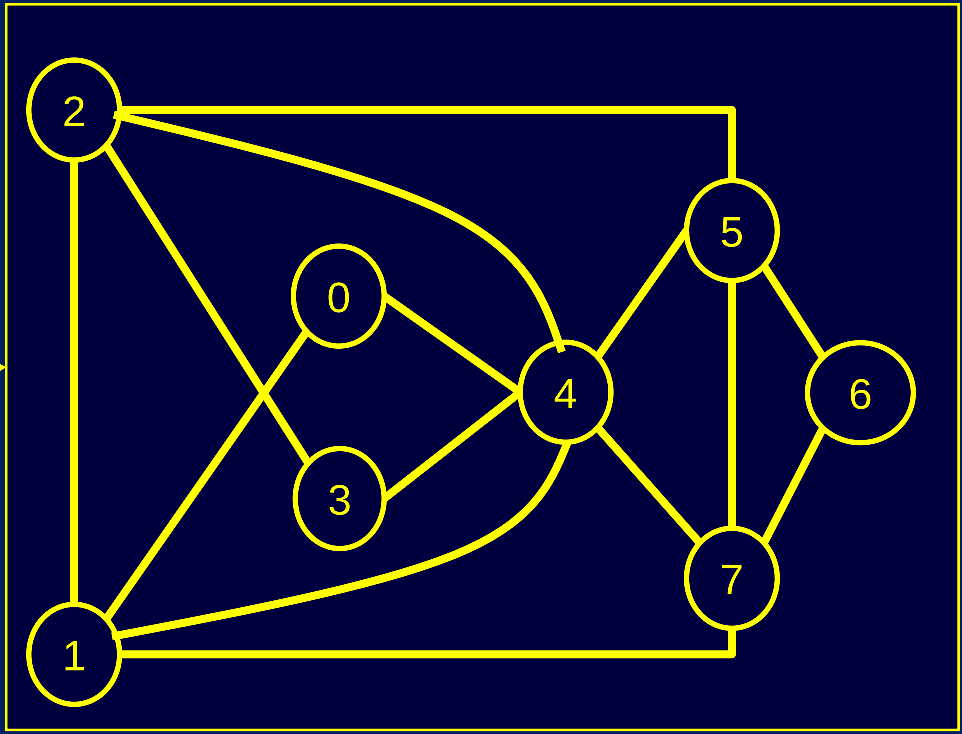
Nodes

0, 1, 2, 3, 4, 5, 6, 7

Edges

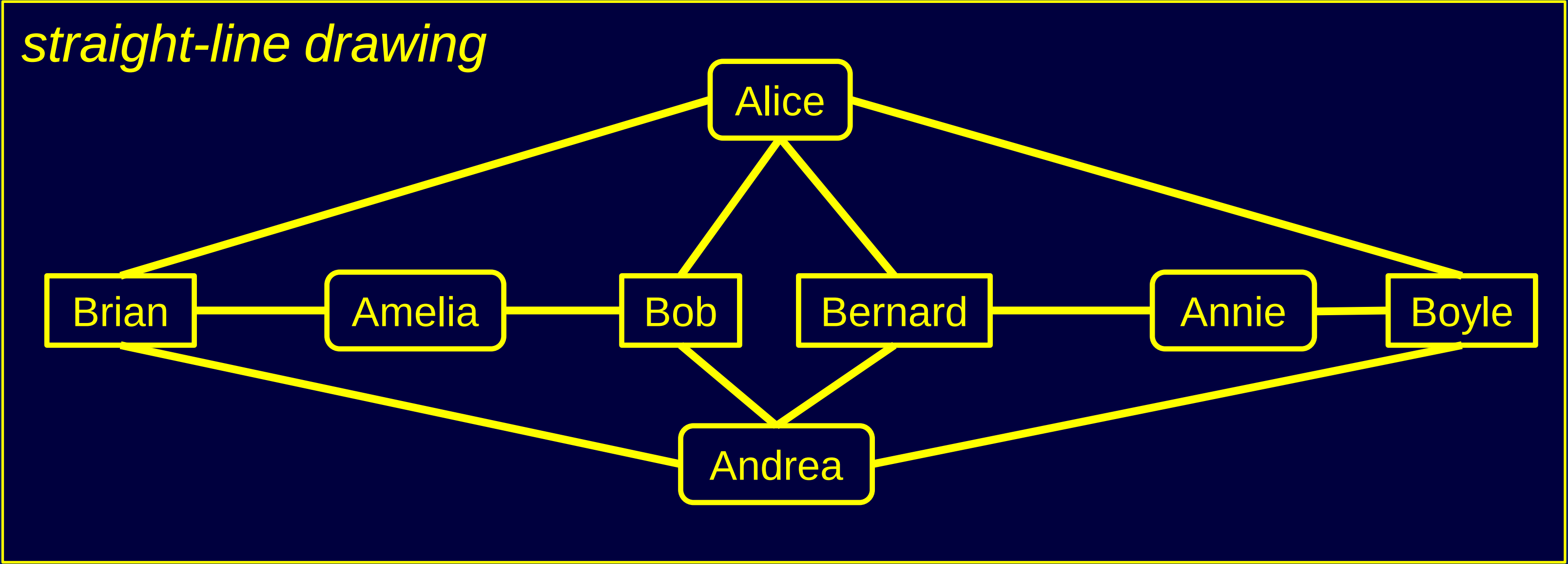
0 – 1
0 – 4
1 – 2
1 – 4
1 – 7
2 – 3
2 – 4
2 – 5
3 – 4
4 – 5
4 – 7
5 – 6
5 – 7
6 – 7

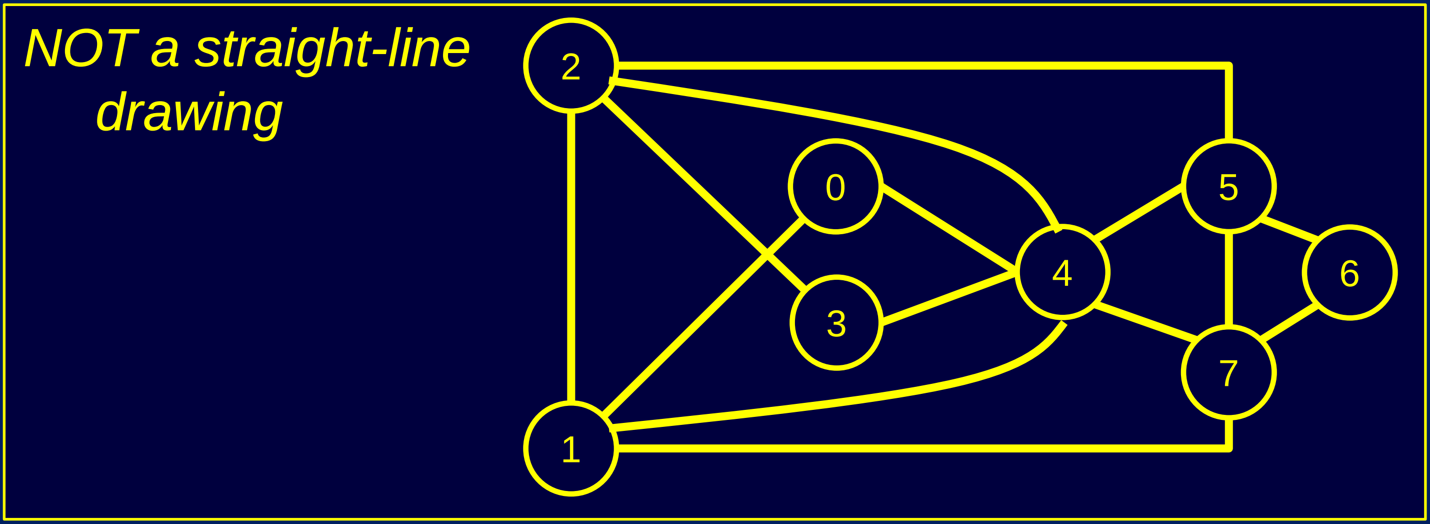
A graph



A drawing of the graph

A graph drawing is a *straight-line drawing* if every edge is a straight line segment.





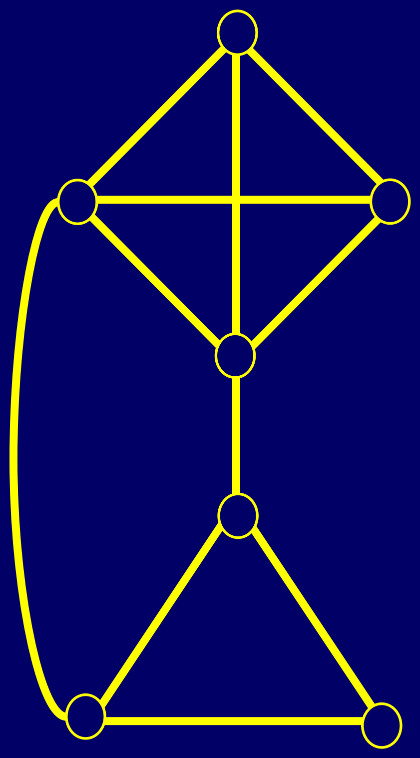
Connectivity of graphs

Connectivity notions are fundamental in any study of graphs or networks

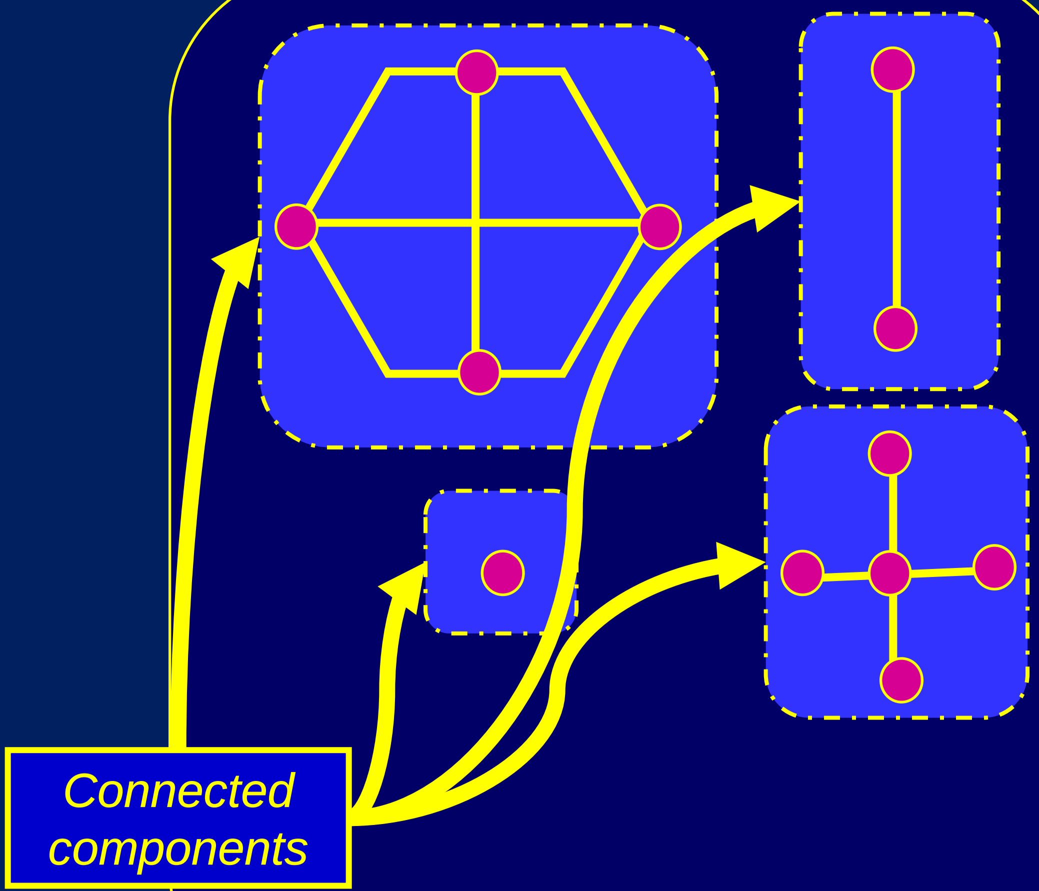
- A graph is *connected* if for every pair u, v of vertices, there is a path between u and v.

- A graph is *k-connected* if there is no set of (k-1) vertices whose deletion disconnects the graph.

  - k = 1: “1-connected” ≡ “connected”
  - k = 2: “2-connected” ≡ “*biconnected*”
  - k = 3: “3-connected” ≡ “*triconnected*”



This graph is connected



This graph is not connected

Connectivity notions are fundamental in any study of networks

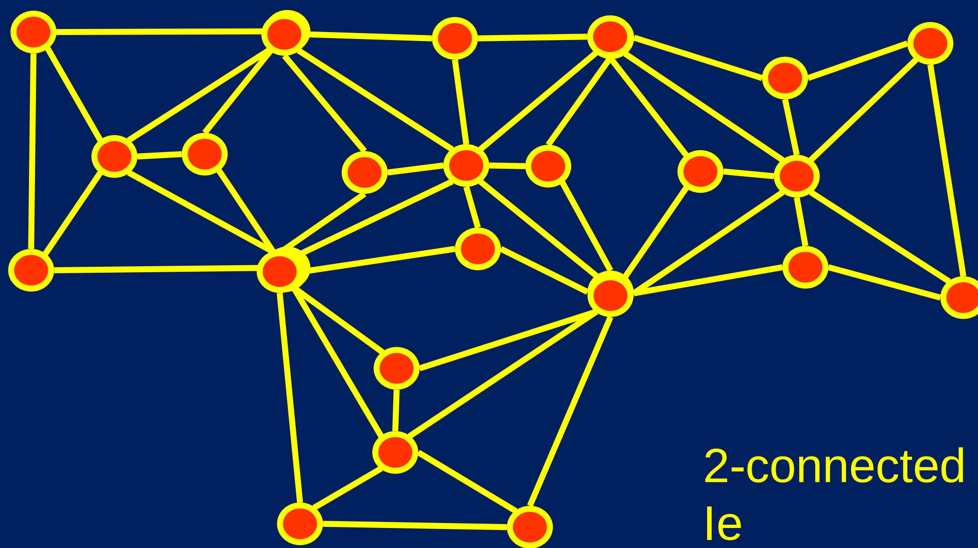
- A graph is *connected* if for every pair u, v of vertices, there is a path between u and v.

- A graph is *k-connected* if there is no set of (k-1) vertices whose deletion disconnects the graph.

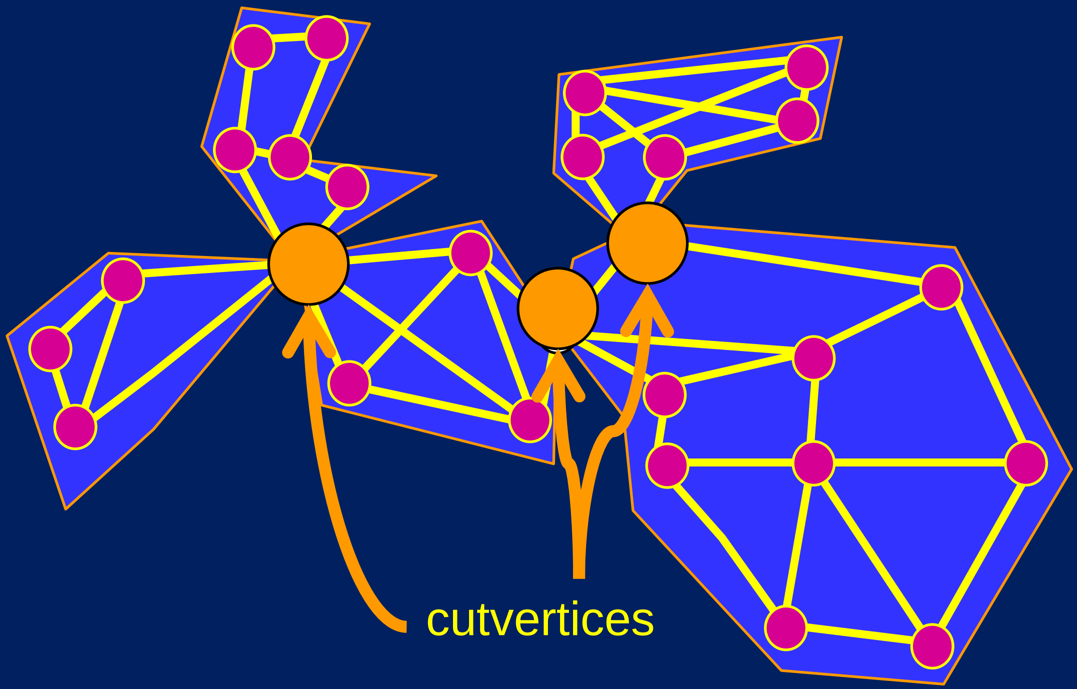
  - k = 1: "1-connected" ≡ "connected"
  - k = 2: "2-connected" ≡ "*biconnected*"
  - k = 3: "3-connected" ≡ "*triconnected*"

"2-connected" ≡ "*biconnected*"

- A *cutvertex* is a vertex whose removal would disconnect the graph.
- A graph without cutvertices is *biconnected*.



2-connected graph,
Ie
Biconnected graph

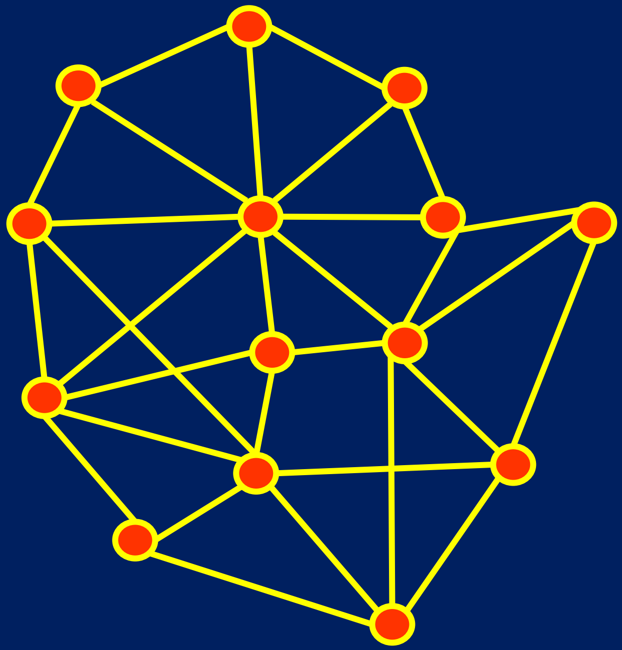


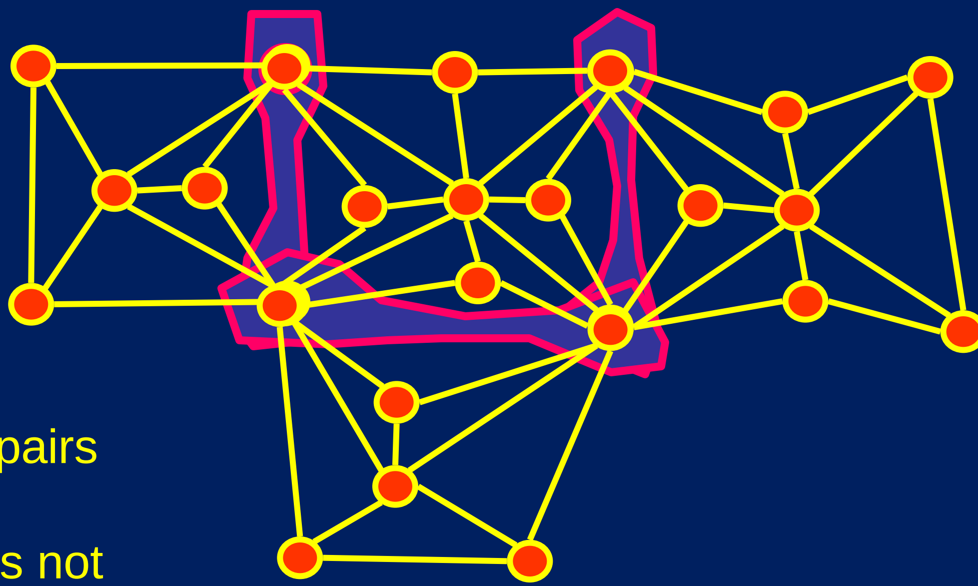
This graph is ***not***
biconnected

“3-connected” ≡ “*triconnected*”

- A *separation pair* is a pair of vertices whose removal would disconnect the graph.
- A graph without separation pairs is *triconnected*.

This graph is triconnected



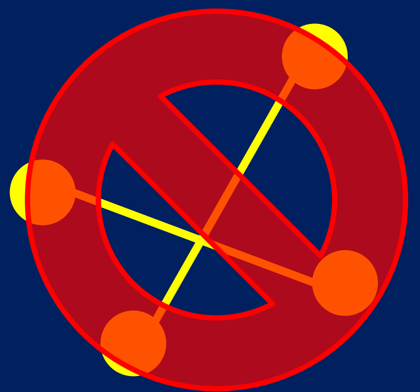


Separation pairs

This graph is not triconnected

1. (b) Review of Planar graphs

A graph is *planar* if it can be drawn without edge crossings.



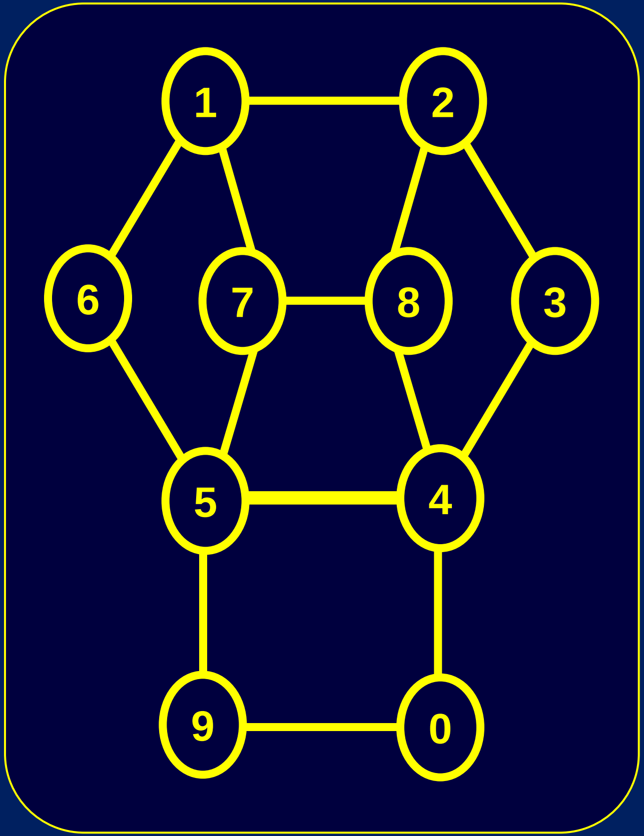
A graph is *planar* if it can be drawn without edge crossings.

Nodes:

- 0,1,2,3,4,5,6,7,8,9

Edges

- 0 – 4
- 0 – 9
- 1 – 2
- 1 – 6
- 1 – 7
- 2 – 3
- 2 – 8
- 3 – 4
- 4 – 5
- 4 – 8
- 5 – 6
- 5 – 7
- 7 – 8

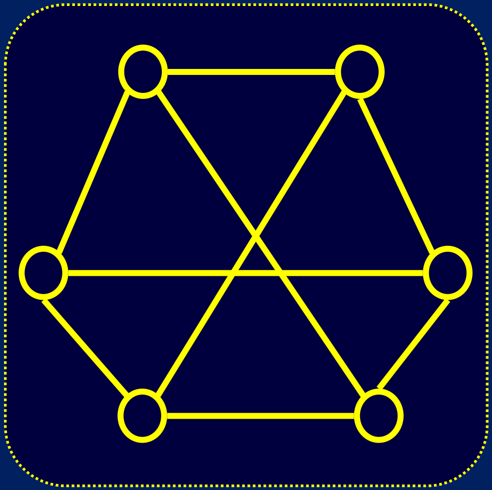


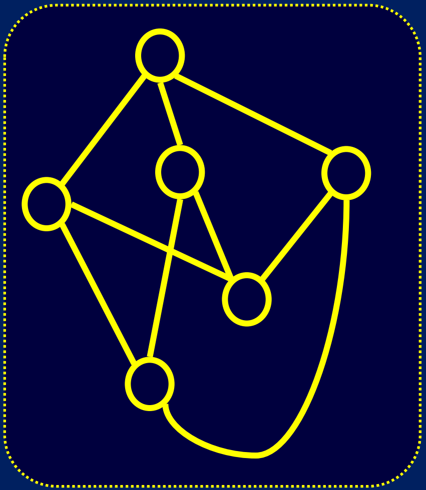
A graph is ***planar*** if it can be drawn without edge crossings.

Non-planar

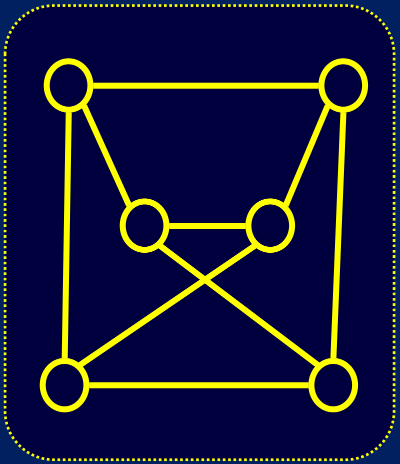
Nodes:
- 0,1,2,3,4,5

Edges
- 0 – 1
- 0 – 3
- 0 – 5
- 1 – 2
- 1 – 4
- 2 – 3
- 2 – 5
- 3 – 4
- 4 – 5



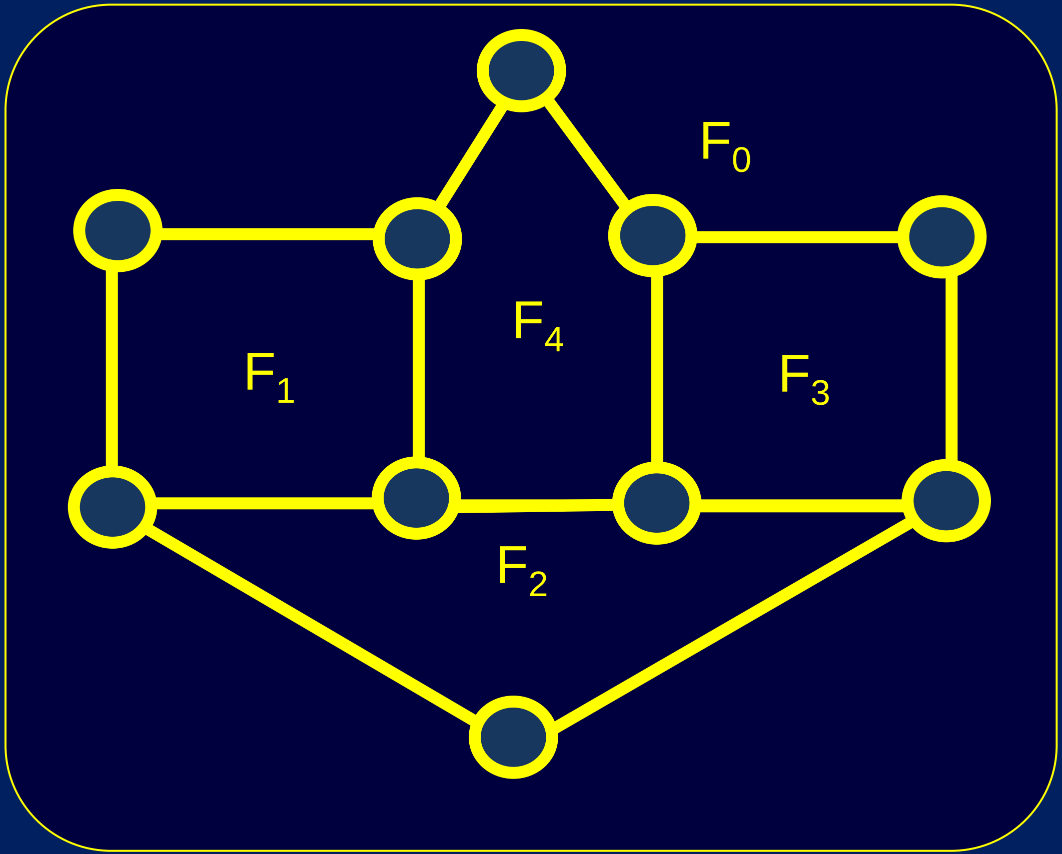


A graph is ***non-planar*** if ***every*** drawing has at least one edge crossing.



There is a lot of theory about planar graphs

A planar drawing divides the plane into *faces*.



F_0 shares a boundary with F_1
F_0 shares a boundary with F_2
F_0 shares a boundary with F_3
F_0 shares a boundary with F_4
F_1 shares a boundary with F_2
F_1 shares a boundary with F_4
F_2 shares a boundary with F_1
F_2 shares a boundary with F_3
F_2 shares a boundary with F_4
F_3 shares a boundary with F_4

The boundary-sharing relationships of the faces defines a *topological embedding* of the graph drawing

Euler formula

If

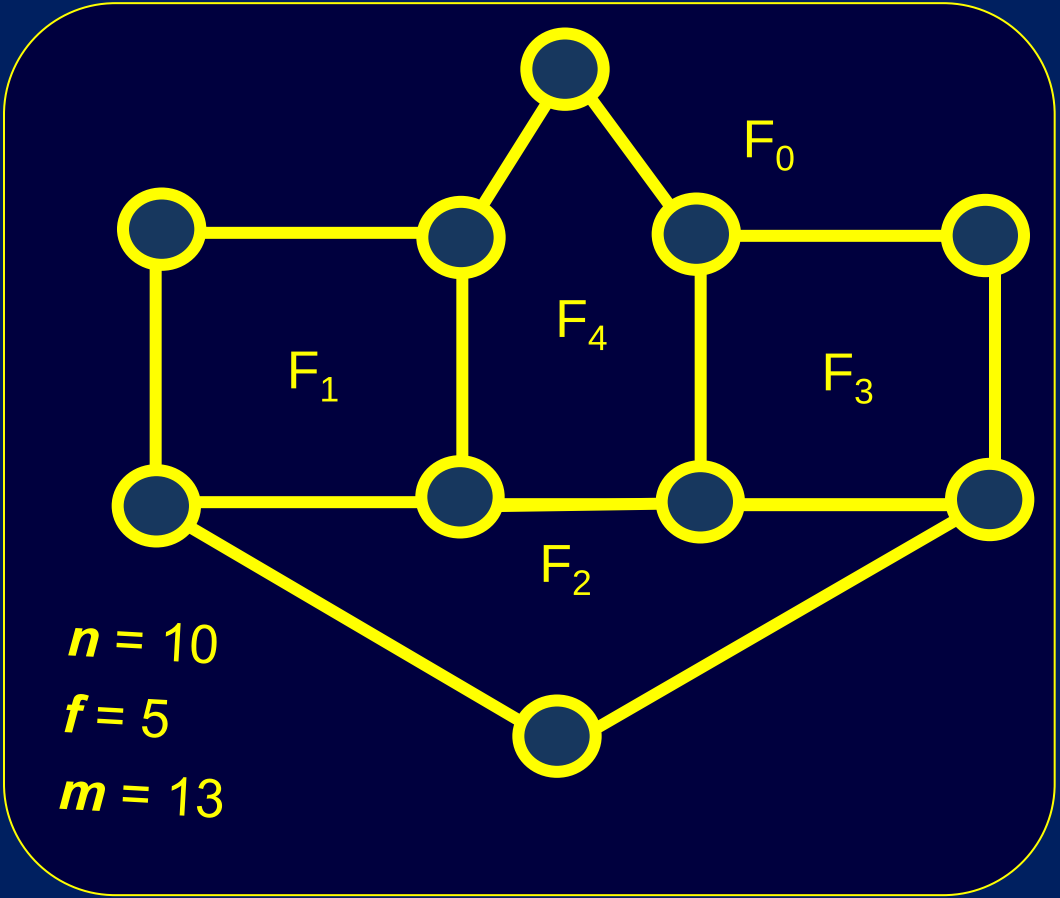
n = #vertices

f = #faces

m = #edges

then

$n+f = m+2$



Corollary $m \leq 3n-6$

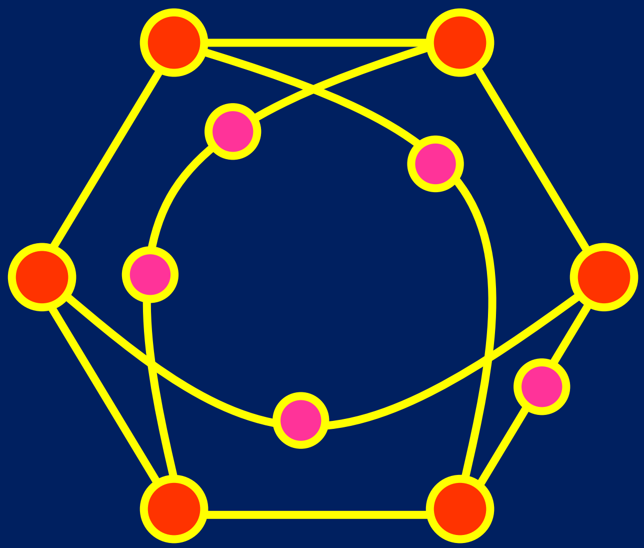
Corollary

If $m = 3n-6$ then every face is a triangle

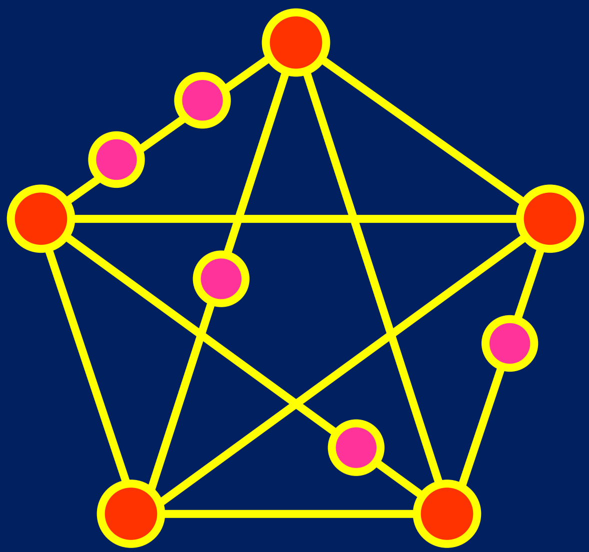
Kuratowski's Theorem (1930)

A graph is planar if and only if it does not contain a subgraph that is a subdivision of K_5 or $K_{3,3}$.

Forbidden subgraphs



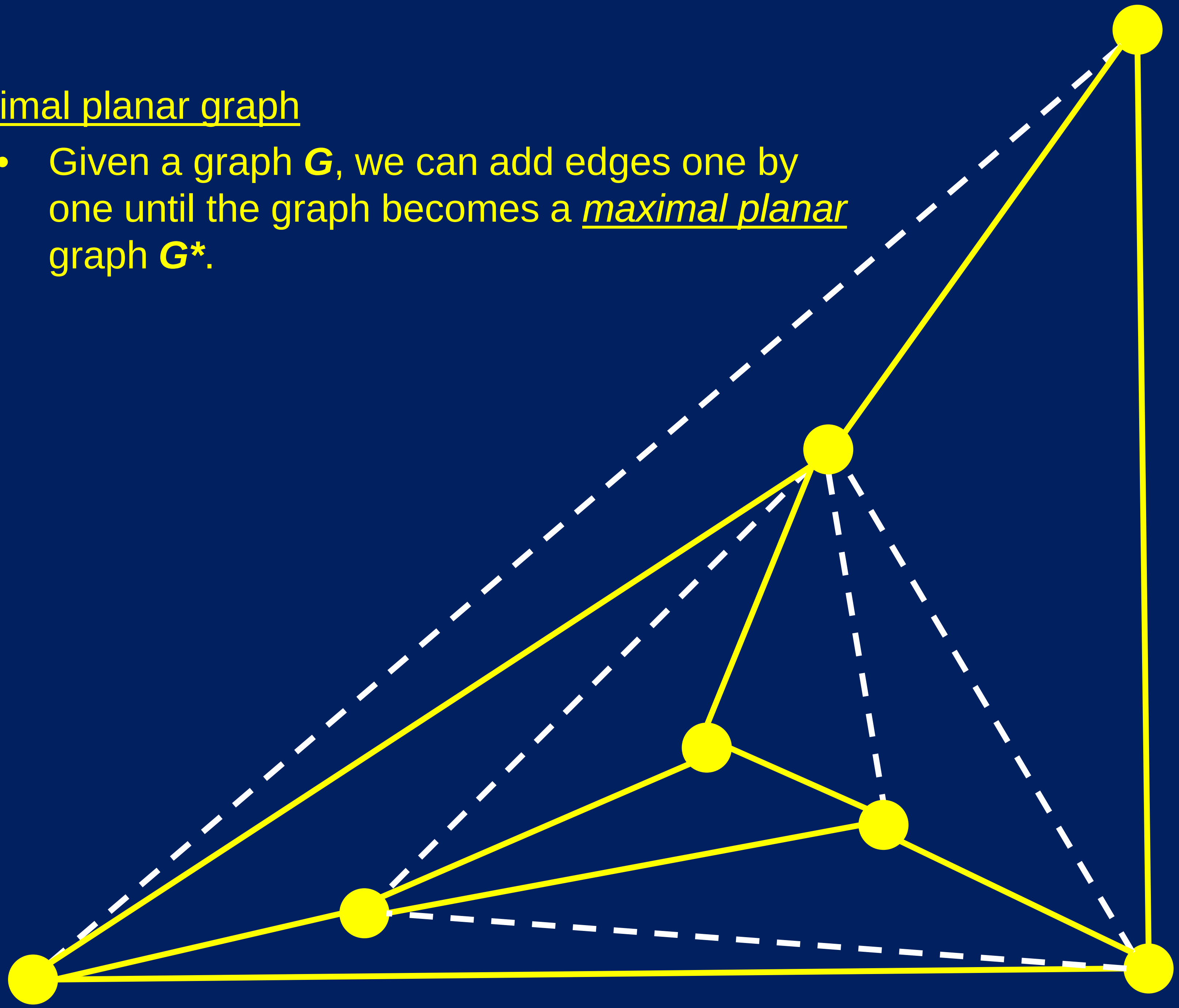
$K_{3,3}$



K_5

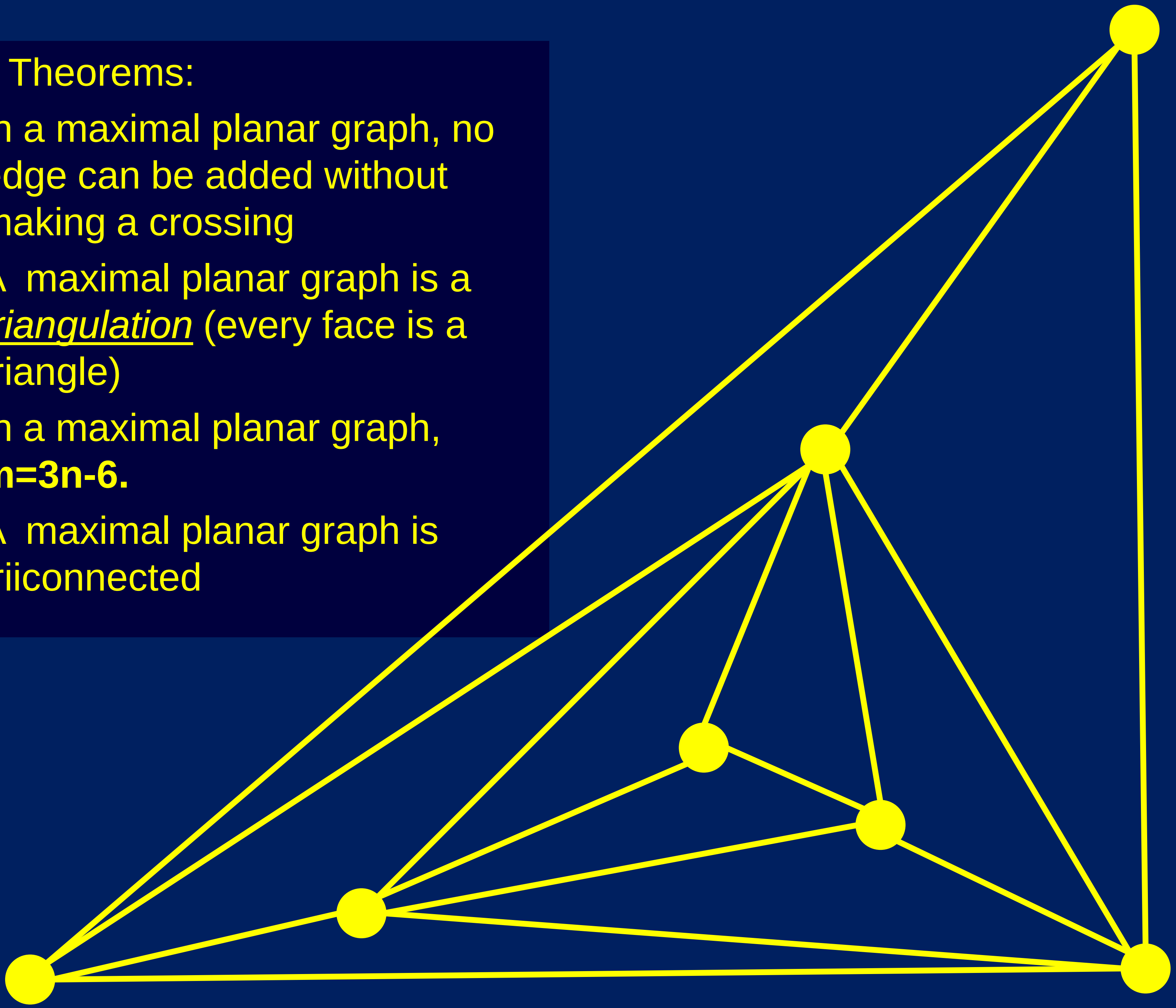
Maximal planar graph

- Given a graph ***G***, we can add edges one by one until the graph becomes a *maximal planar* graph ***G****.



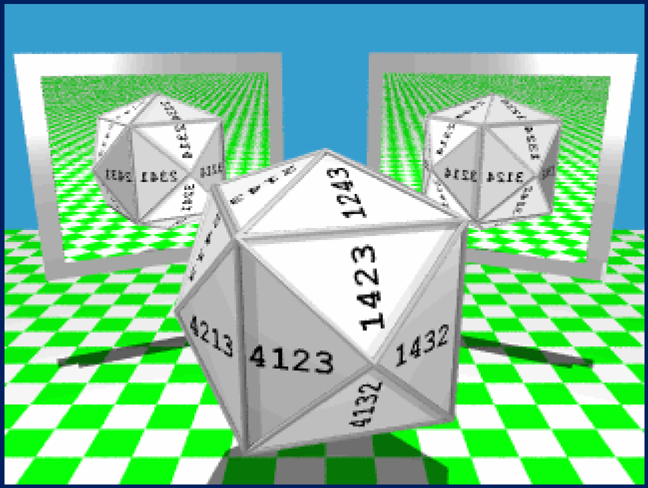
Easy Theorems:

- In a maximal planar graph, no edge can be added without making a crossing
- A maximal planar graph is a *triangulation* (every face is a triangle)
- In a maximal planar graph, **m=3n-6.**
- A maximal planar graph is triiconnected



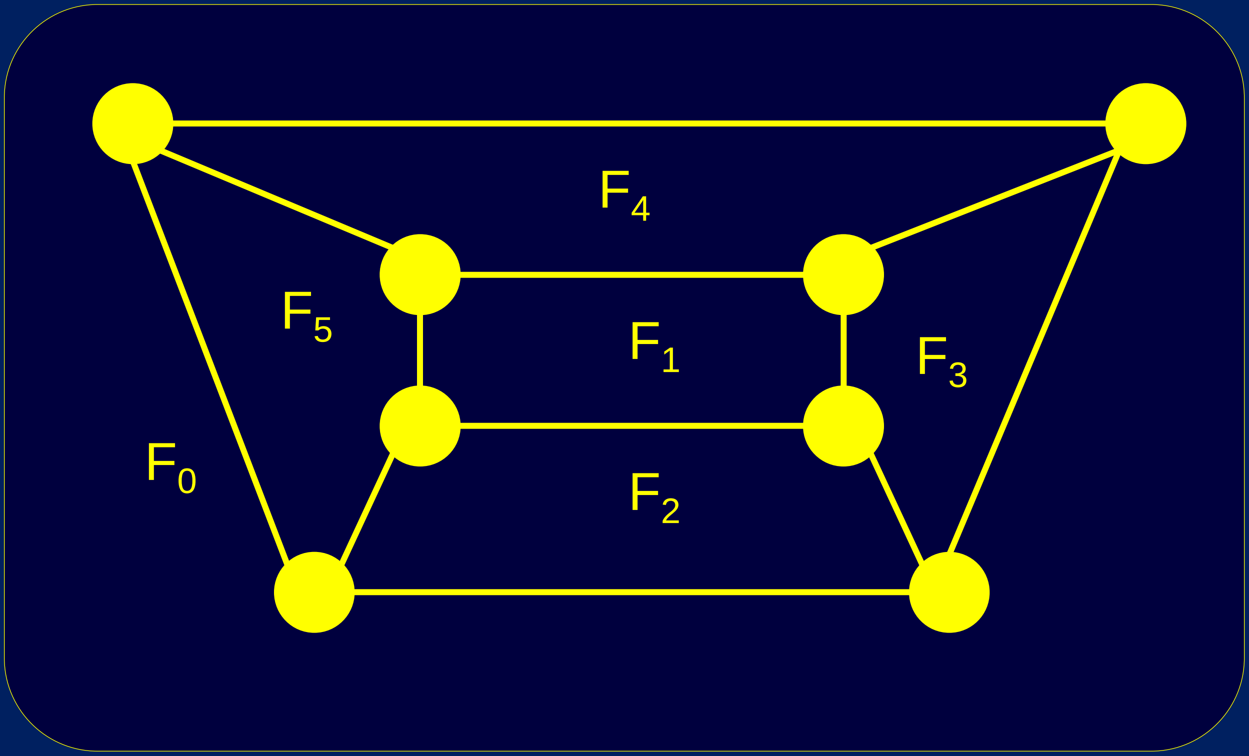
Steinitz Theorem (1922)

Every triconnected planar graph is the skeleton of a convex polyhedron



Whitney's Theorem (1933)

There is only one topological embedding of a triconnected planar graph (on the sphere).



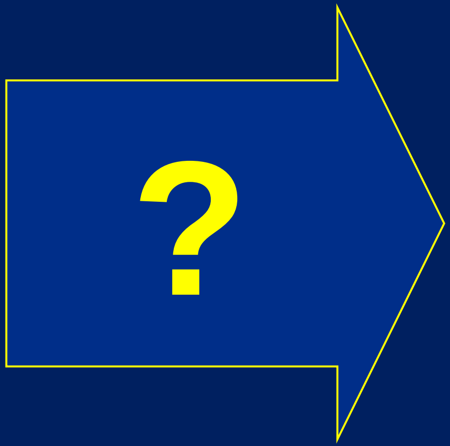
2. How to draw a planar graph

The classical *graph drawing* problem:

– ***How to draw a graph?***

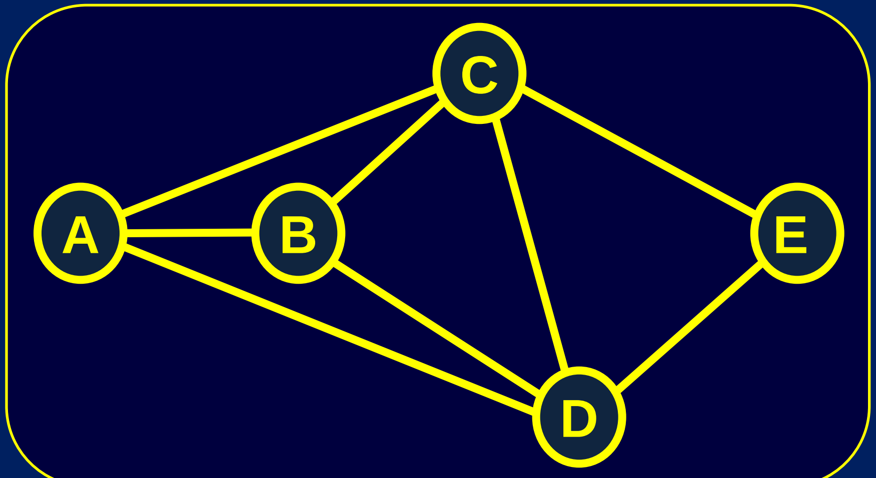
The input is a graph with no geometry

A - B, C, D
B - A, C, D
C - A, B, D, E
D - A, B, C, E
E - C, D



The output is a drawing of the graph;

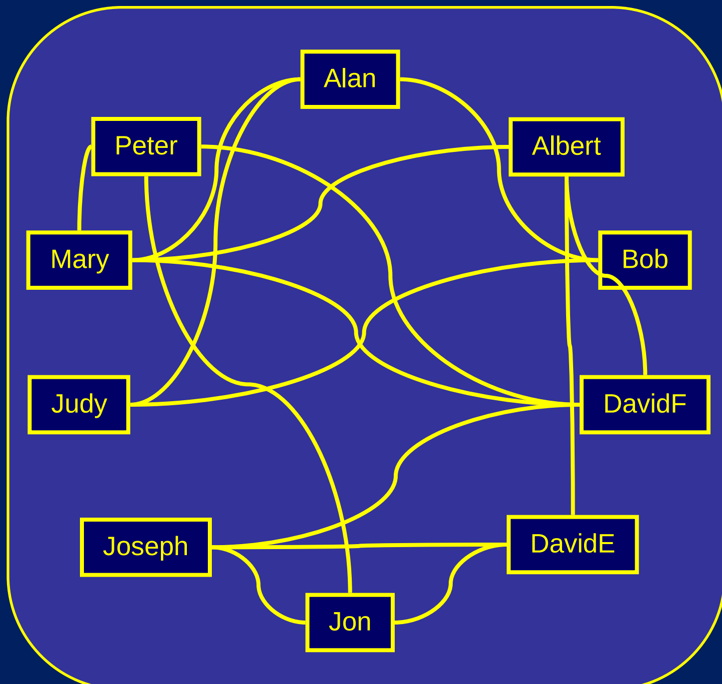
the drawing should be easy to understand, easy to remember, beautiful.



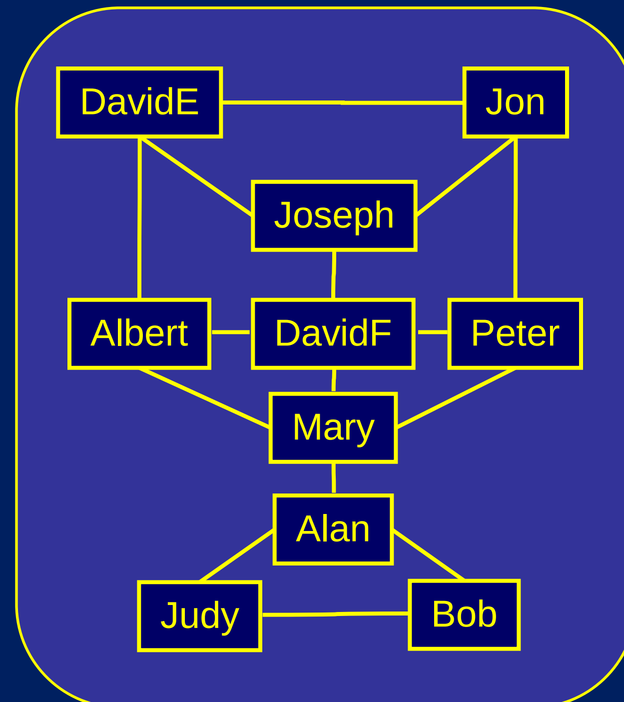
Question: What makes a *good* drawing of a graph?

Answer: Many things, including

- lack of edge crossings (planar drawings are good!)
- straightness of edges (straight-line drawings are good)



Bad picture



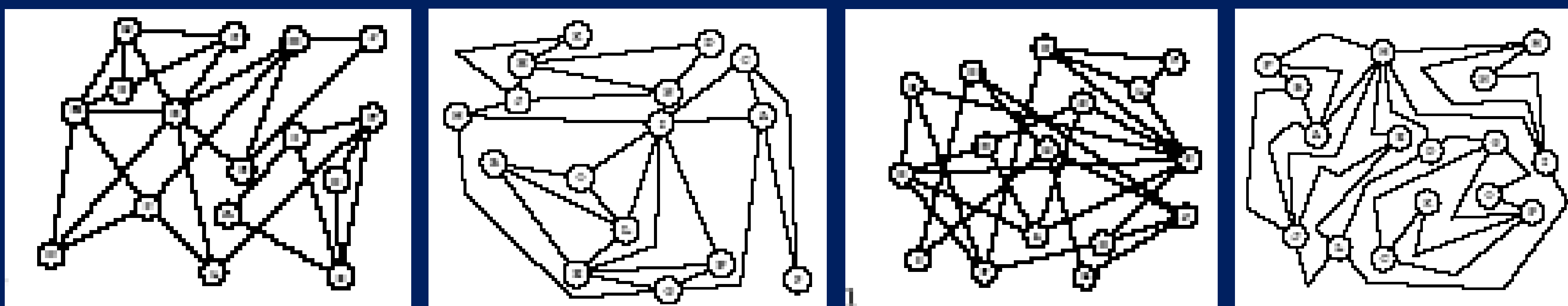
Good picture

__~1979 Intuition (Sugiyama et al.):__

– Planar straight-line drawings make good pictures

1997+: *Science confirms the intuition*

- Human experiments by Purchase and others

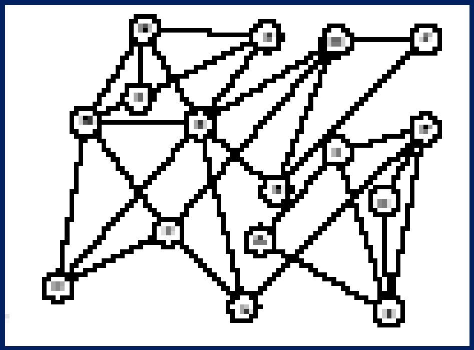


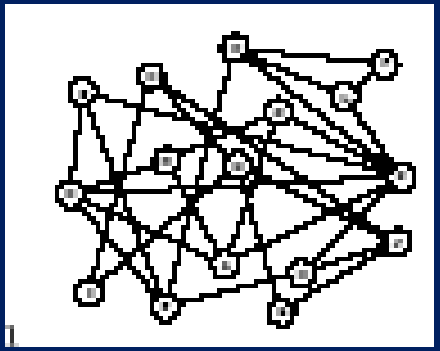
4 • H.C.Purchase, R.F.Cohen and M.I.James

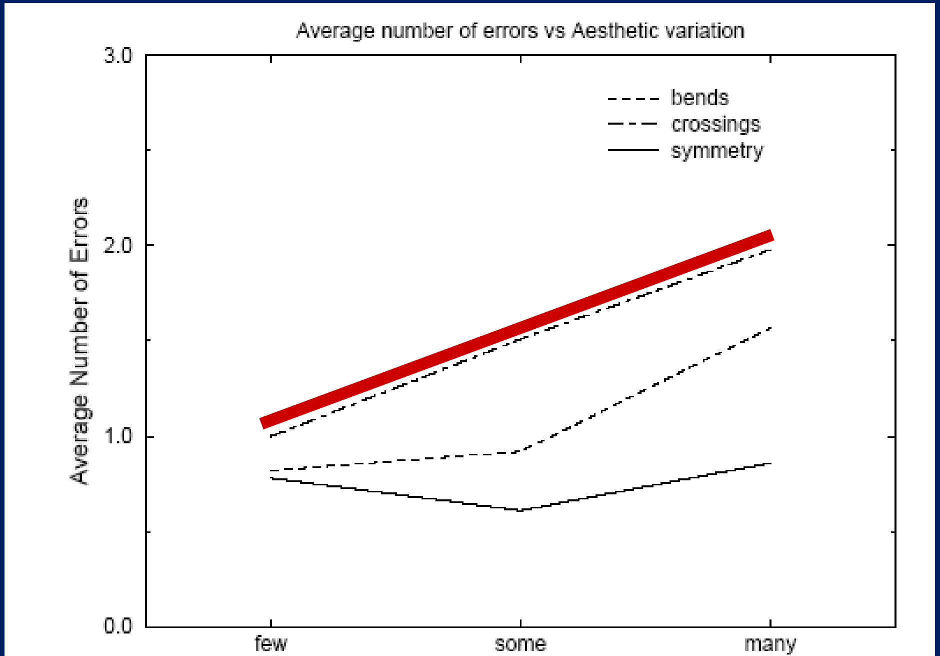
(1) How long is the shortest path between two given nodes?

(2) What is the minimum number of nodes that must be removed in order to disconnect two given nodes such that there is no path between them?

(3) What is the minimum number of arcs that must be removed in order to disconnect two given nodes such that there is no path between them?







bends	6	18	30
crossings	6	24	42
symmetry	4.6	25.7	51

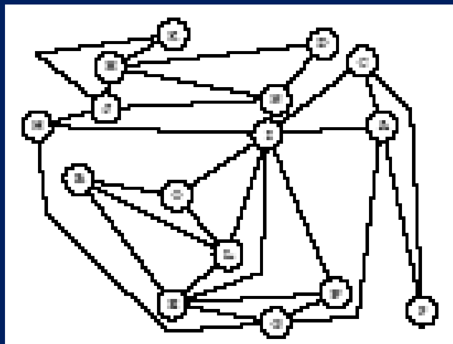
Aesthetic Variation

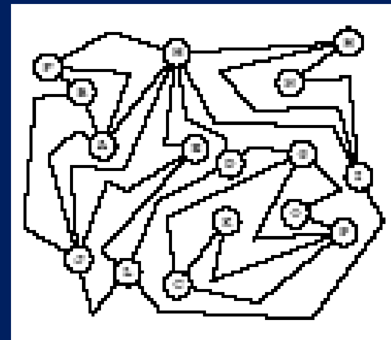
Fig. 3. Results for the dense graph

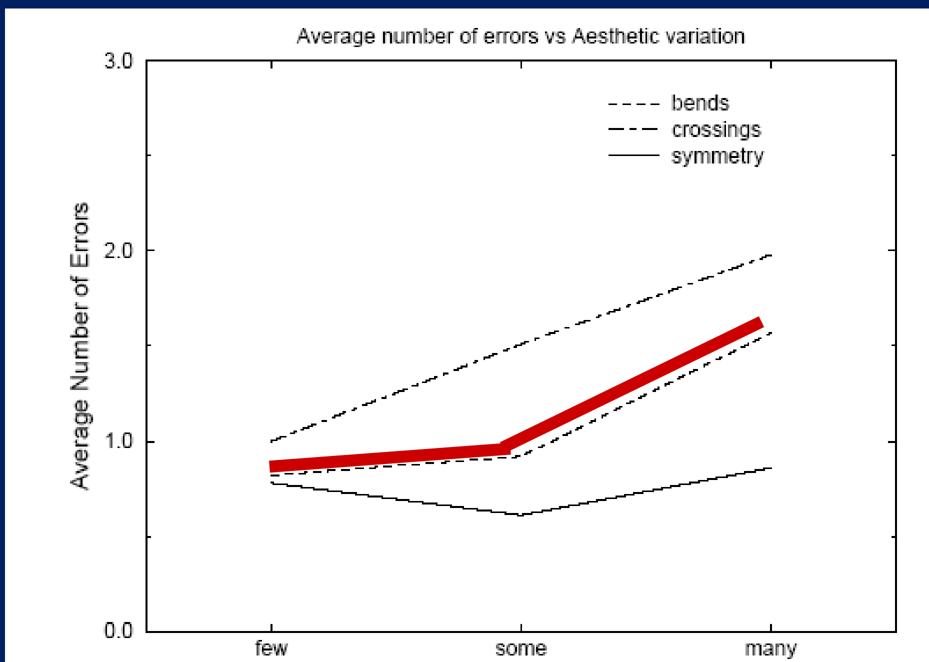
Purchase et al.,1997:

Significant correlation between *edge crossings* and human understanding

- *More edge crossings means more human errors in understanding*







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Aesthetic Variation

Fig. 3. Results for the dense graph

<u>Purchase et al., 1997</u>:

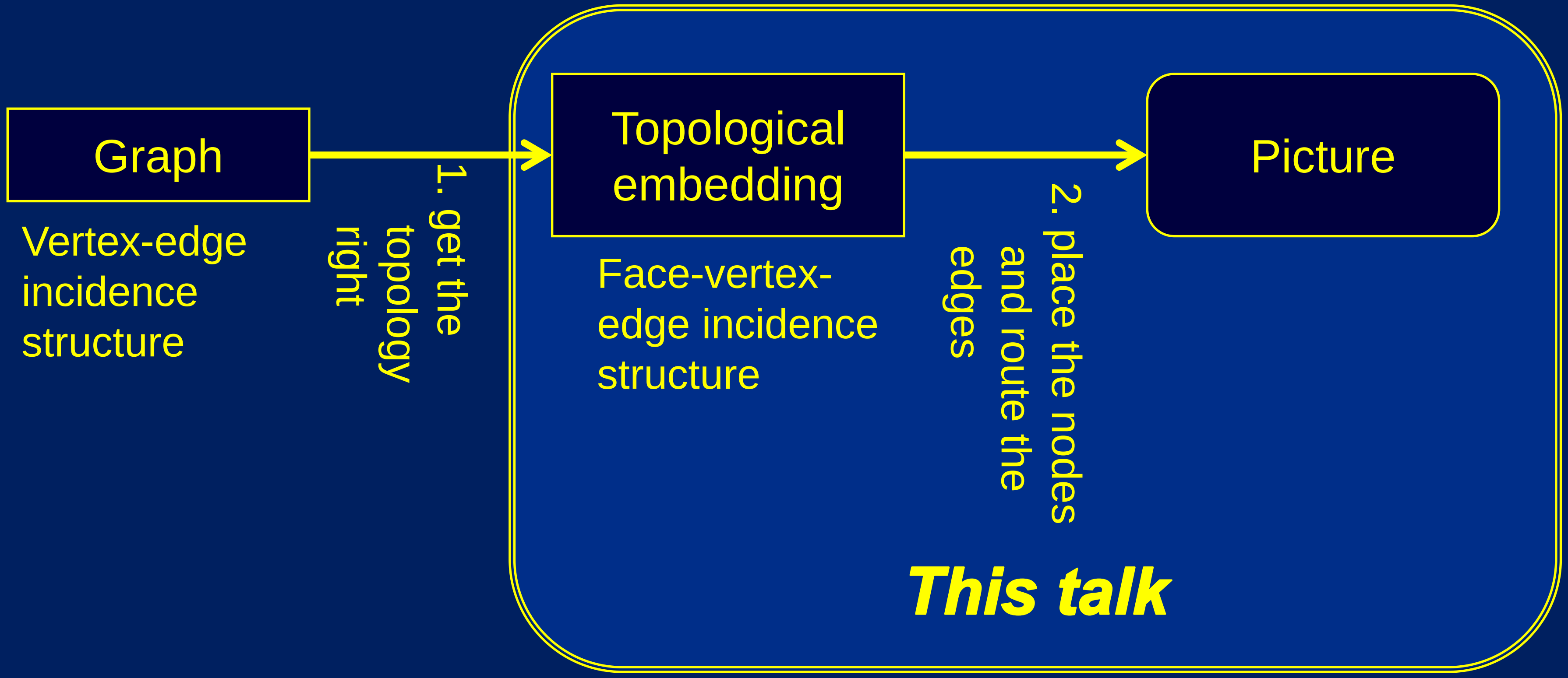
Significant correlation between <u>*straightness of edges*</u> and human understanding

- *More bends mean more human errors in understanding*

How to make a planar drawing of a planar graph?

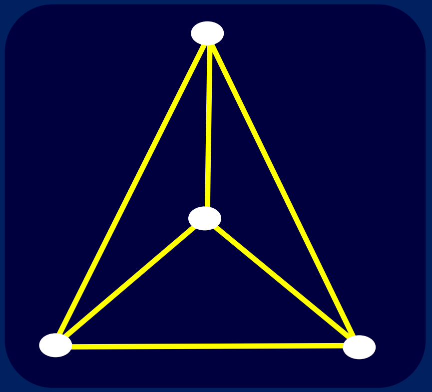
How to make a planar drawing of a planar graph:

1. Get the topology right
2. Place the nodes and route the edges

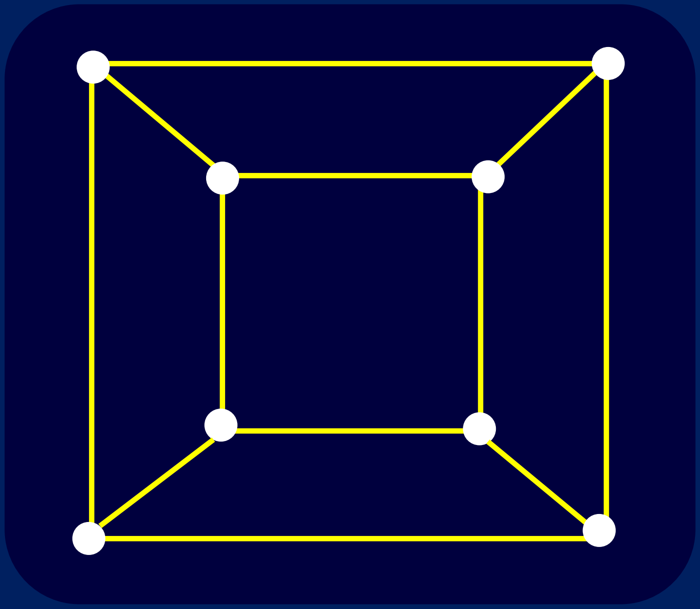


Straight-line drawings

- Each edge is a straight line segment



This talk is about planar straight-line drawings



Important note: a straight-line drawing of a graph G=(V,E) can be specified with a mapping

$p: V \rightarrow R^2$

that gives a position p(u) in R^2 for each vertex u in V.

2. How to draw a planar graph?

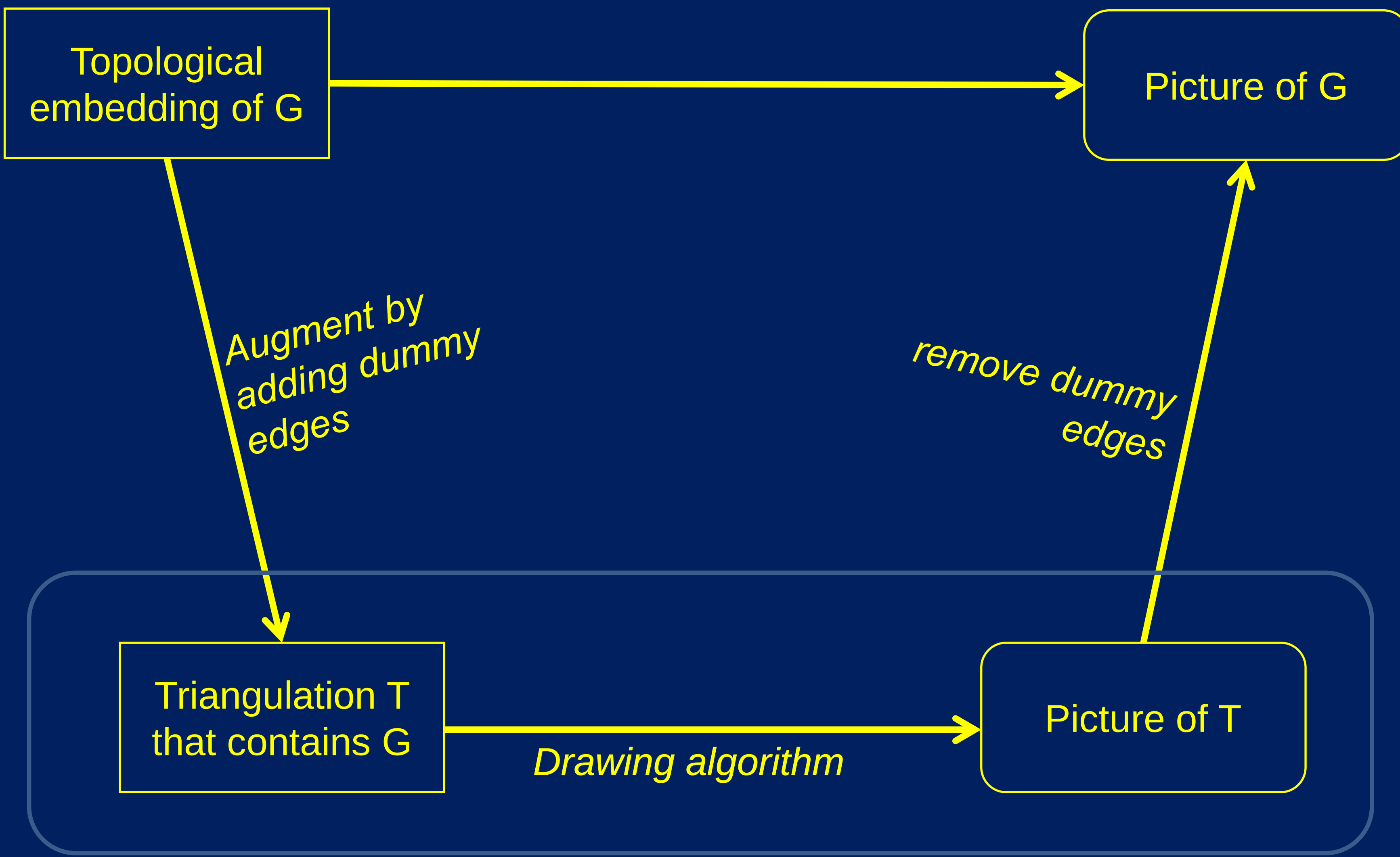
a) Before Tutte: 1920s – 1950s

Fáry's Theorem Every topological embedding of a planar graph has a straight-line planar drawing.

Proved independently by Wagner (1936), Fary (1948) and Stein (1951)

Wikipedia proof of Fáry's Theorem

First note that it is enough to prove it for triangulations.

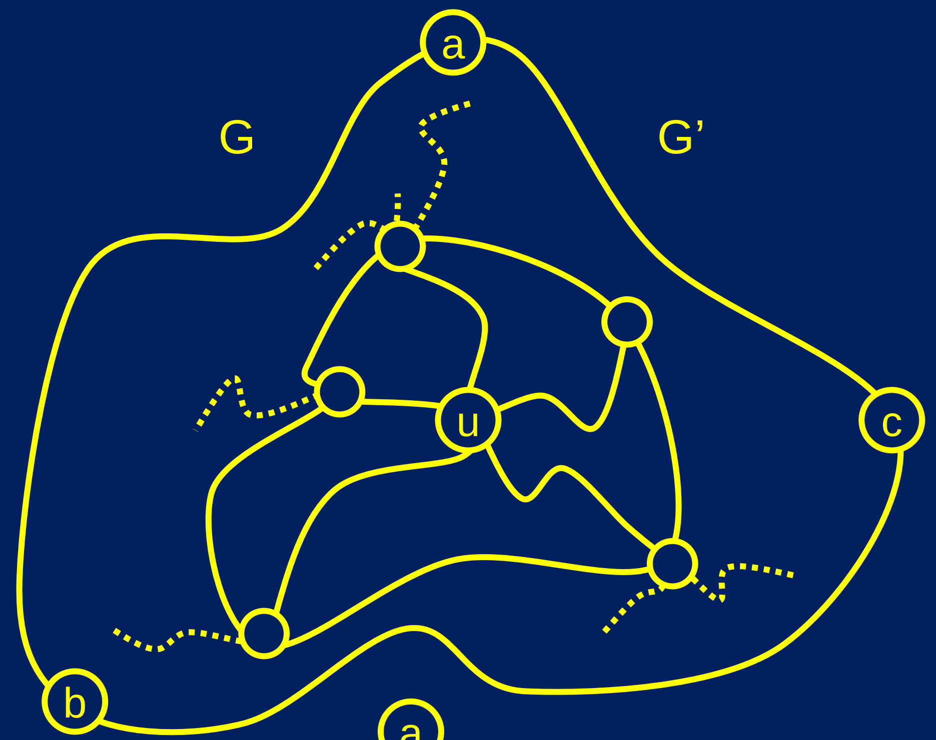


We prove Fáry's theorem by induction on the number of vertices.

If G has only three vertices, then it is easy to create a planar straight-line drawing.

Suppose G has n>3 vertices and 3n-6 edges, and that the outer face of G is the triangle <abc>.

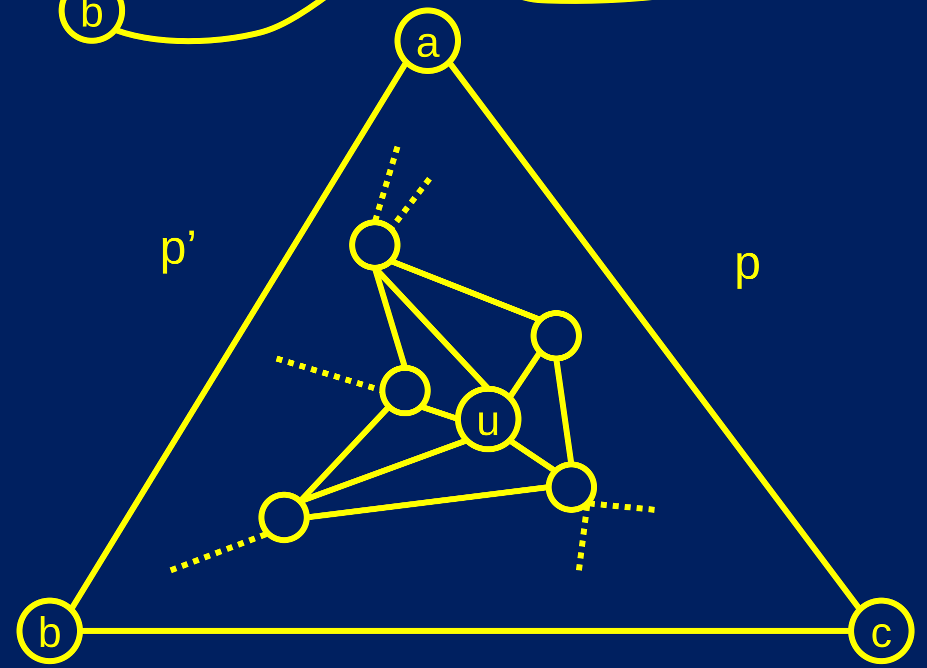
Since every vertex has degree at least 3, one can show that there is a vertex ***u*** not on the outside face with degree at most 5.



Delete ***u*** from G to form G'; this gives a face F of G of size at most 5.

Since G' has n-1 vertices, by induction it has a planar straight-line drawing p'.

Since F has at most 5 vertices, it is star-shaped, and we can place the vertex u in the kernel of F to give a planar straight-line drawing p of G



2. How to draw a planar graph?

   a) Before Tutte: 1920s – 1950s

   b) Tutte

      W. Tutte, How to Draw a Graph, *Proceedings of the London Mathematical Society* 13, pp743 – 767, 1960

Tutte's barycentre algorithm

Input:

- A graph G = (V,E)

Output

- A straight-line drawing p

Step 1. Choose a subset A of V

Step 2. Choose p location p (a) = (x_a, y_a) for each vertex a ∈ A

Step 3. For all u ∈ V-A,

$$p(u) = (\sum p(v))/ \deg(u),$$

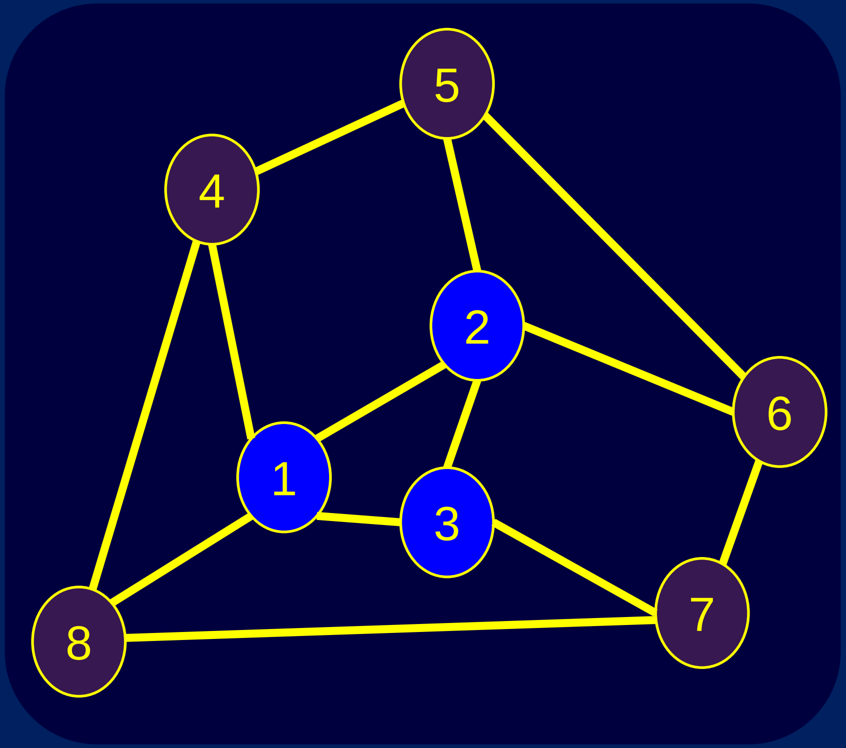
where the sum is over all neighbors v of u

Vertex u is placed at the *barycenter* of its neighbors

This is two sets of equations, one for x coordinates and one for y coordinates

Tutte’s barycenter algorithm

1. Choose a set A of vertices.
2. Choose a location p(a) for each a∈A
3. For each vertex ***u*** ∈V-A, place ***u*** at the barycentre of its graph- theoretic neighbors.



Example

Step 1. A = {4, 5, 6, 7, 8}

Step 2. For all i = 4, 5, 6, 7, 8,
choose x_i and y_i in some way.

Step 3. Find x_1, y_1, x_2, y_2, x_3, and y_3
such that:

$$x_1 = \frac{1}{4}\left(x_2 + x_3 + x_4^* + x_8^*\right)$$

$$x_2 = \frac{1}{4}\left(x_1 + x_3 + x_5^* + x_6^*\right)$$

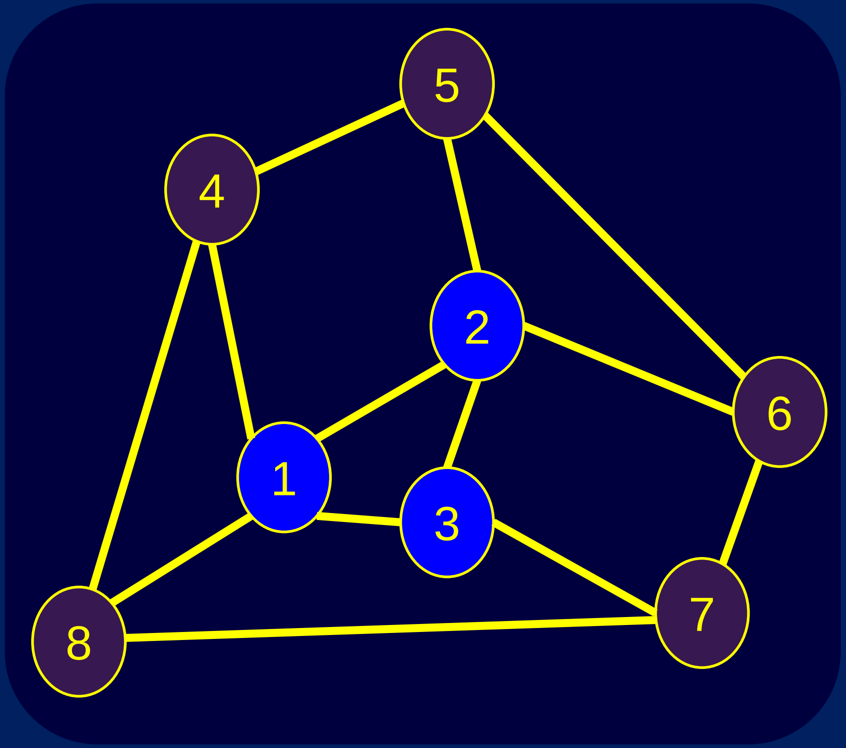
$$x_3 = \frac{1}{3}\left(x_1 + x_2 + x_7^*\right)$$

and

$$y_1 = \frac{1}{4}\left(y_2 + y_3 + y_4^* + x_8^*\right)$$

$$y_2 = \frac{1}{4}\left(y_1 + y_3 + y_5^* + y_6^*\right)$$

$$y_3 = \frac{1}{3}\left(y_1 + y_2 + y_7^*\right)$$



Step 1. A = {4, 5, 6, 7, 8}

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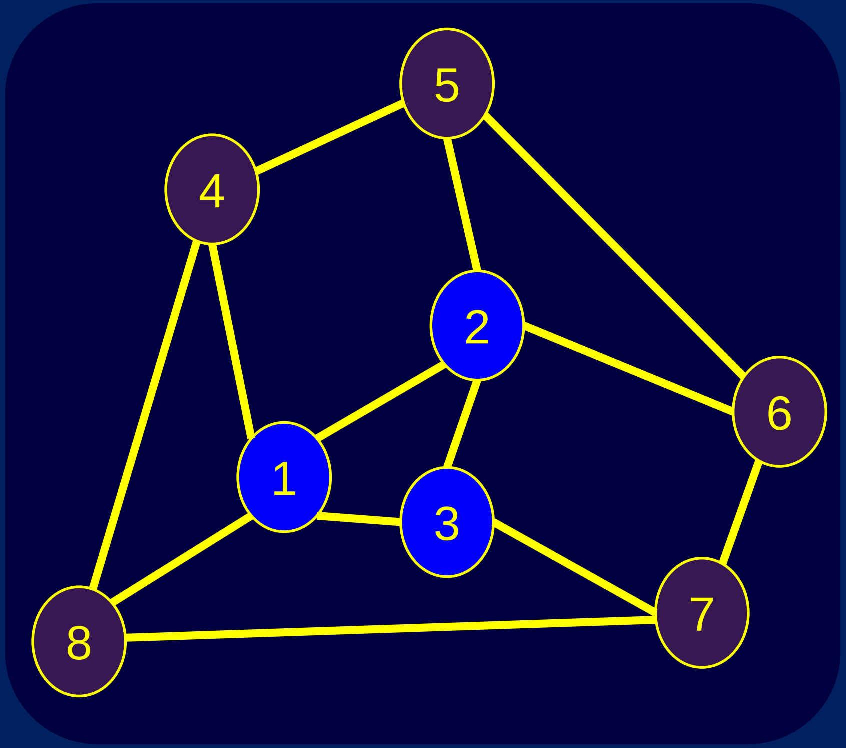


$$x_1 = \frac{1}{4}\left(x_2 + x_3 + x_4^* + x_8^*\right)$$

$$x_2 = \frac{1}{4}\left(x_1 + x_3 + x_5^* + x_6^*\right)$$

$$x_3 = \frac{1}{3}\left(x_1 + x_2 + x_7^*\right)$$

and

$$y_1 = \frac{1}{4}\left(y_2 + y_3 + y_4^* + x_8^*\right)$$

$$y_2 = \frac{1}{4}\left(y_1 + y_3 + y_5^* + y_6^*\right)$$

$$y_3 = \frac{1}{3}\left(y_1 + y_2 + y_7^*\right)$$

$\rightarrow$

$$4y_1 - y_2 - y_3 = y_4^* + x_8^* = d_1$$

$$-y_1 + 4y_2 - y_3 = y_5^* + y_6^* = d_2$$

$$y_1 + y_2 + 3y_3 = y_5^* = d_3$$

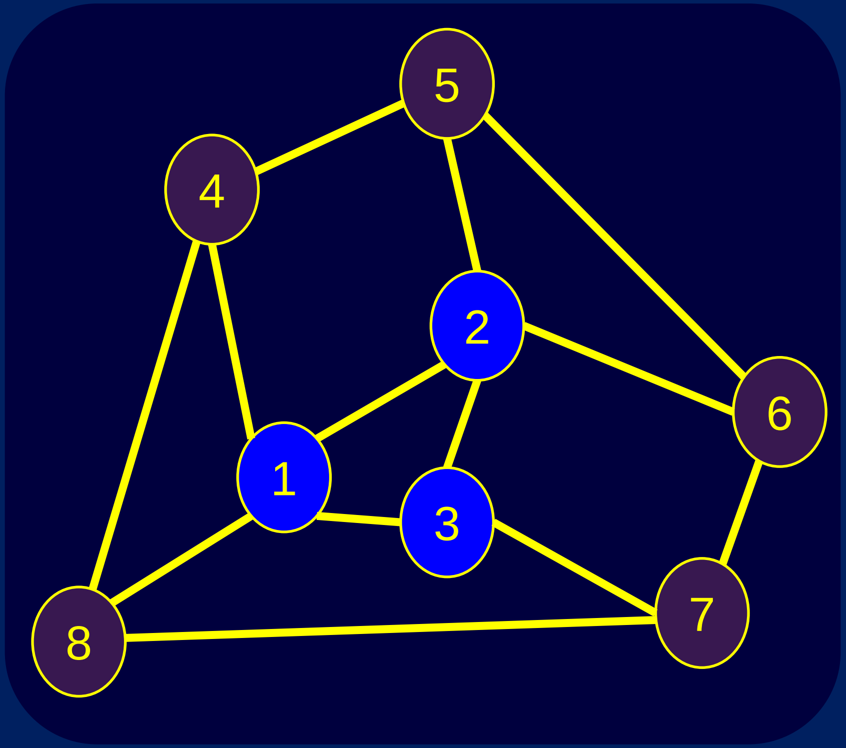
$\rightarrow$

$$\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

Step 1. A = {4, 5, 6, 7, 8}

Step 2. For all i = 4, 5, 6, 7, 8, choose x_i and y_i in some way.

Step 3. Find x_1, y_1, x_2, y_2, x_3, and y_3 such that:



$$x_1 = \frac{1}{4}\left(x_2 + x_3 + x_4^* + x_8^*\right)$$
$$x_2 = \frac{1}{4}\left(x_1 + x_3 + x_5^* + x_6^*\right)$$
$$x_3 = \frac{1}{3}\left(x_1 + x_2 + x_7^*\right)$$

$$x = Mc$$

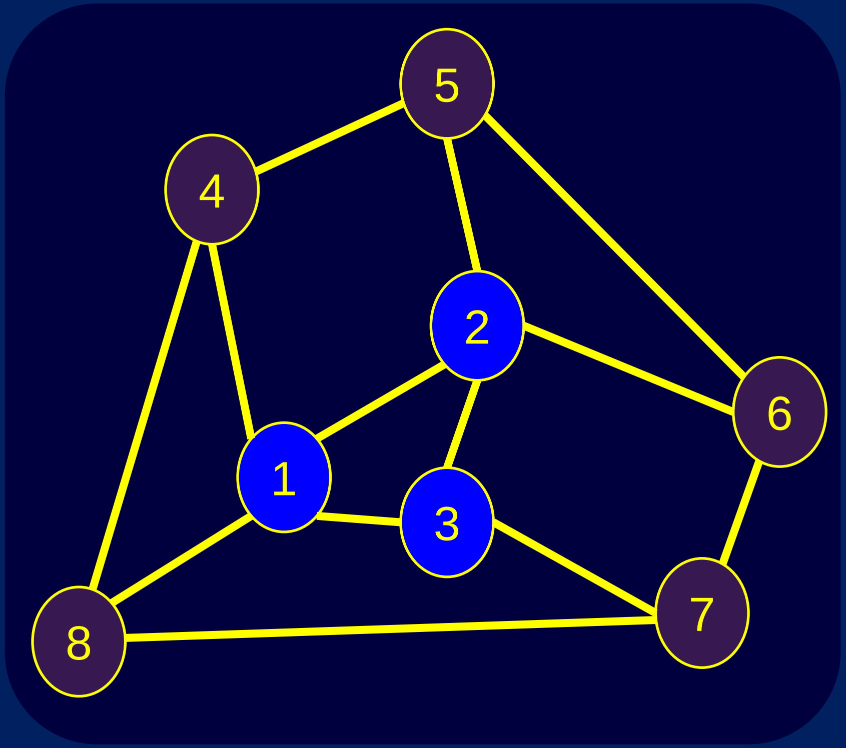
and

$$y_1 = \frac{1}{4}\left(y_2 + y_3 + y_4^* + x_8^*\right)$$
$$y_2 = \frac{1}{4}\left(y_1 + y_3 + y_5^* + y_6^*\right)$$
$$y_3 = \frac{1}{3}\left(y_1 + y_2 + y_7^*\right)$$

$$y = Md$$

$$M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

Tutte's barycentre algorithm



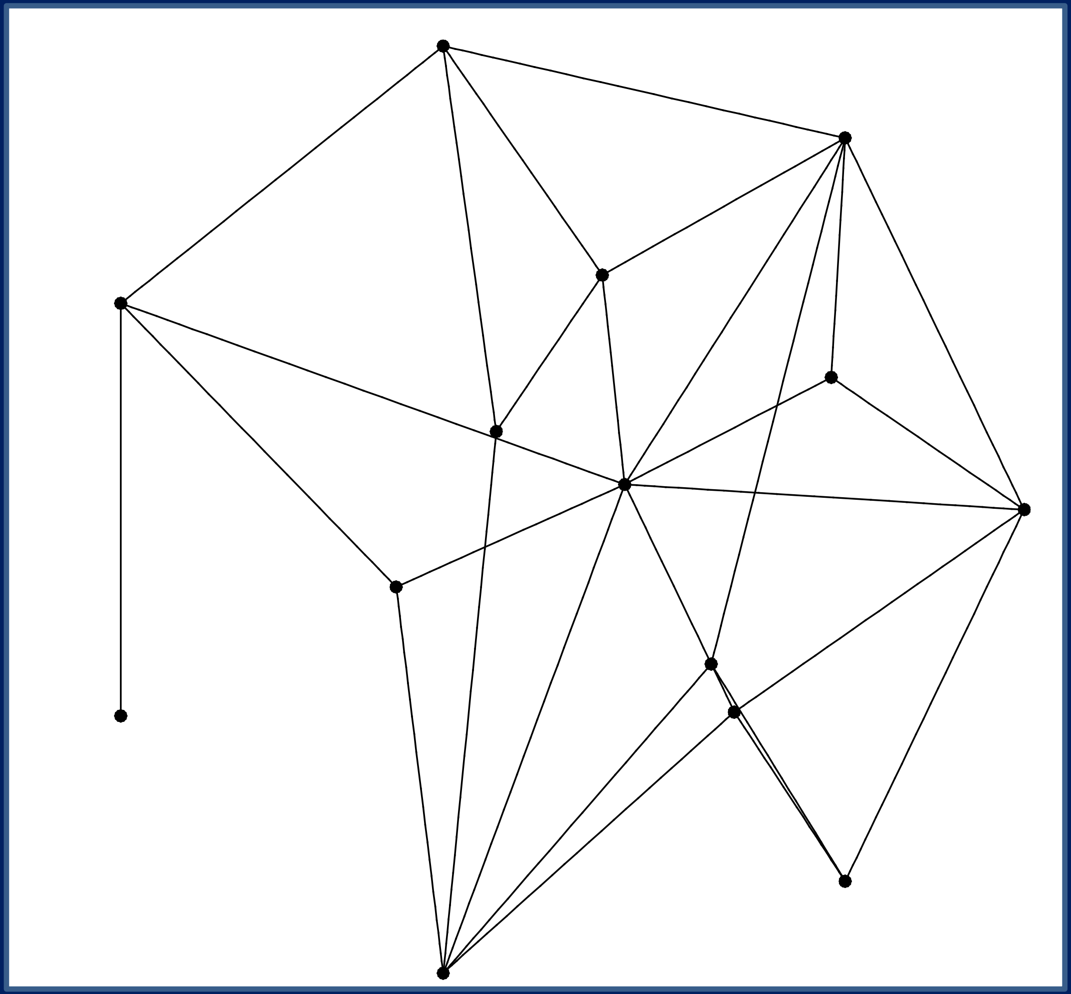
- The essence of the algorithm is in inverting the matrix M
  - Can be done in time $O(n^3)$
  - This is a special matrix: *Laplacian submatrix*.
  - Many software packages can solve such equations efficiently,

$$\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 3 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

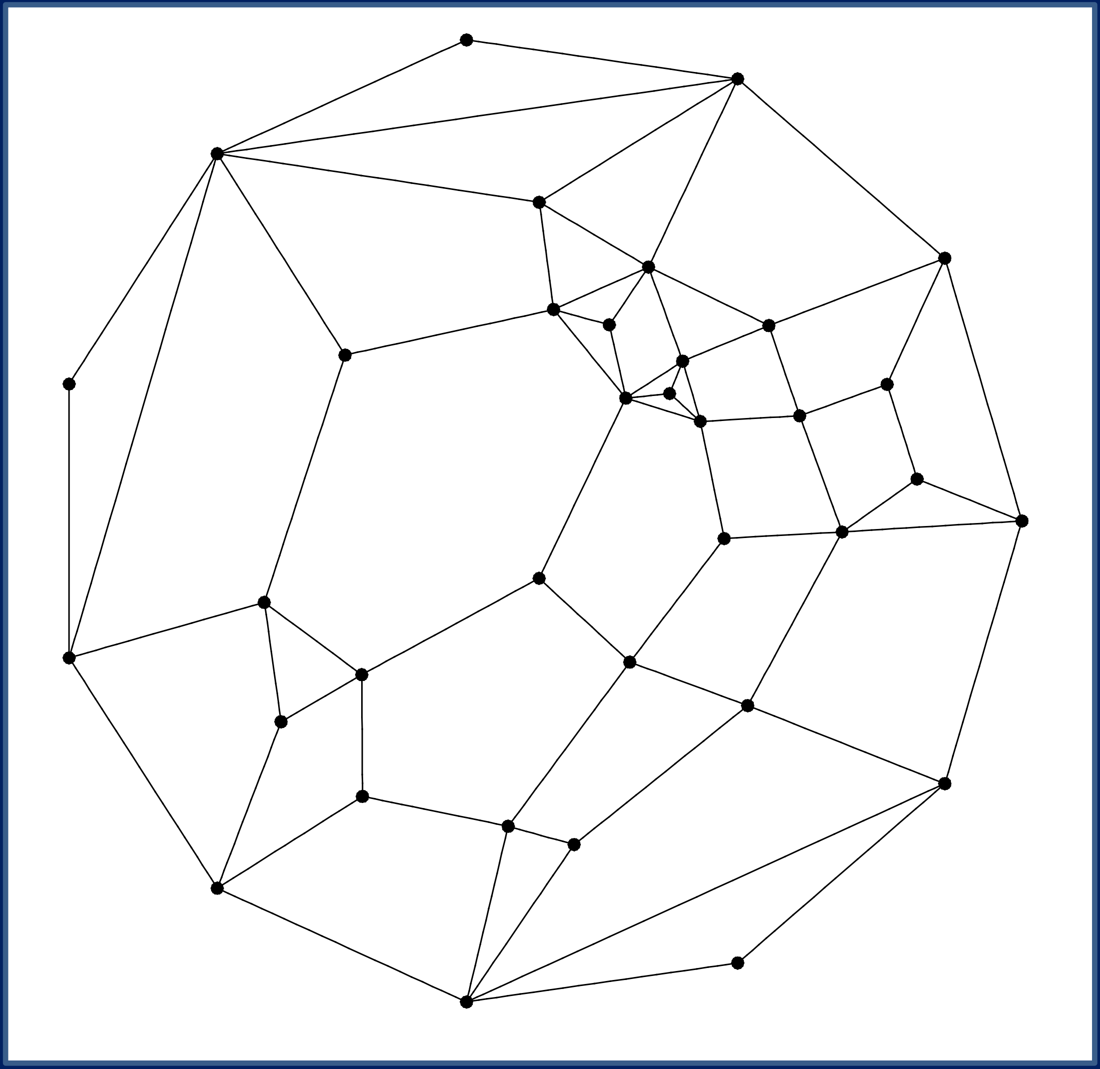
Tutte's barycentre algorithm

- Example output on a non-planar graph



Tutte’s barycentre algorithm

- Example output on a *planar* graph



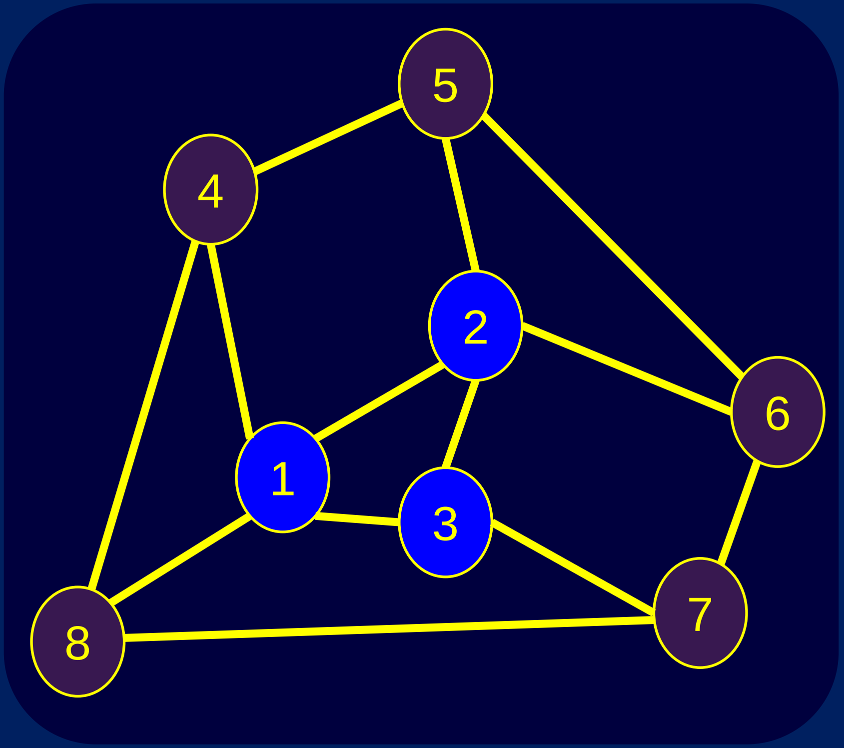
Tutte’s barycenter algorithm for
triconnected planar graphs

Tutte's barycenter algorithm **for triconnected planar graphs**

1. Choose A to be *the outside face of the graph*.

2. Choose the location p(a) for each a∈A to be *at the vertices of a convex polygon*.

3. For each vertex ***u*** ∈V-A, place ***u*** at the barycentre of its graph-theoretic neighbors.



Note: For planar graphs, the Laplacian matrix is sparse, and can be inverted fast.

Tutte's amazing theorems (1960)

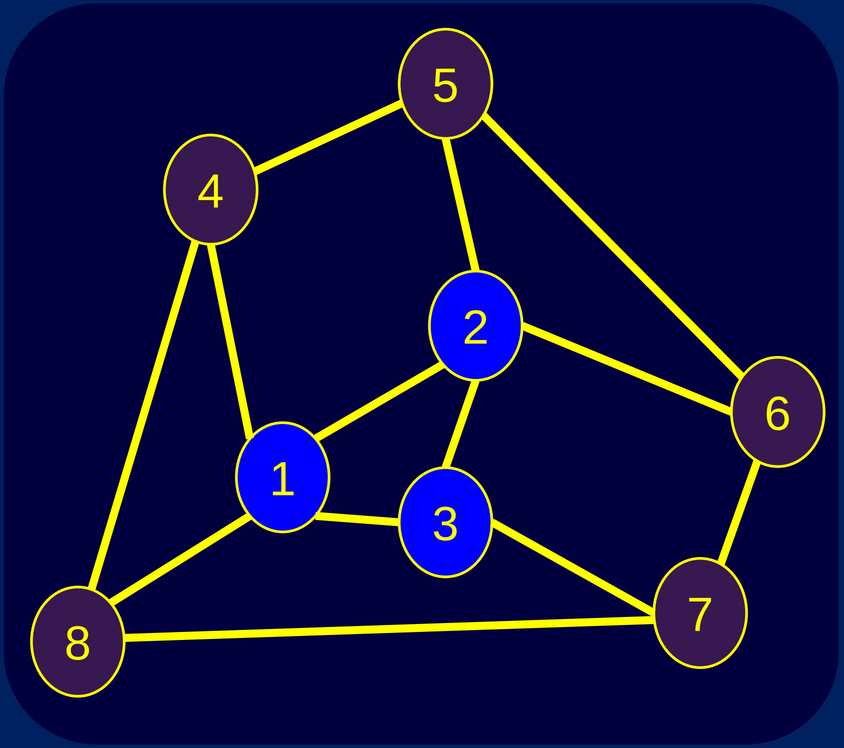
If the input graph is planar and triconnected, then the drawing output by the barycentre algorithm is planar, and every face is convex.

The *energy* view of Tutte's barycentre algorithm

Tutte's barycenter algorithm:

The energy view

1. Choose a set A of vertices.
2. Choose a location p(a) for each a∈A
3. Place all the other vertices to minimize *energy*.



What is the *energy* of a drawing p?

- For each edge e = (u,v), denote the distance between u and v in the drawing p by d(u,v), ie,

  $d(u,v) = ((x_u - x_v)^2 + (y_u - y_v)^2)^{0.5}$
- The *energy in the edge e* is $d(u,v)^2 = (x_u - x_v)^2 + (y_u - y_v)^2$
- The *energy in the drawing p* is the sum of the energy in its edges, ie,

  $\Sigma\, d(u,v)^2 = \Sigma\, (x_u - x_v)^2 + (y_u - y_v)^2$

  where the sum is over all edges (u,v).

Tutte's barycenter algorithm:

<u>The energy view</u>

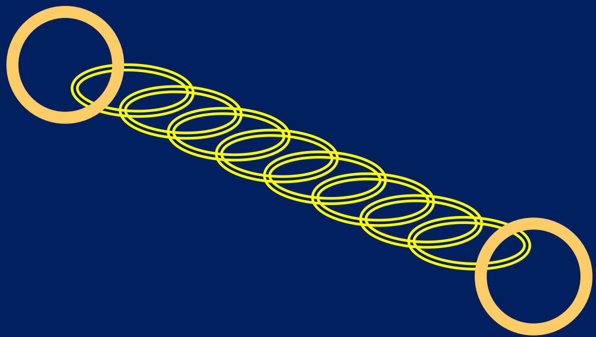
1. Represent each vertex by a steel ring, and represent each edge by a spring of natural length zero connecting the rings at its endpoints.

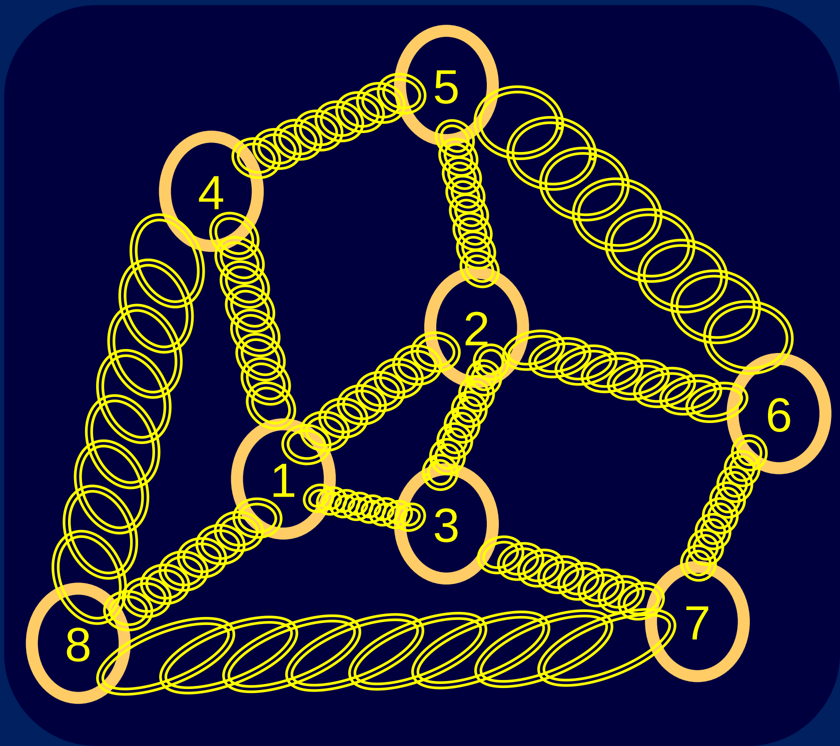
2. Choose a set A of vertices.

3. For each a∈A, nail the ring representing a to the floor at some position.

4. The vertices in V-A will move around a bit,

When the movement stops, take a photo of the layout; this is the drawing.





How to minimize energy:

- We need to choose a location (x(u),y(u)) for each u in V-A to minimize Σ $(x_u - x_v)^2 + (y_u - y_v)^2$
- Note that the minimum is unique, and occurs when the partial derivative wrt x_u and y_u is zero for each u in V-A.

$$\frac{\partial}{\partial x_u}\left(\sum_{(u,v)}(x_u - x_v)^2 + (y_u - y_v)^2\right) = \sum_{(u,v)} 2(x_u - x_v) = 0$$

$$\Leftrightarrow$$

$$x_u = \left(\sum_{(u,v)} x_v\right) / \deg(u)$$

Barycentre equations

How good is Tutte's barycentre algorithm?

Efficiency:

- In theory it is not bad: $O(n^{1.5})$ for planar graphs
- In practice it is fast, using numerical methods for Laplacians

Elegance:

- Very simple algorithm
- Easy to implement
- Numerical software available for the hard parts

Effectiveness:

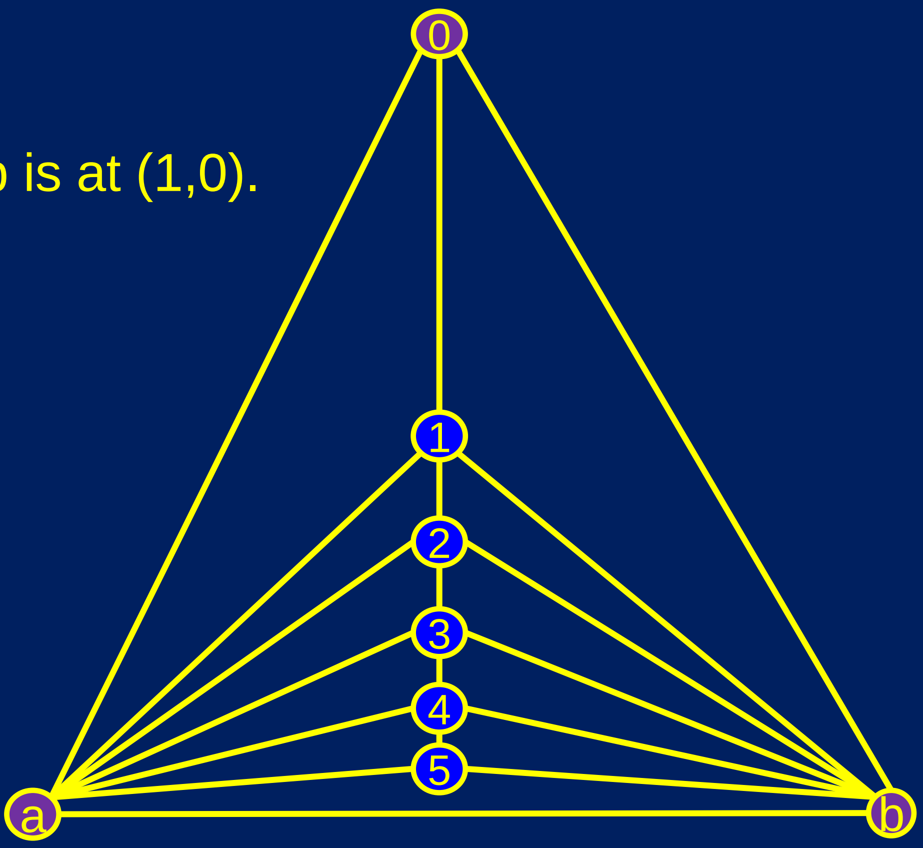
- Planar graphs drawn planar
- Straight-line edges

But unfortunately →

But unfortunately:

- Tutte's algorithm gives *poor vertex resolution* in many cases

Example:



Vertex 0 is at (0.5, 0), a is at (0,0), b is at (1,0).

For j>0, vertex j is at (x_j,y_j).

From the barycentre equations:

$$y_j = (y_{j-1} + y_{j+1}) / 4.$$

Also:

$$y_j > y_{j-1} > y_{j-2} > \ldots$$

Thus $y_j < y_j\text{-}1 / 2$

Thus $0 < y_j < 2^{-j}$

Aside:

Commercial graph drawing software needs good resolution.

How good is Tutte's barycentre algorithm?

Efficiency:

- OK

Elegance:

- Excellent

Effectiveness:

- So-so

2. How to draw a planar graph?

a) Before Tutte: 1920s – 1950s

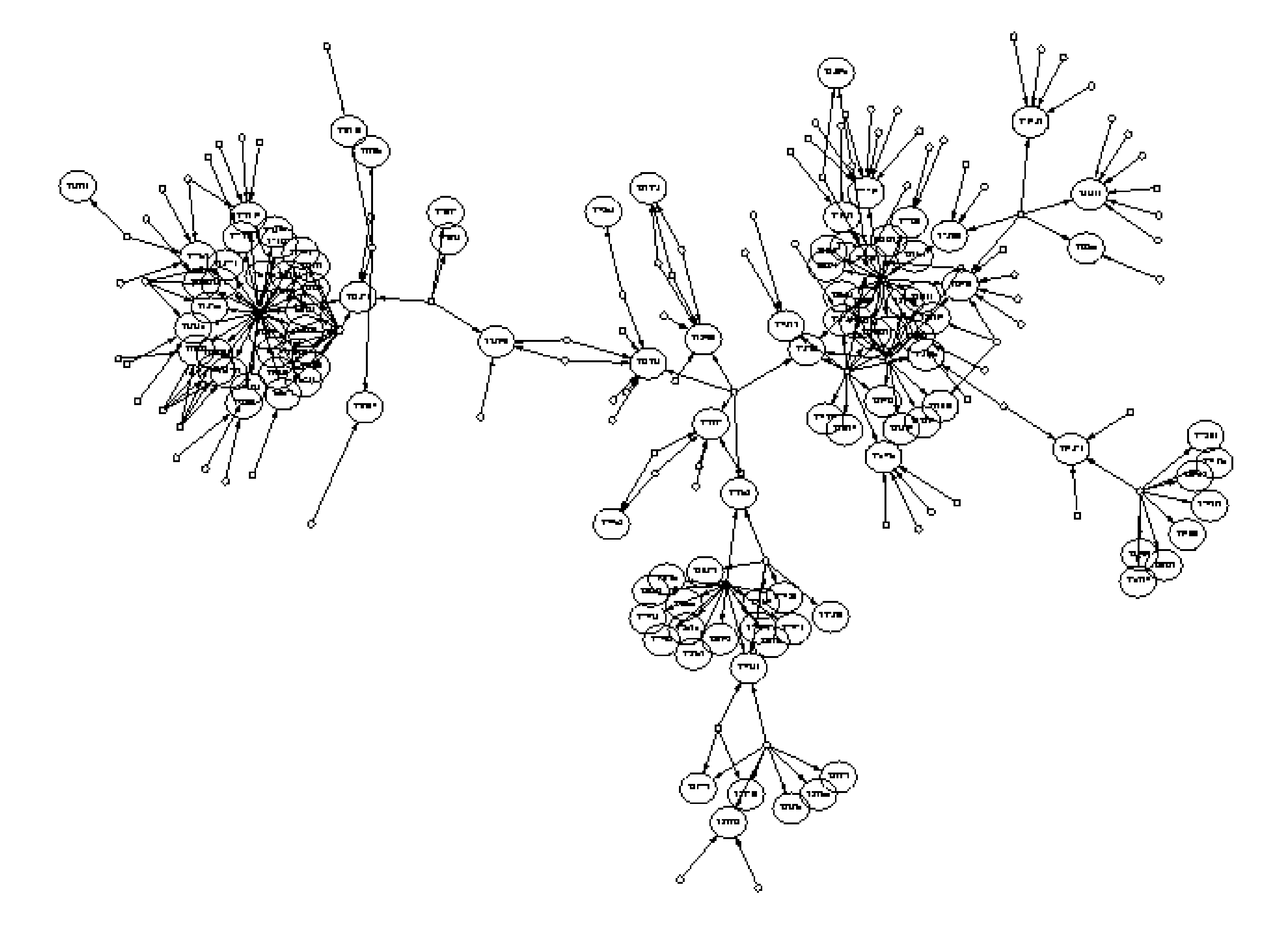
b) Tutte: 1960s

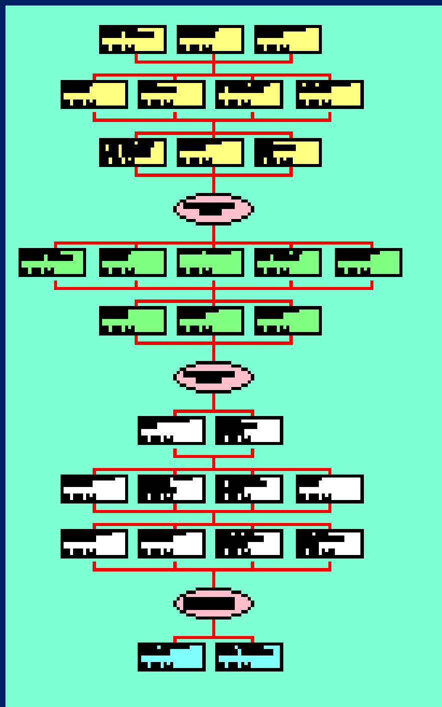
c) After Tutte: 1970s – 1990s

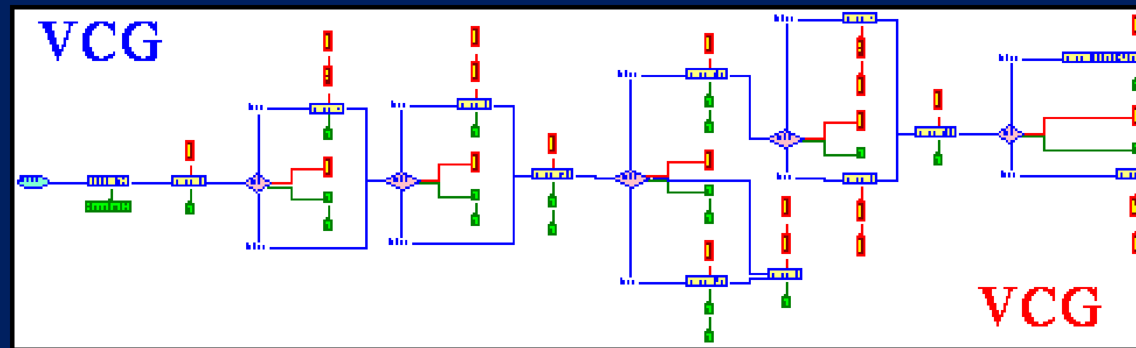
After Tutte: 1970s – 1990s

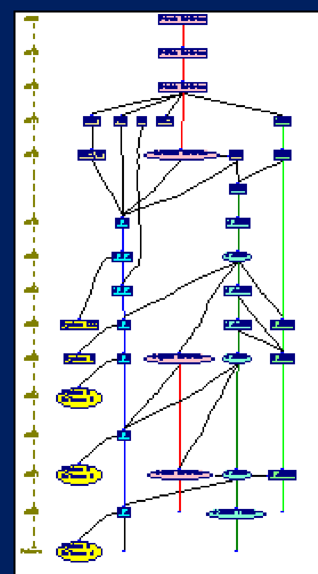
Sometime in the 1980s, the motivation for graph drawing changed from *Mathematical curiosity* to *visual data mining*.

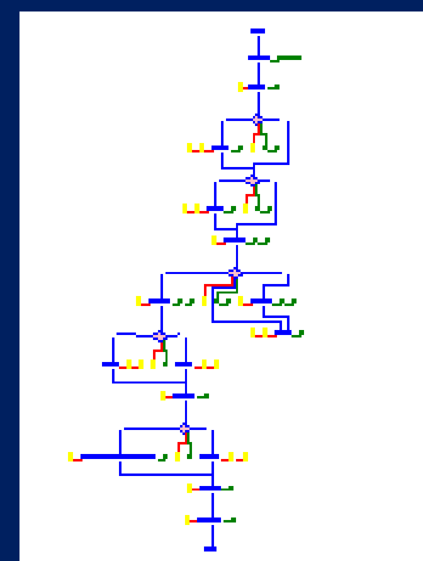
Software

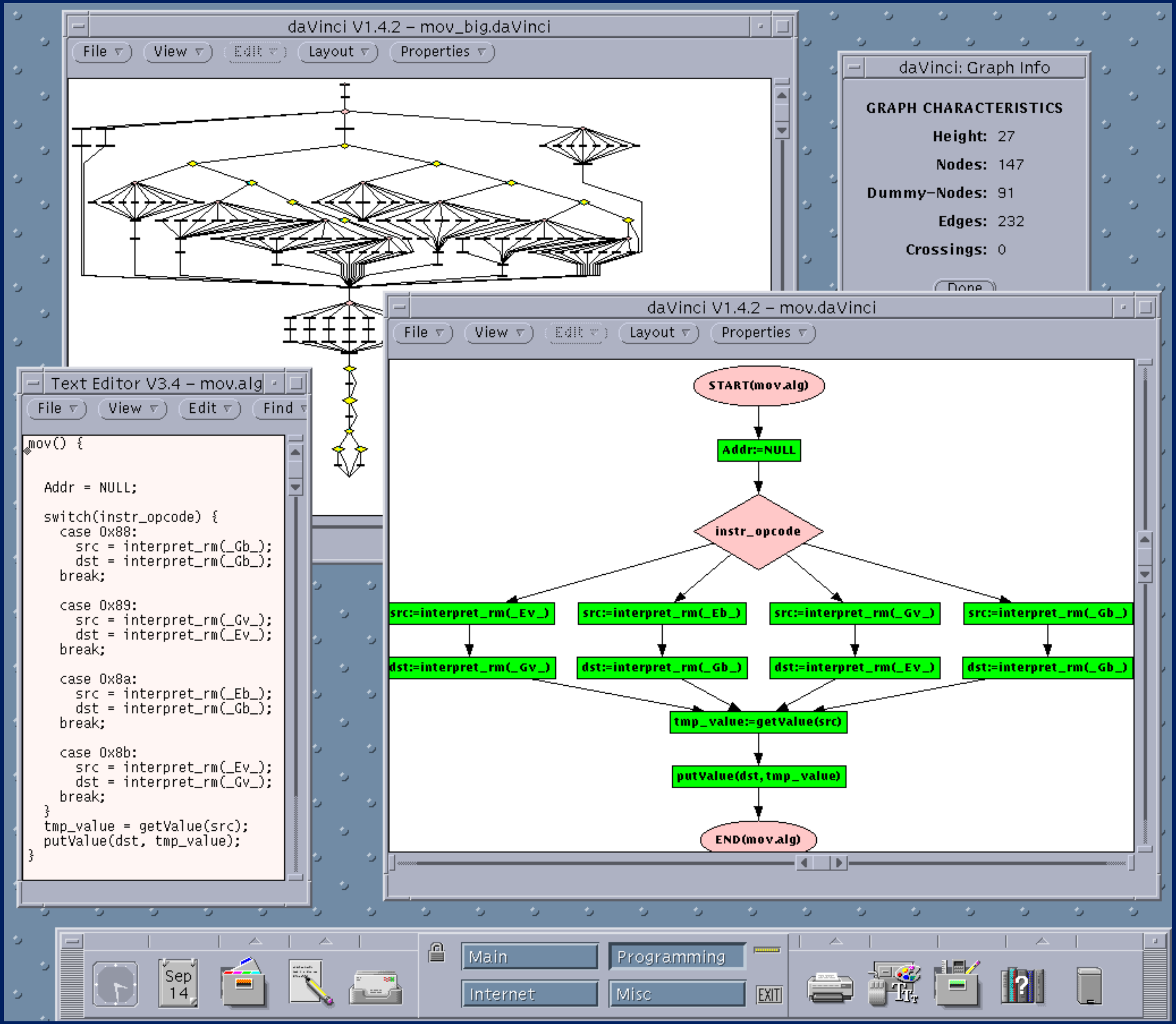




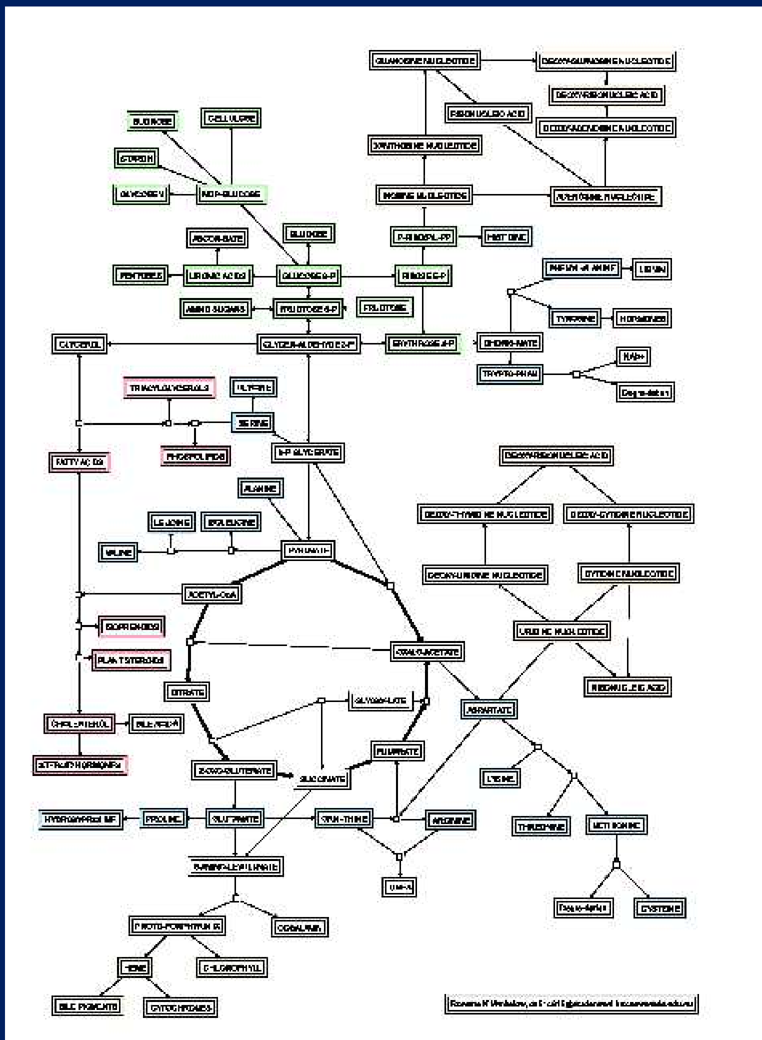

VCG
VCG



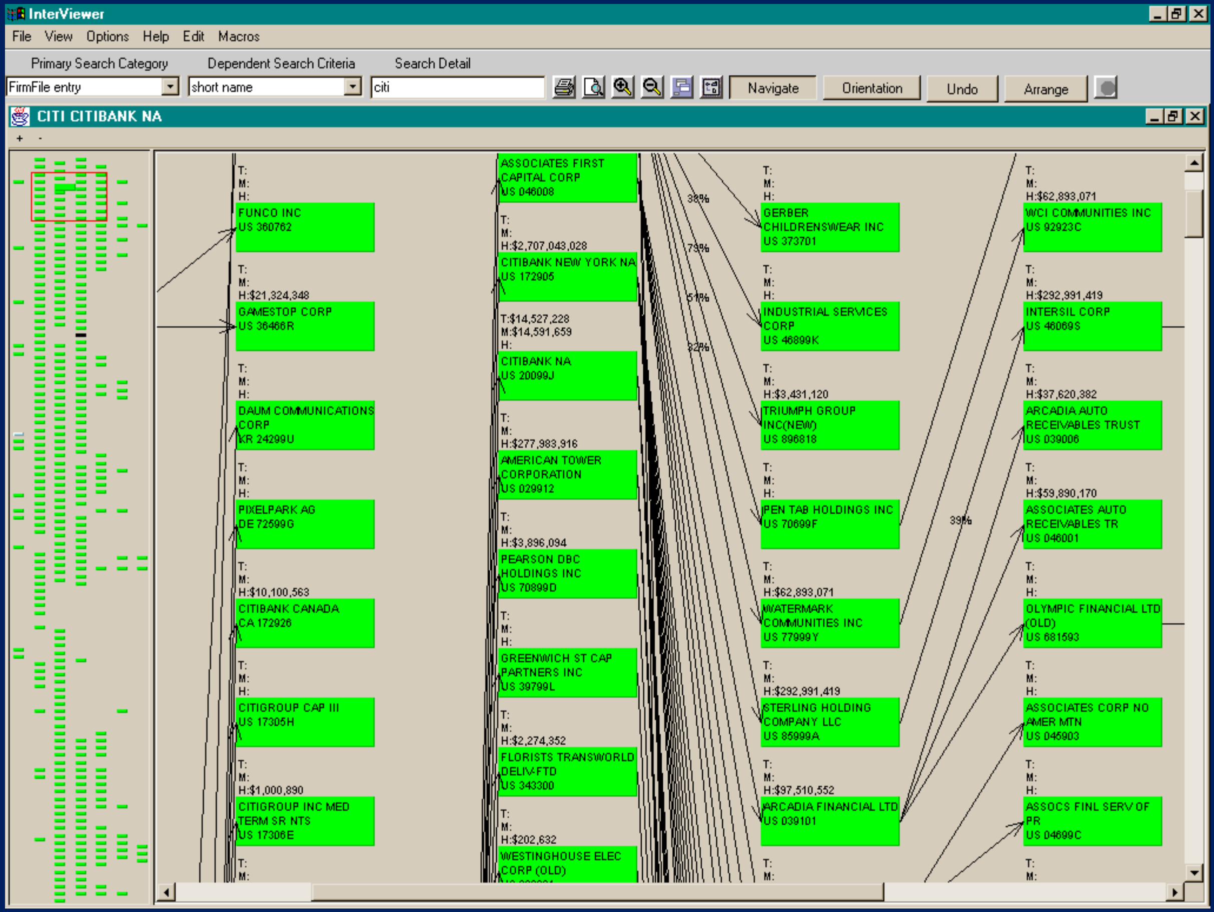



daVinci V1.4.2 – mov_big.daVinci
File
View
Edit
Layout
Properties
daVinci: Graph Info
GRAPH CHARACTERISTICS
Height: 27
Nodes: 147
Dummy-Nodes: 91
Edges: 232
Crossings: 0
Done
daVinci V1.4.2 – mov.daVinci
START(mov.alg)
Addr:=NULL
instr_opcode
src:=interpret_rm(_Ev_)
src:=interpret_rm(_Eb_)
src:=interpret_rm(_Gv_)
src:=interpret_rm(_Gb_)
dst:=interpret_rm(_Gv_)
dst:=interpret_rm(_Gb_)
dst:=interpret_rm(_Ev_)
dst:=interpret_rm(_Gb_)
tmp_value:=getValue(src)
putValue(dst,tmp_value)
END(mov.alg)
Text Editor V3.4 – mov.alg
Find
mov() {
Addr = NULL;
switch(instr_opcode) {
case 0x88:
src = interpret_rm(_Gb_);
dst = interpret_rm(_Gb_);
break;
case 0x89:
src = interpret_rm(_Gv_);
dst = interpret_rm(_Ev_);
break;
case 0x8a:
src = interpret_rm(_Eb_);
dst = interpret_rm(_Gb_);
break;
case 0x8b:
src = interpret_rm(_Ev_);
dst = interpret_rm(_Gv_);
break;
}
tmp_value = getValue(src);
putValue(dst, tmp_value);
}
Sep
14
Main
Programming
Internet
Misc
EXIT

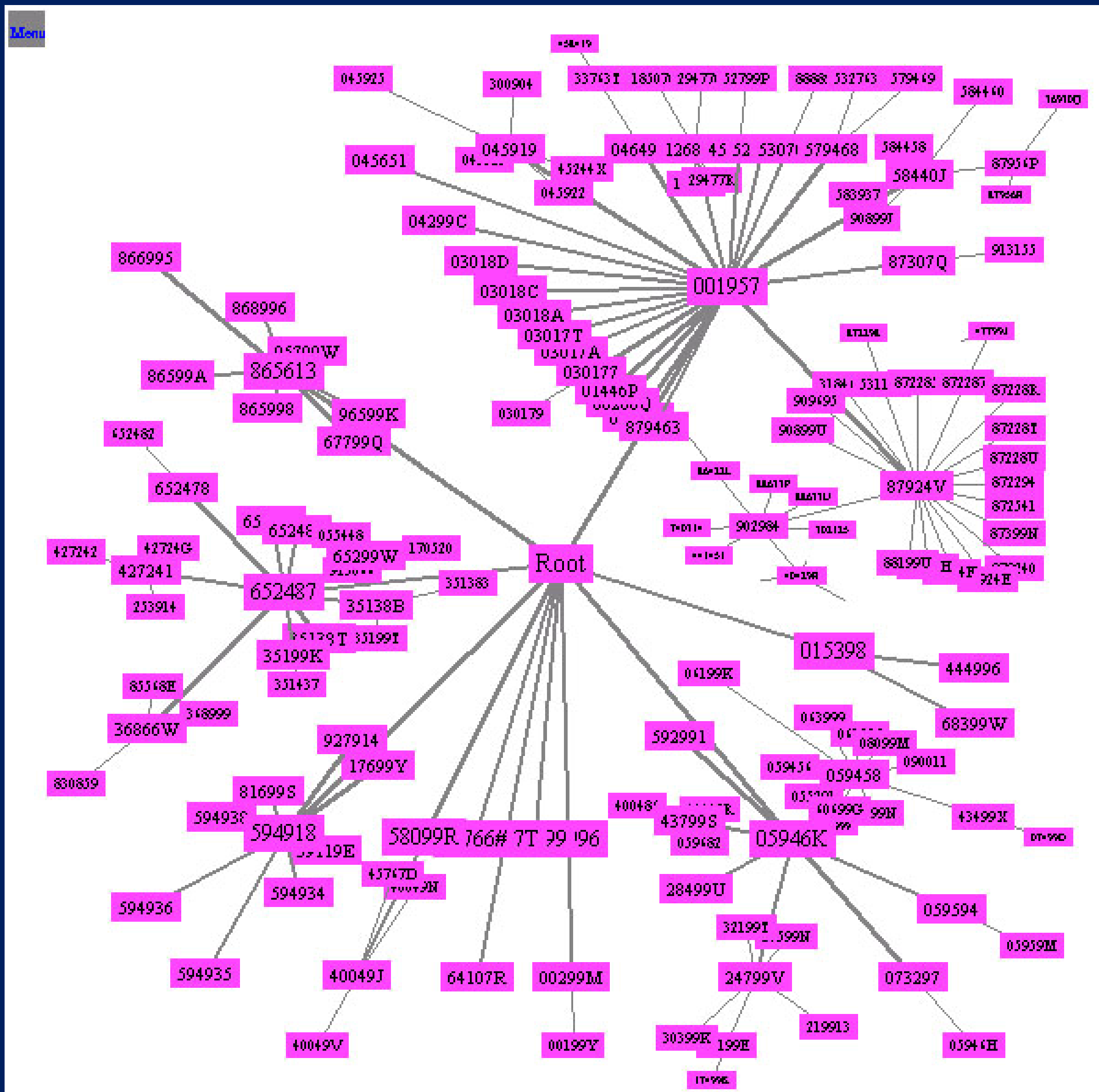
(DaVinci)



Biology



Risk Exposure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045919
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04299C
03018D
03018C
03018A
03017T
030177
030179
879463
87307Q
913155
58440J
584458
583937
90899J
87956P
909495
90899U
87228K
87228I
87228U
872294
872541
87399N
88199U
866995
868996
86599A
865998
96599K
67799Q
652482
652478
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42724G
427241
253914
65299W
170520
351383
35138B
35199I
35199K
351437
83548E
368999
36866W
830859
927914
17699Y
81699S
594936
594934
594935
40049J
40049V
58099R
64107R
00299M
00199Y
444996
68399W
04199K
592991
063999
08099M
090011
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219913
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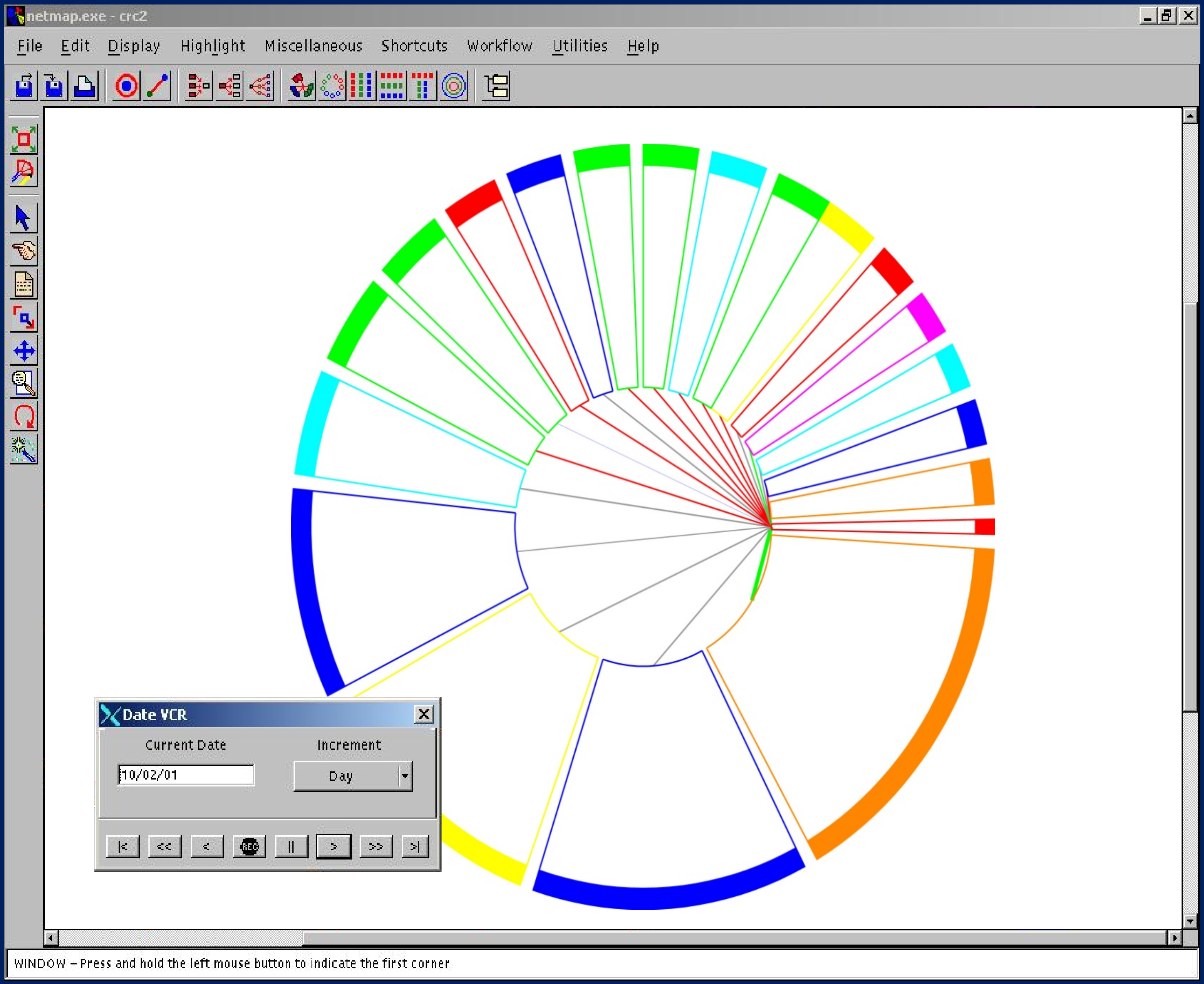
073297



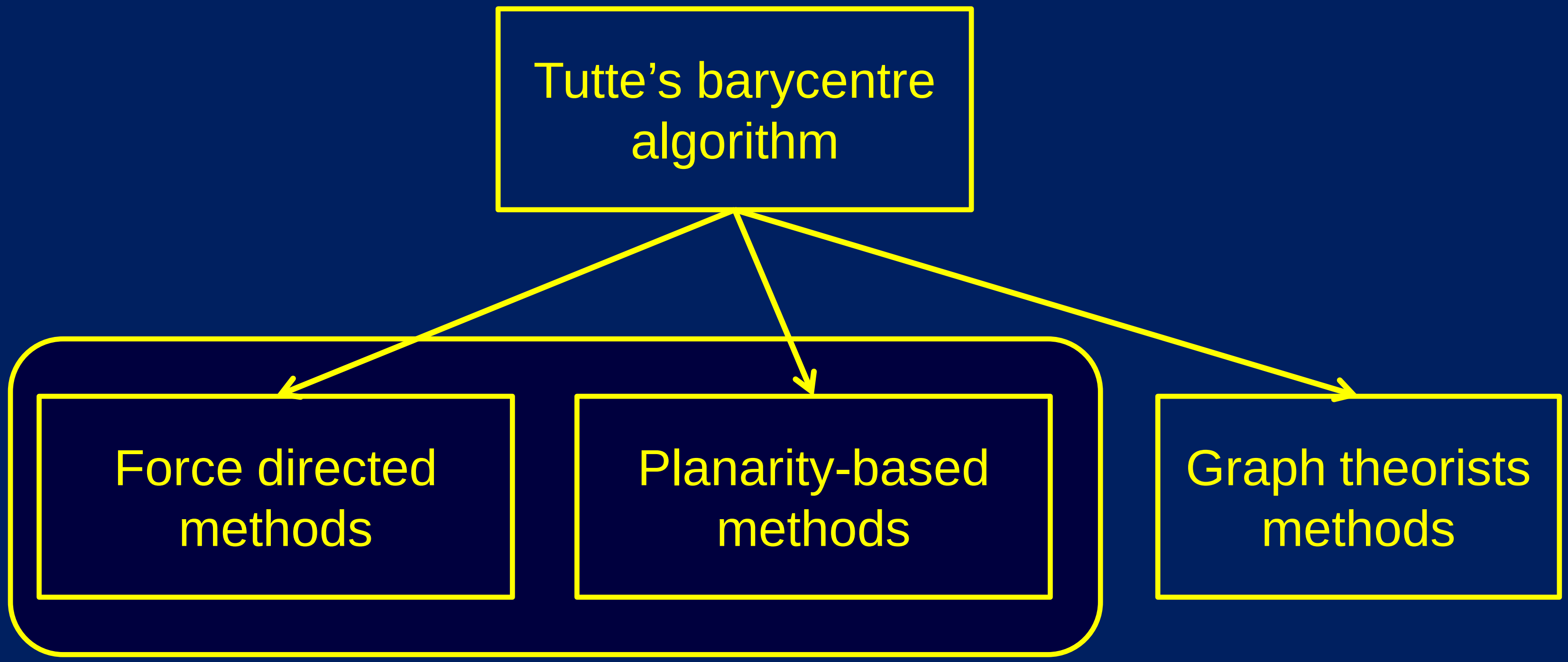
Money movement

From the 1980s, industrial demand for graph drawing algorithms has grown

- Software engineering: CASE systems, reverse engineering
- Biology: PPI networks, gene regulatory networks
- Physical networks: network management tools
- Security: risk management, money movements, social network analysis
- Customer relationship management: value identification

Many companies buy graph drawing algorithms, many code them.

Currently the international market for graph drawing algorithms is in the hundreds of millions of dollars per year.


Tutte's barycentre algorithm
Force directed methods
Planarity-based methods
Graph theorists methods

Planarity based methods after Tutte

Planarity based methods after Tutte

R.C. Read (1979, 1980)

1. Efficient?
   - Yes, linear time algorithm
2. Elegant?
   - Yes, follows proof of Fáry’s theorem
3. Effective?
   - Maybe ...
     - Straight-line planar drawings of planar graphs
     - But, unfortunately, output has poor vertex resolution

Planarity based methods after Tutte

<u>Chiba-Nishizeki-Yamanouchi (1984)</u>

1. Efficienct?
   - Yes, linear time algorithm
2. Elegant?
   - Yes, a simple divide&conquer approach
3. Effective?
   - Maybe ...
     - Straight-line planar drawings of planar graphs
     - Convex faces for well connected input
     - But, unfortunately, output has poor vertex resolution

Planarity based methods after Tutte

Breakthrough in 1989:

de Fraysseix-Pach-Pollack Theorem (1989)
Every planar graph has a planar straight-line ***grid*** (that is, vertices are at integer grid points) drawing on a *2n X 4n* grid.

Notes:

- This gives a minimum distance of *screensize/4n* between vertices, that is, good resolution.
- Chrobak gave a linear-time algorithm to implement this theorem.

The deFraysseix-Pach-Pollack Theorem gave much hope for planarity-based methods, and many refinements appeared 1990 – 2000.

de Fraysseix-Pach-Pollack-Chrobak Algorithm

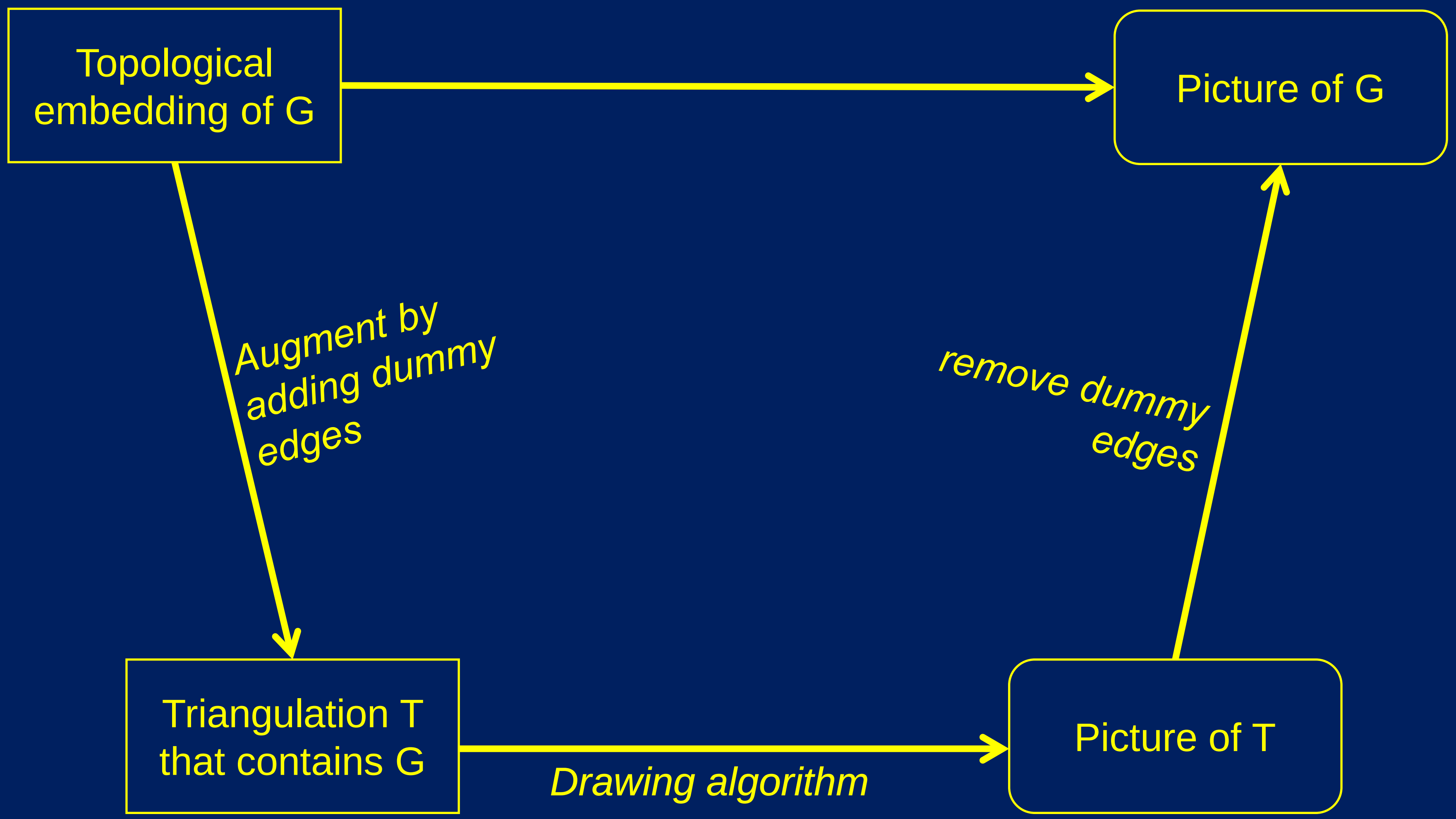
1. Add dummy edges to make the graph into a triangulation

2. Construct an ordering $u_1, u_2, \ldots, u_n$ of the vertices , called the *canonical ordering*.

3. Draw the graph, adding one vertex at a time, in order $u_1, u_2, \ldots, u_n$

Wikipedia proof of Fáry's Theorem

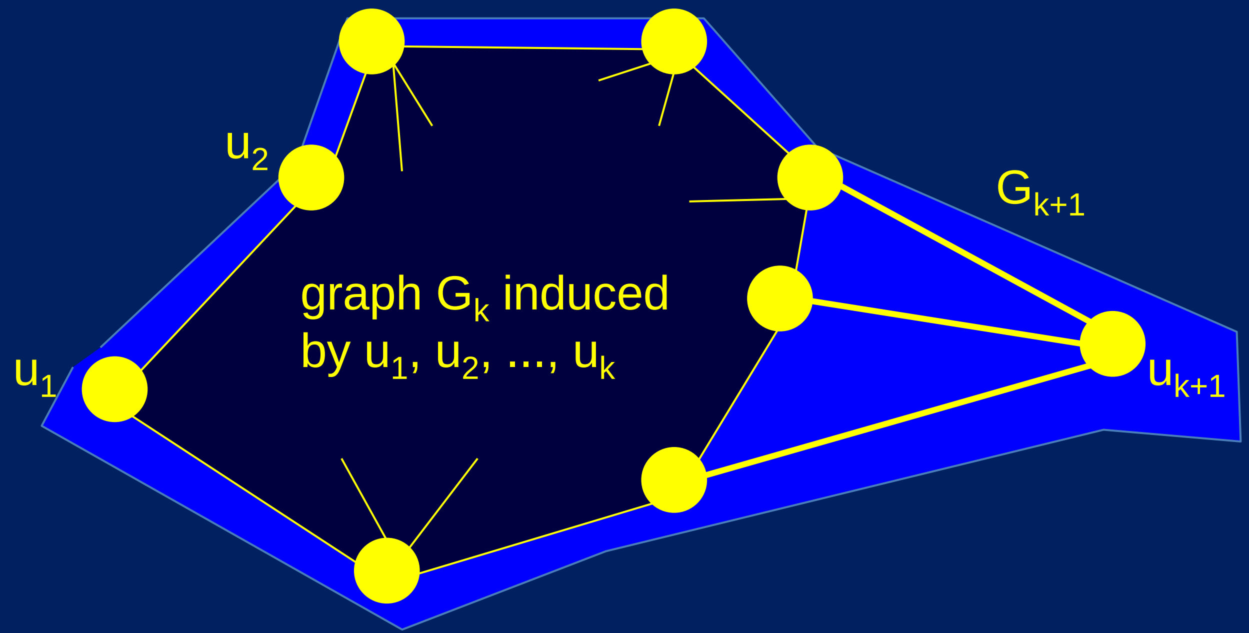
Step 1: Add dummy edges to make the graph into a triangulation



Step 2: Construct an ordering u_1, u_2, … , u_n of the vertices , called the *canonical ordering*.

A canonical ordering is an ordering u_1, u_2, ..., u_n of the vertices of a triangulation having the property that, for each k, 3 <= k < n, the graph G_k induced by u_1, u_2, ..., u_k has the following properties

- G_k is biconnected
- G_k contains the edge (u_1, u_2)on its outer face,
- Any vertices in G_k adjacent to u_{k+1} are on the outer face of G_k
- The vertices in G_k adjacent to u_{k+1} form a path along the outer face of G_k

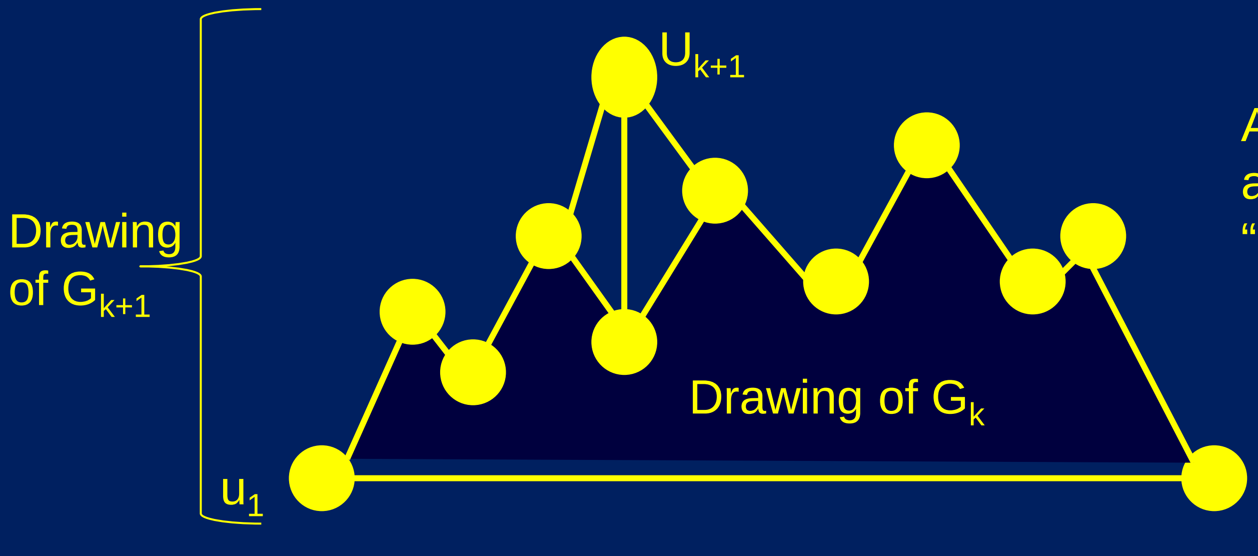


Step 3: Draw the graph, adding one vertex at a time in order $u_1, u_2, \ldots, u_n$

a) Start with the edge (u_1, u_2) at $y=0$

b) For each $k>1$:

- add u_{k+1} on $y=k$
- Choose x coordinate of u_{k+1} so that there are no edge crossings.



At each stage, there is a drawing of G_k as a "*terrain*".

Some details of deFraysseix-Pach-Pollack-Chrobak algorithm are needed to show

- It runs in linear time
- It is possible to avoid edge crossings
- Each vertex lies on an integer grid of size at most 4nX2n

The deFraysseix-Pach-Pollack-Chrobak algorithm

Efficiency:

- Yes, linear time

Elegance:

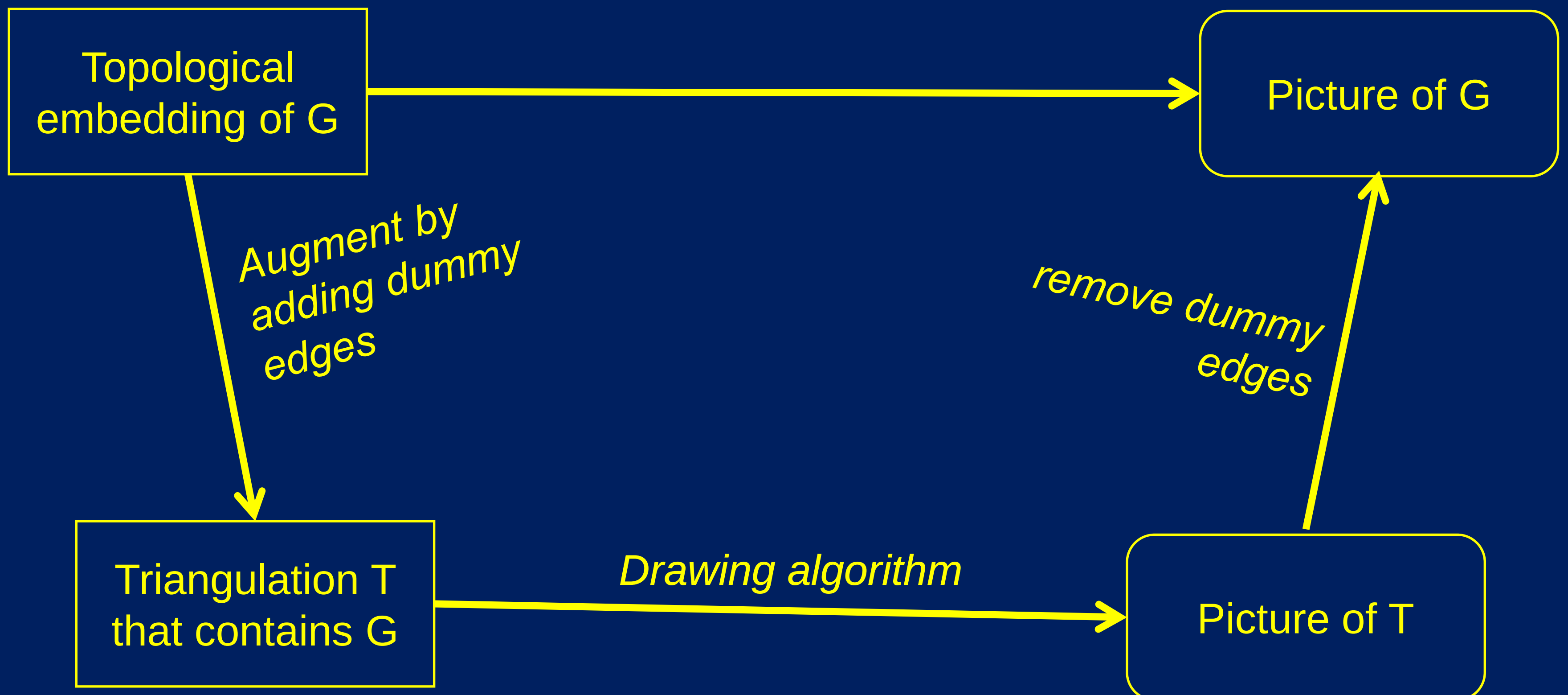
- Not bad; can be coded by a student in a week or so.

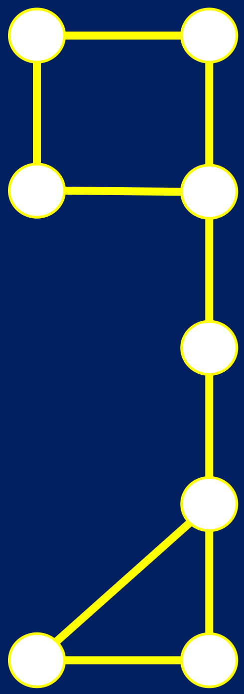
Effectiveness:

- Looks good
  - Straight-line edges
  - No edge crossings
  - Good vertex resolution

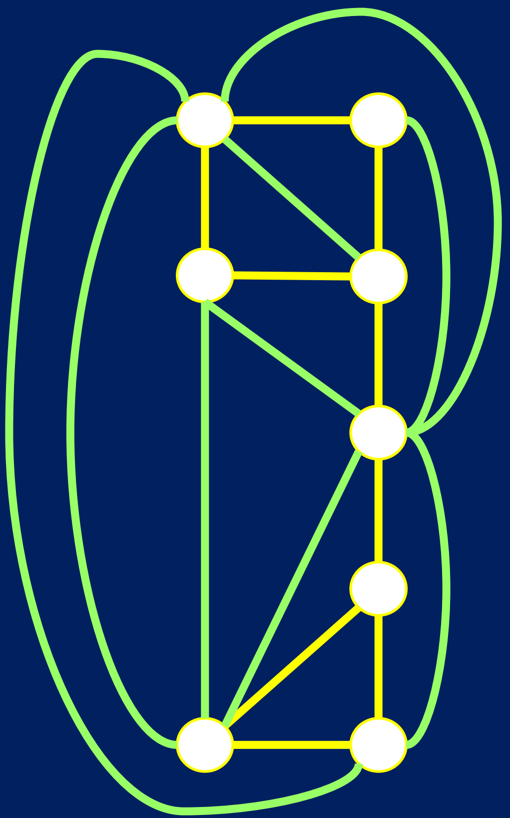
The deFraysseix-Pach-Pollack-Crobak algorithm gave much hope for planarity-based methods, and many refinements appeared 1990 – 2000.

But, unfortunately, we found that the first step (increasing connectivity by triangulation) gives some problems.

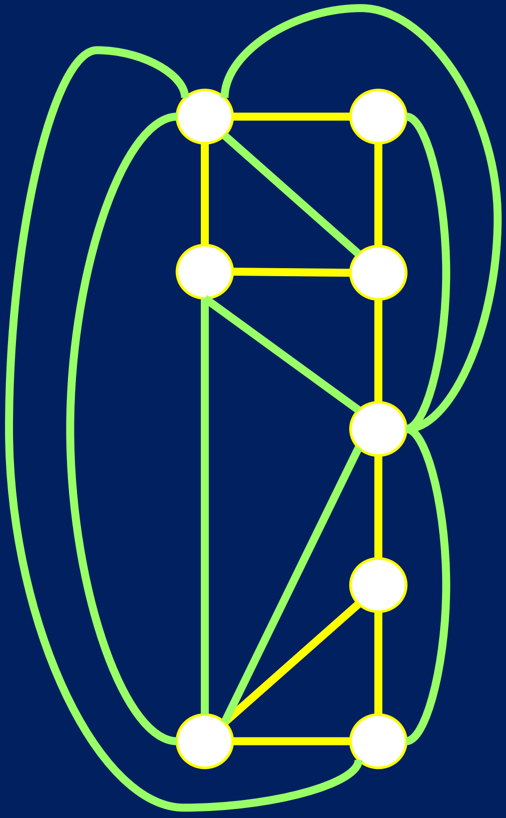




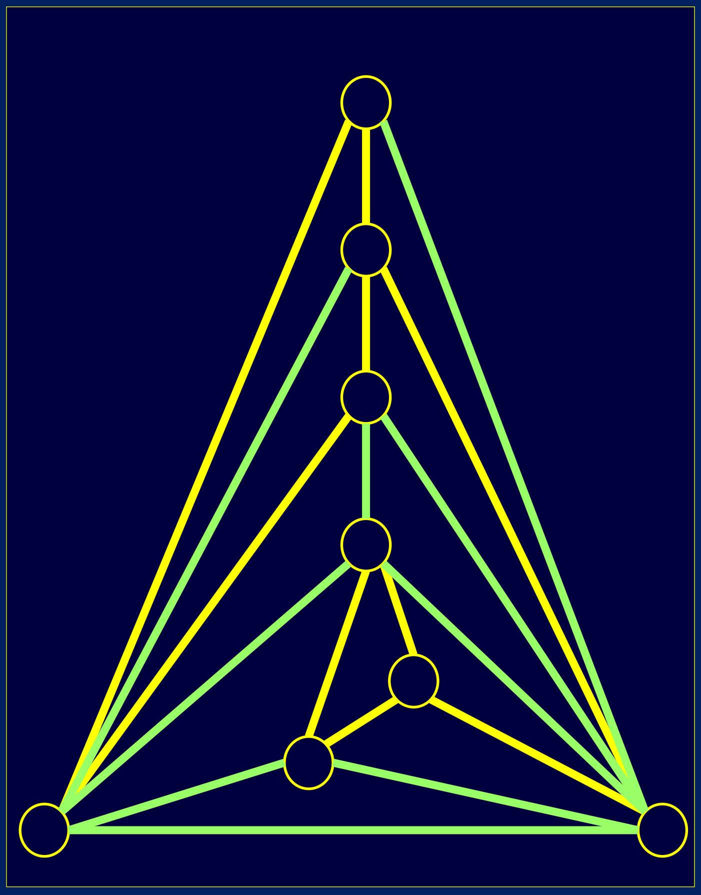
1. Add dummy edges to triangulate



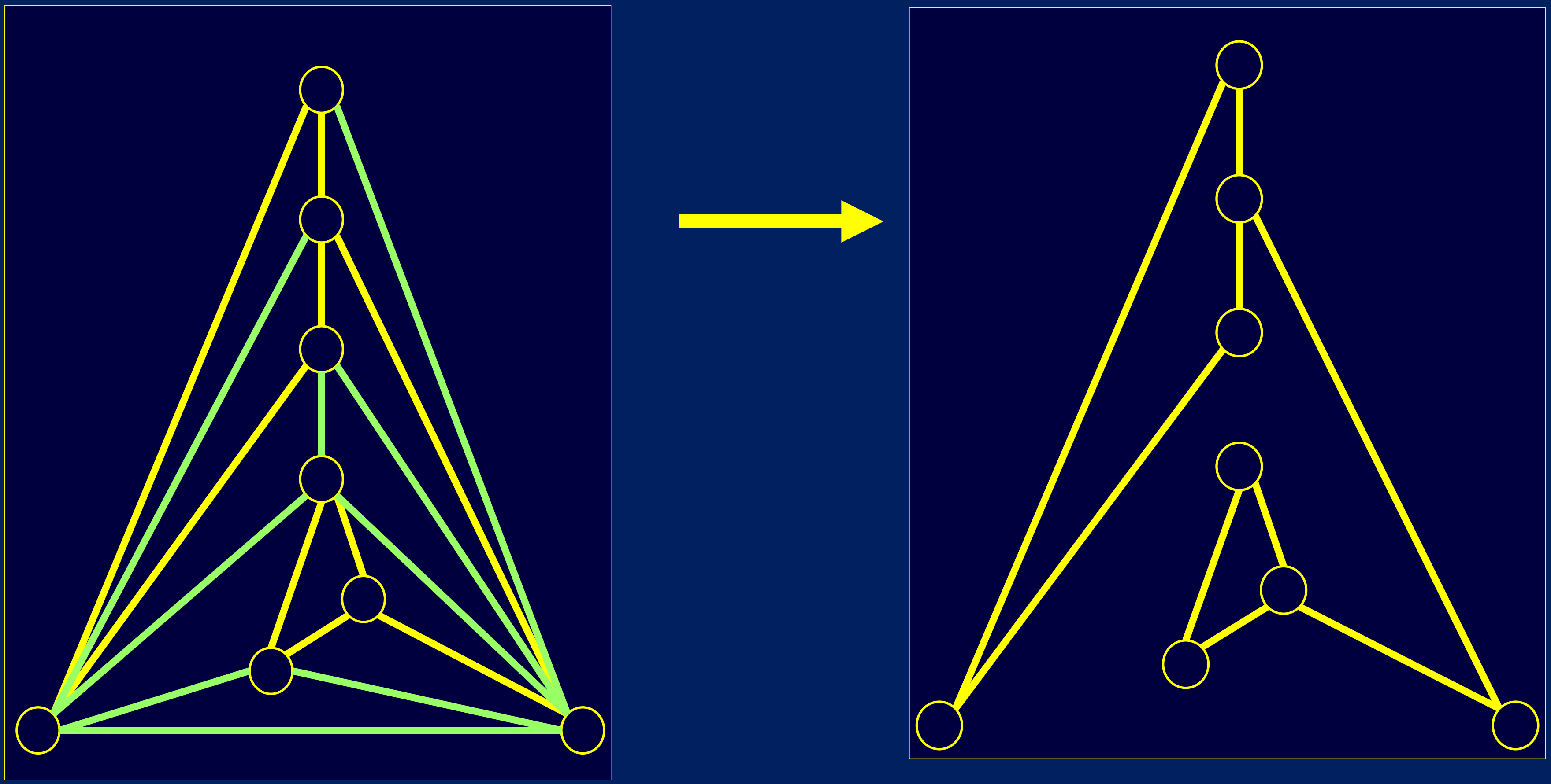
Planarity based methods



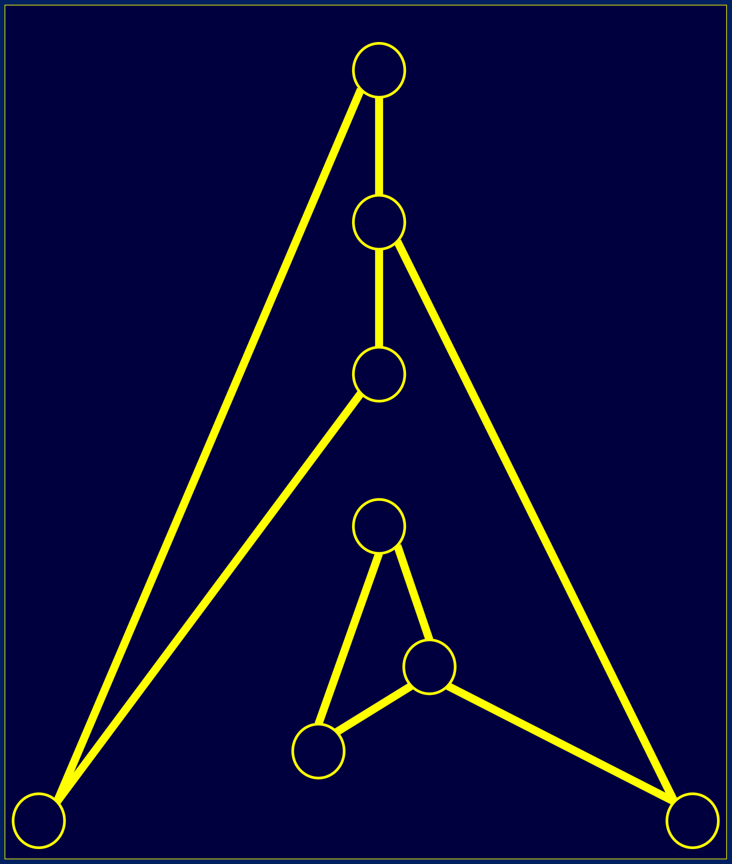
2. Draw the augmented graph.



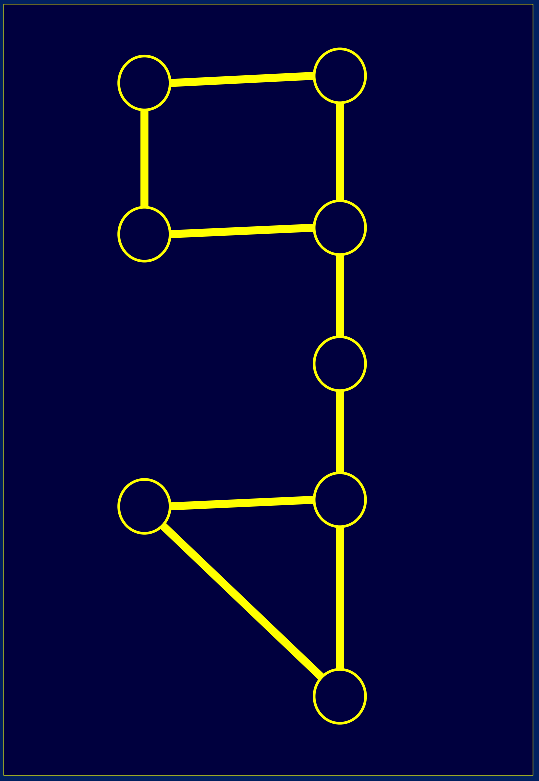
3. Delete the dummy edges



Note: the resulting drawing is ugly.



A better drawing

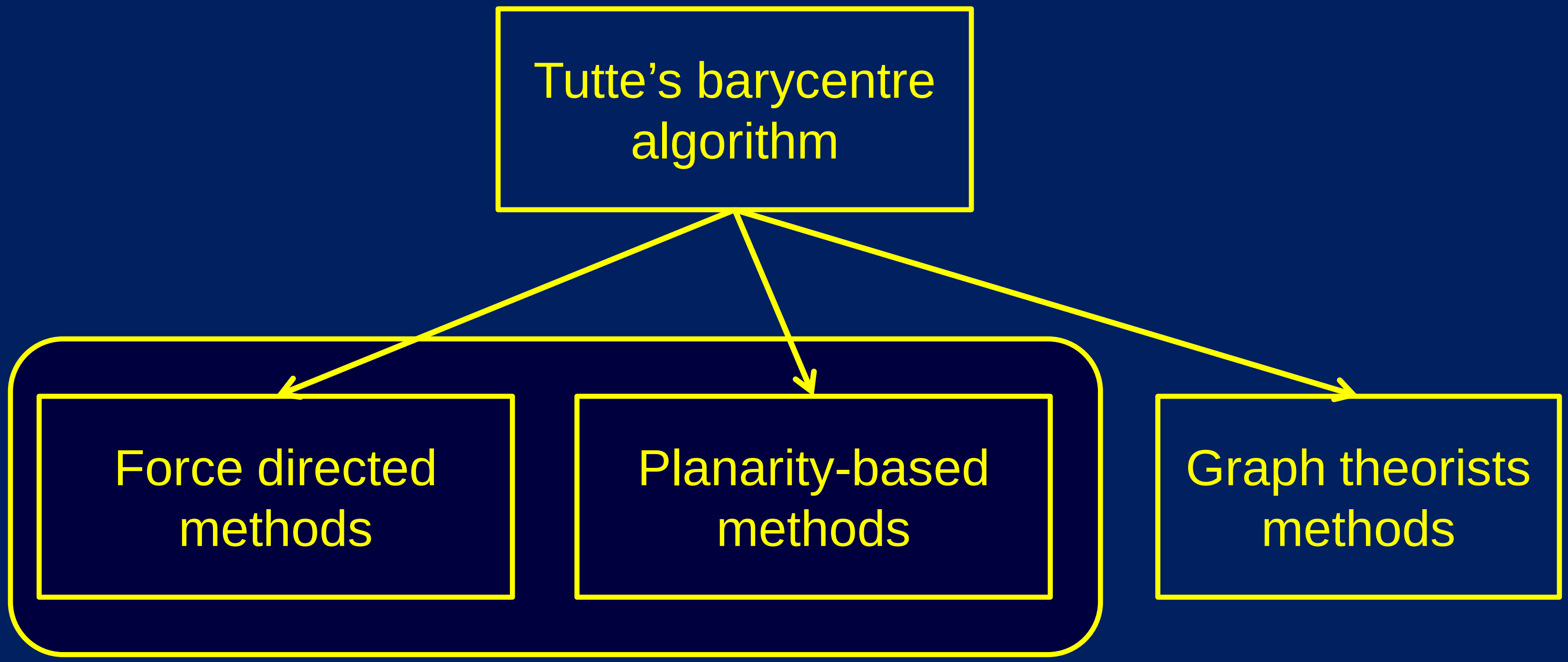


Current state-of-the-art for planarity based methods:

- There are many small improvements to the deFraysseix-Pach-Pollack-Chrobak algorithm.
- But none have overcome all the connectivity augmentation problem.

- *Almost no planarity based methods* have been adopted in commercial software …

- Despite the fact that planarity is the single most important aesthetic criterion.



Tutte’s barycentre algorithm
Force directed methods
Planarity-based methods
Graph theorists methods

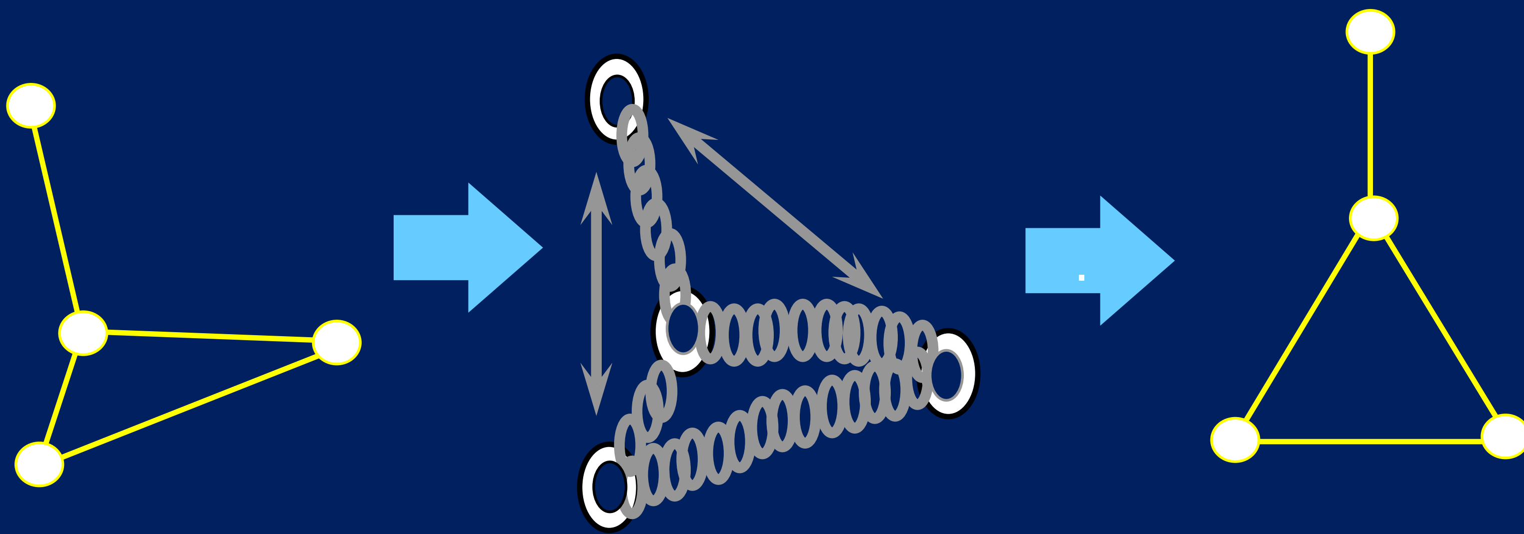
Energy/force methods after Tutte

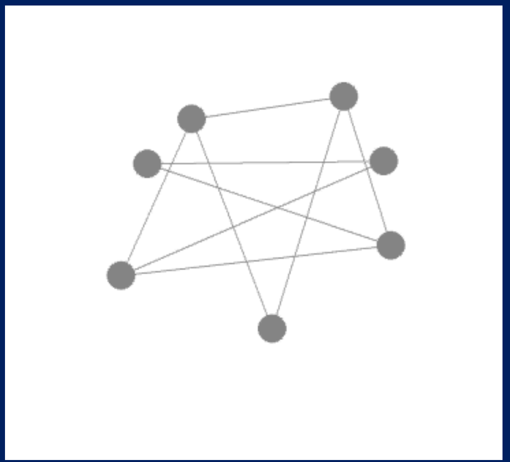
To improve Tutte's barycentre algorithm, we need to prevent vertices from becoming very close together.

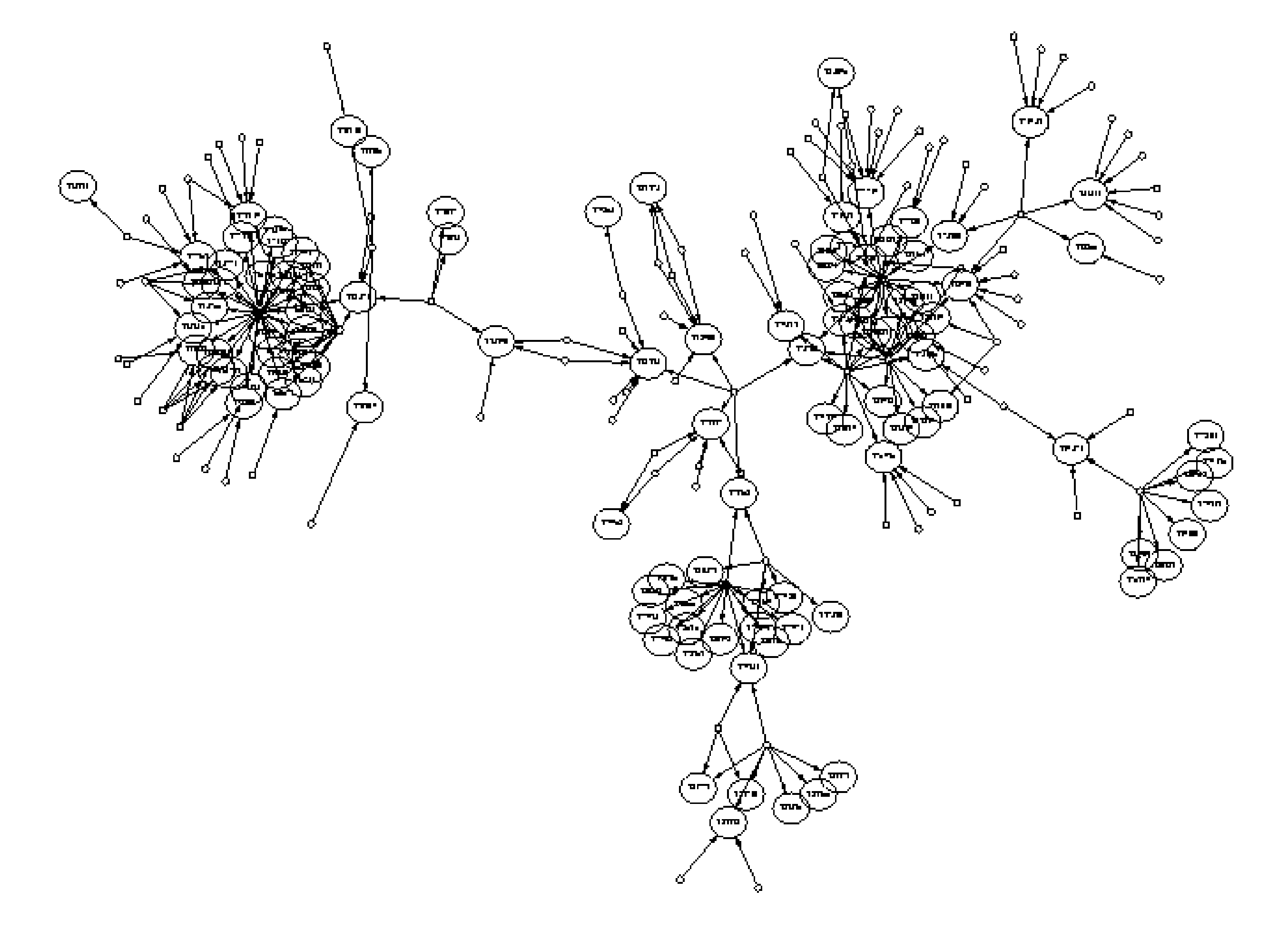
This can be done with forces:-

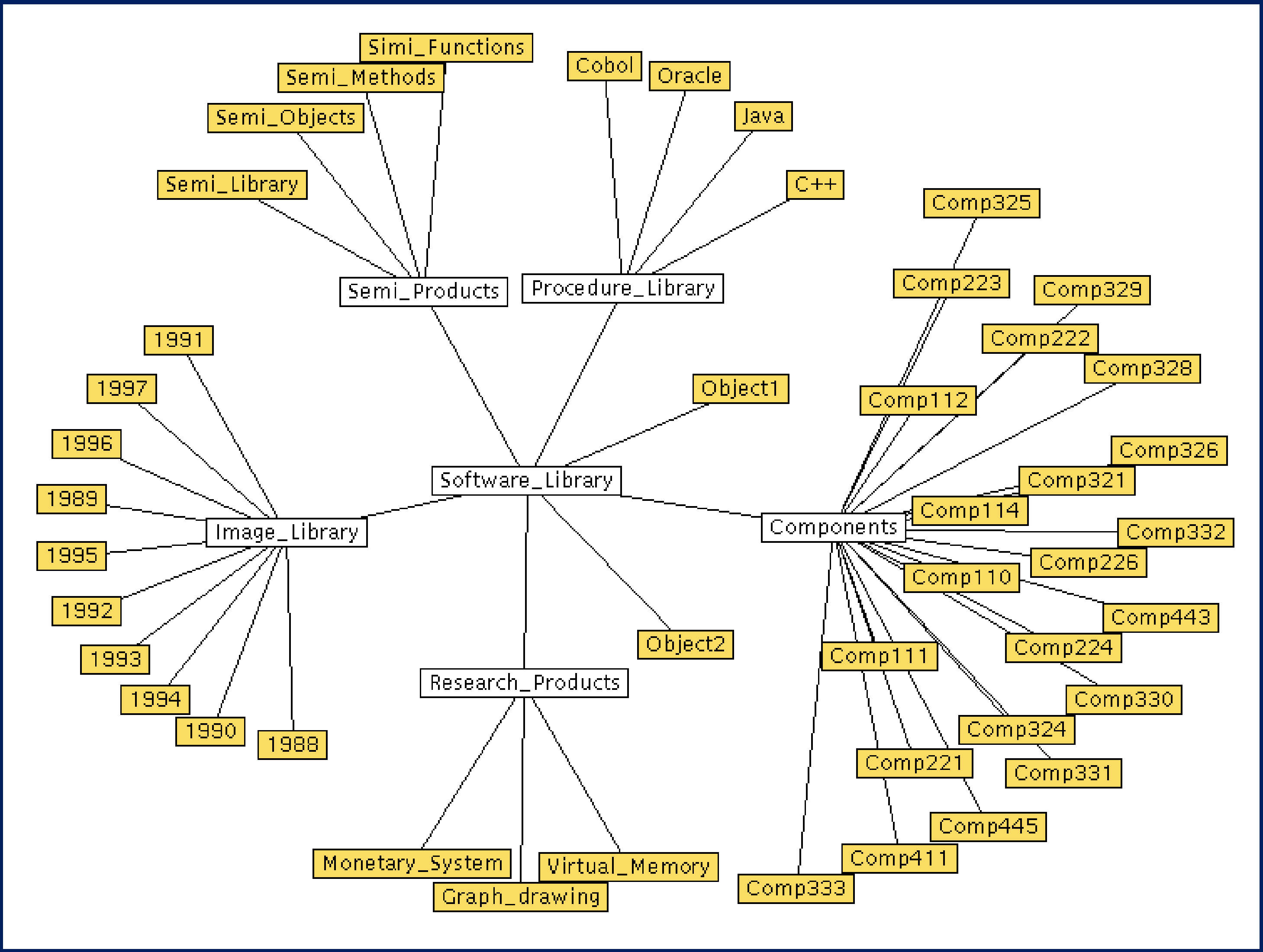
1. Use springs of nonzero natural length

2. Use an inverse square law repulsive force between nonadjacent vertices.





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Semi_Objects
Semi_Library
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Image_Library
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Research_Products
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Comp324
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Comp411
Comp333
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Virtual_Memory
Graph_drawing

Force exerted by a vertex v on a vertex u:

If u and v are adjacent:

$$f_{spring}(u,v) = k_{uv} \, |d(u,v) - q_{uv}|$$

where

- k_{uv} is constant, it is the *strength* of the spring between u and v
- $d(u,v)$ is the Euclidean distance between u and v
- q_{uv} is constant, it is the *natural length* of the u-v spring

If u and v are not adjacent:

$$f_{nonajac}(u,v) = r_{uv} / d(u,v)^2$$

where

- r_{uv} is constant, it is the *strength* of the repulsive force

Total force on a vertex u:

$$F(u) = \Sigma\, f_{spring}(u,v) \;+ \Sigma\, f_{nonajac}(u,w)$$

where

- The first sum is over all vertices v adjacent to u
- The second sum is over all vertices w not adjacent to u

A minimum energy configuration satisfies

$$F(u) = 0$$

for each vertex u.

This is a system of *nonlinear* equations.

Note

1. In general, the solution to this system of equations is not unique, that is, there are local minima that may not be global.
2. Many methods to solve this system of equations are available. Some methods are fast, some are slow, depending on the equations.

Force-based techniques can be constrained in various ways.

The constants in the force definitions

$$f_{spring}(u,v) = k_{uv} \, |d(u,v) - q_{uv}|$$

$$f_{nonajac}(u,v) = r_{uv} / d(u,v)^2$$

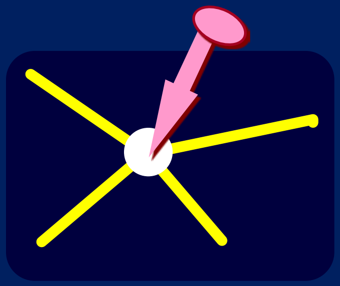
can be chosen to reflect the relationships in the domain.

For example

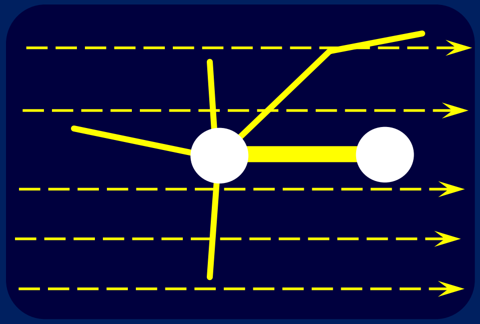
- If the edge between u and v is important, then we can choose k_{uv} to be large and q_{uv} to be small.

Force-based techniques can be constrained in various ways.

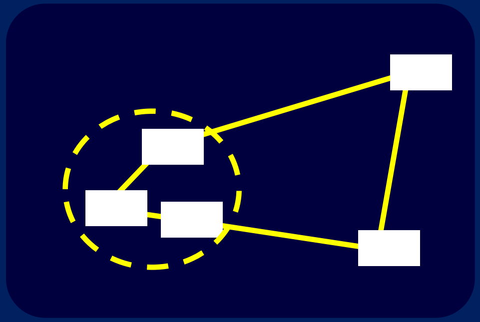
Nails can be used to hold a node in place.



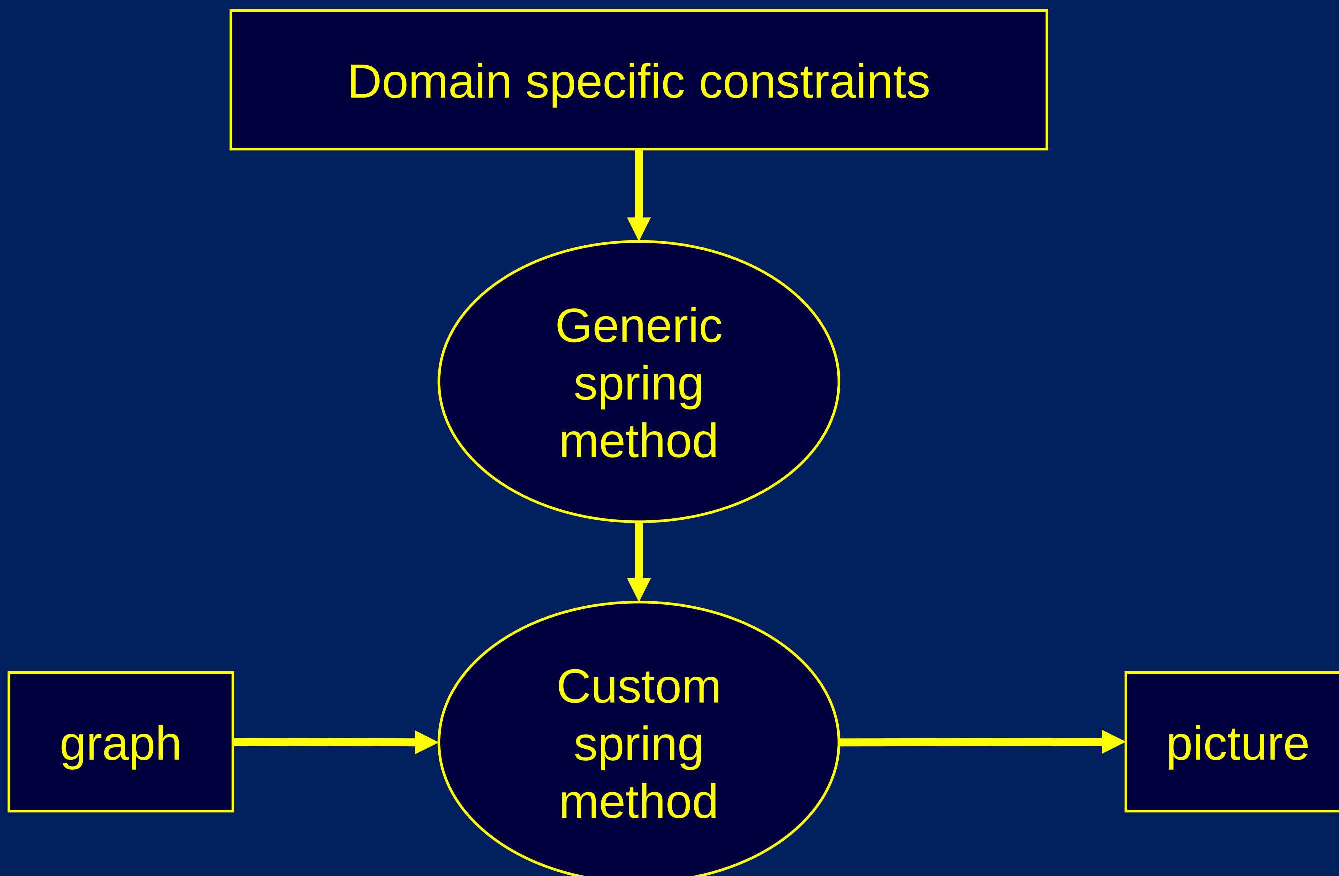
Magnetic fields and *magnetized springs* can be used to align nodes in various ways.

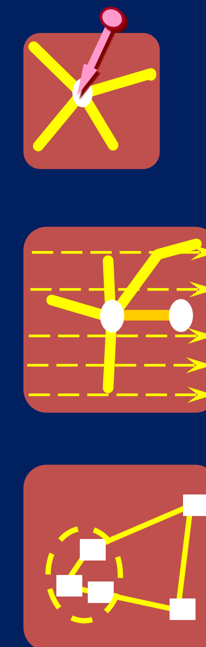


Attractive forces can be used to keep clusters together.



These constraints are very useful in customizing the general spring method to a specific domain.





Example:

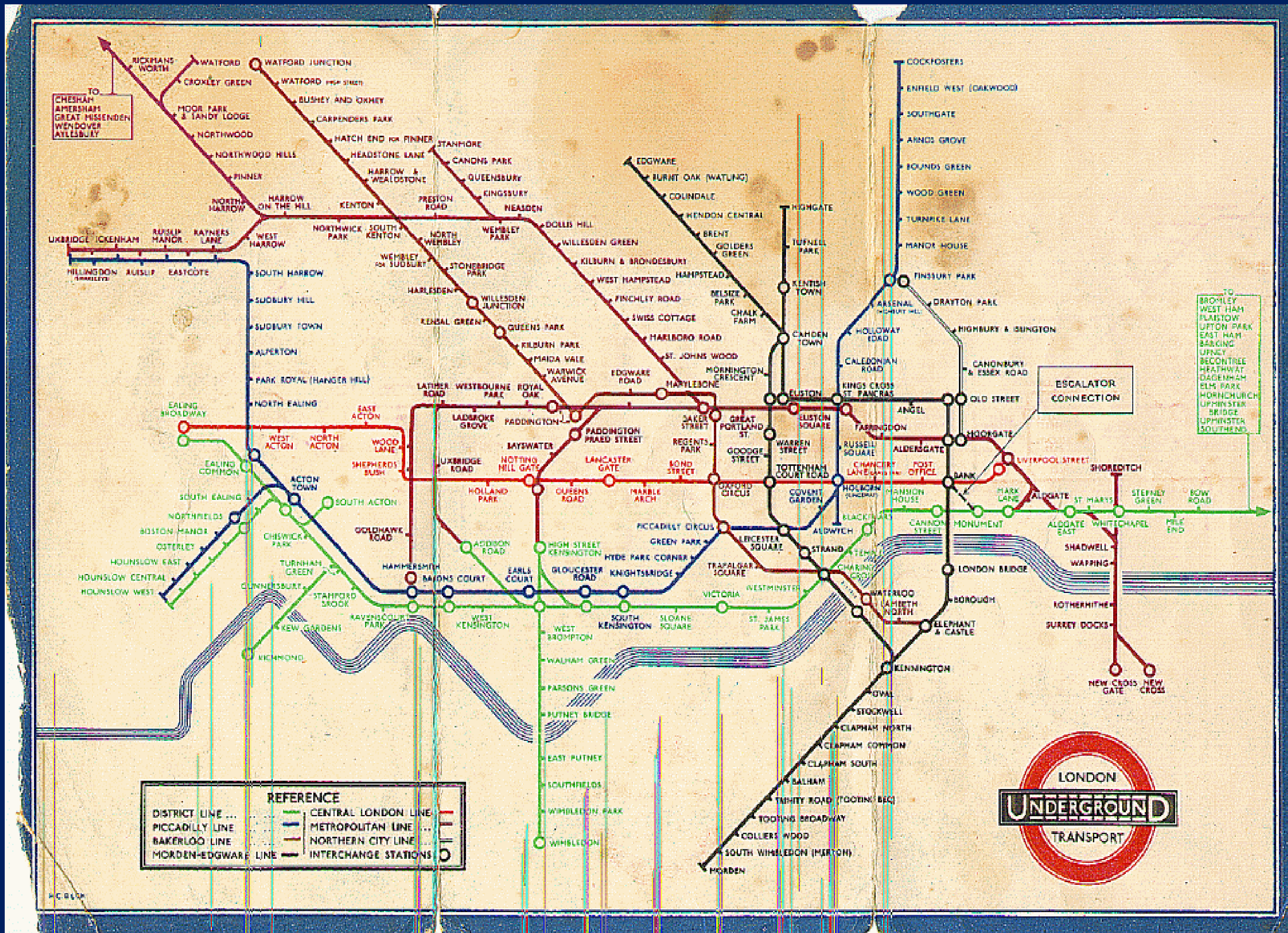
<u>Metro Maps</u>

- *Damian Merrick*
- *SeokHee Hong*
- *Hugo do Nascimento*

The Metro Map Problem

- Existing metro maps, produced by professional graphic artists, are ***excellent examples*** of network visualization

- *Can we produce good metro maps* ***automatically?***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MORDEN-EDGWARE LINE
CENTRAL LONDON LINE
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COCKFOSTERS
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ARNOS GROVE
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WOOD GREEN
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WIMBLEDON
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PARSONS GREEN
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RICHMOND
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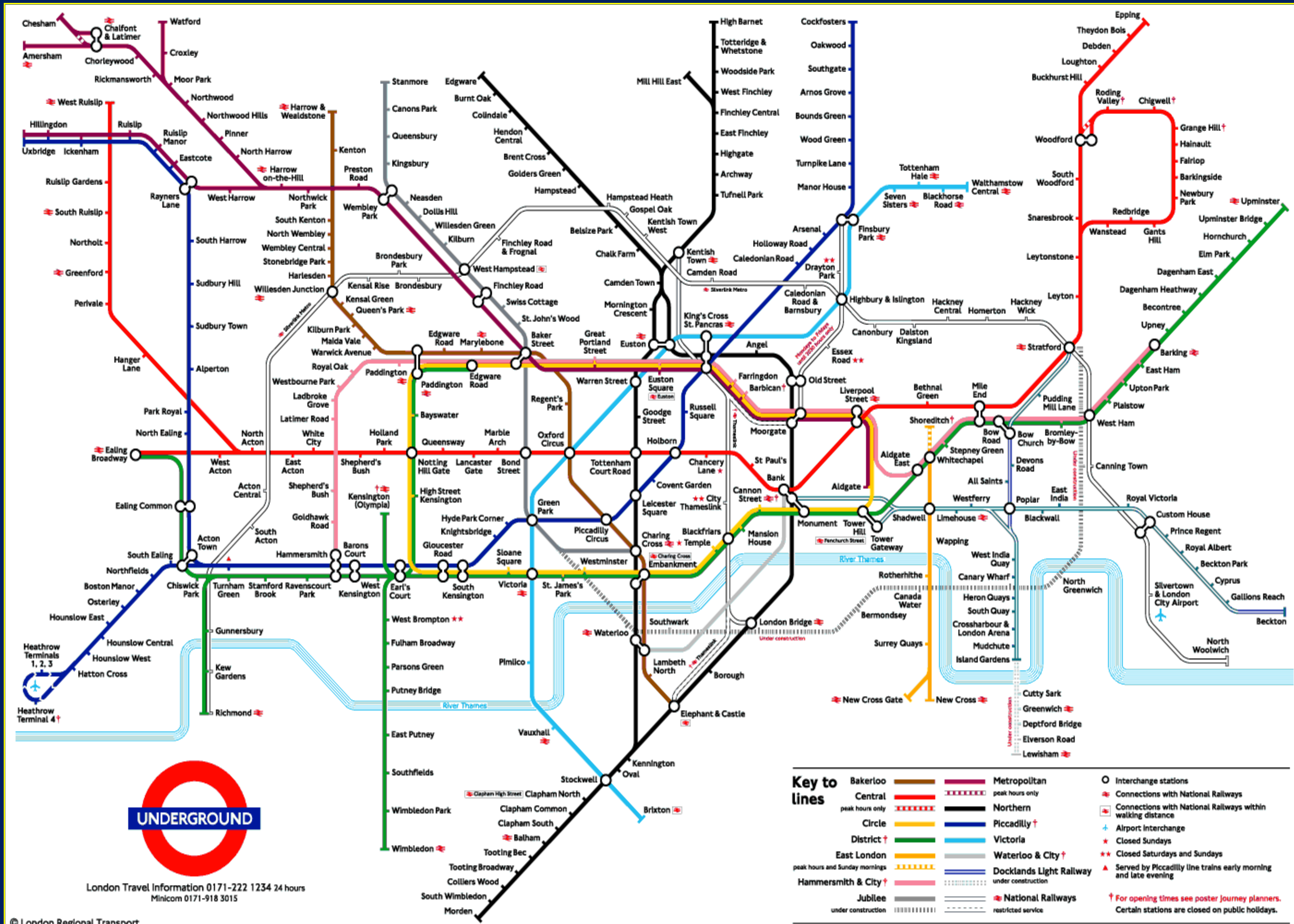
ROTHERHITHE
SURREY DOCKS

H. Beck, 1931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Kingsbury
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Wembley Park
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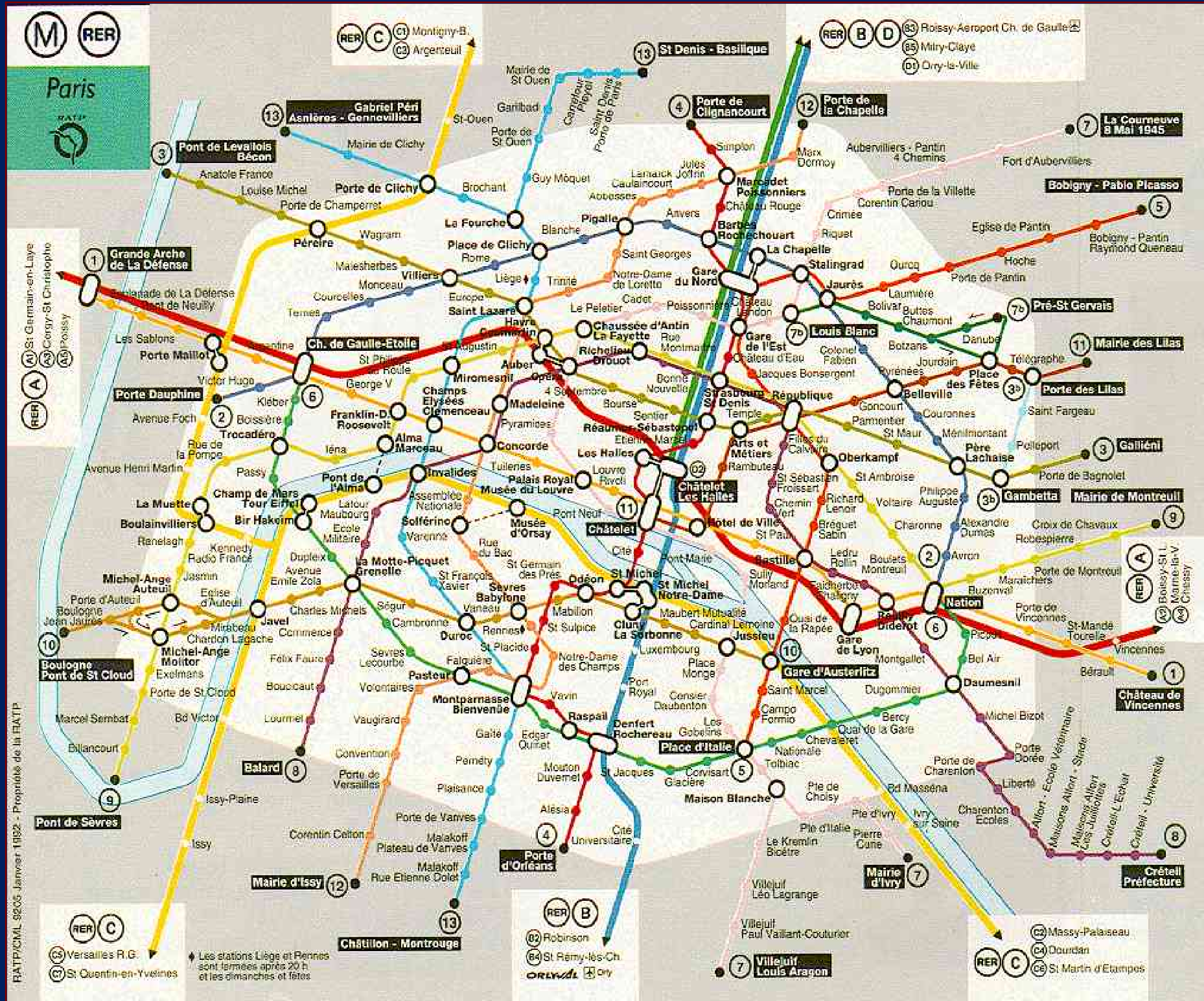

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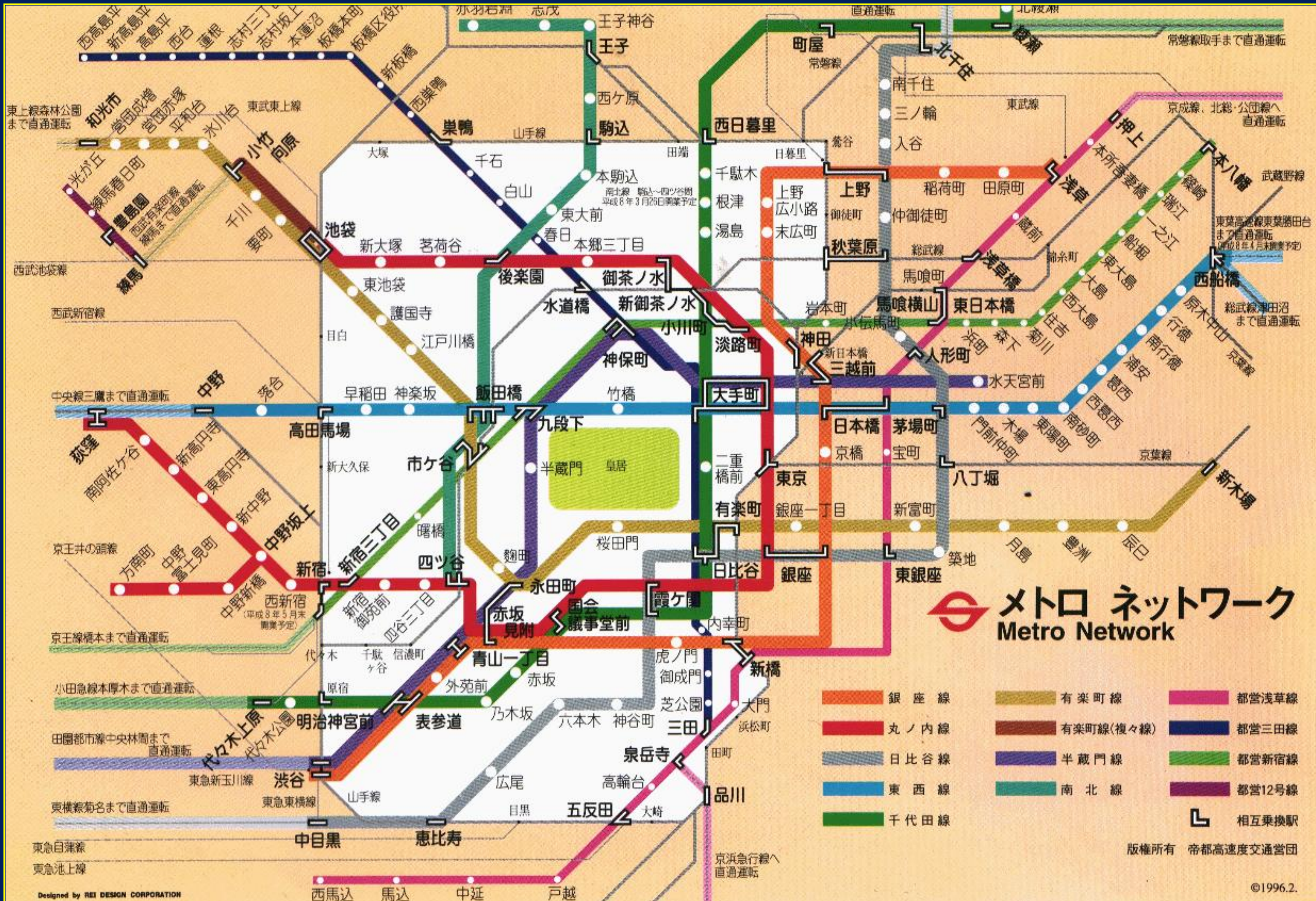
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本所吾妻橋
蔵前
浅草橋
仲御徒町
秋葉原
末広町
上野広小路
湯島
根津
千駄木
西日暮里
町屋
綾瀬
常磐線取手まで直通運転
京成線、北総・公団線へ直通運転
東葉高速線東葉勝田台まで直通運転
総武線津田沼まで直通運転
和光市
営団成増
営団赤塚
平和台
氷川台
小竹向原
千川
要町
池袋
新大塚
茗荷谷
東池袋
護国寺
江戸川橋
飯田橋
早稲田
神楽坂
竹橋
市ケ谷
四ツ谷
麹町
永田町
赤坂見附
国会議事堂前
霞ケ関
桜田門
内幸町
虎ノ門
御成門
芝公園
大門
浜松町
三田
田町
泉岳寺
高輪台
品川
五反田
大崎
戸越
中延
馬込
西馬込
中野
落合
高田馬場
荻窪
南阿佐ケ谷
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東高円寺
新中野
中野坂上
中野富士見町
中野新橋
方南町
西新宿
新宿
新宿三丁目
新宿御苑前
四谷三丁目
曙橋
青山一丁目
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練馬春日町
豊島園
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西武有楽町線
西武池袋線
西武新宿線
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京王井の頭線
京王線橋本まで直通運転
小田急線本厚木まで直通運転
田園都市線中央林間まで直通運転
東急新玉川線
東急東横線
東横線菊名まで直通運転
東急目蒲線
東急池上線
京浜急行線へ直通運転
山手線
常磐線
東武線
東武東上線
武蔵野線
総武線
京成線


SYDNEY SUBURBAN AREA
SYDNEY HARBOUR
BOTANY BAY
HORNSBY
Normanhurst
Thornleigh
Pennant Hills
Beecroft
Cheltenham
Epping
Eastwood
Denistone
West Ryde
Meadowbank
Rhodes
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North Strathfield
Waitara
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CHATSWOOD
Artarmon
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NORTH SYDNEY
Milsons Point
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TOWN HALL
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Museum
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FAIRFIELD
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Villawood
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Chester Hill
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Yagoona
BANKSTOWN
Punchbowl
Wiley Park
Lakemba
Belmore
Campsie
Canterbury
Hurlstone Park
Dulwich Hill
Marrickville
Green Square
Mascot
Domestic
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Wolli Creek
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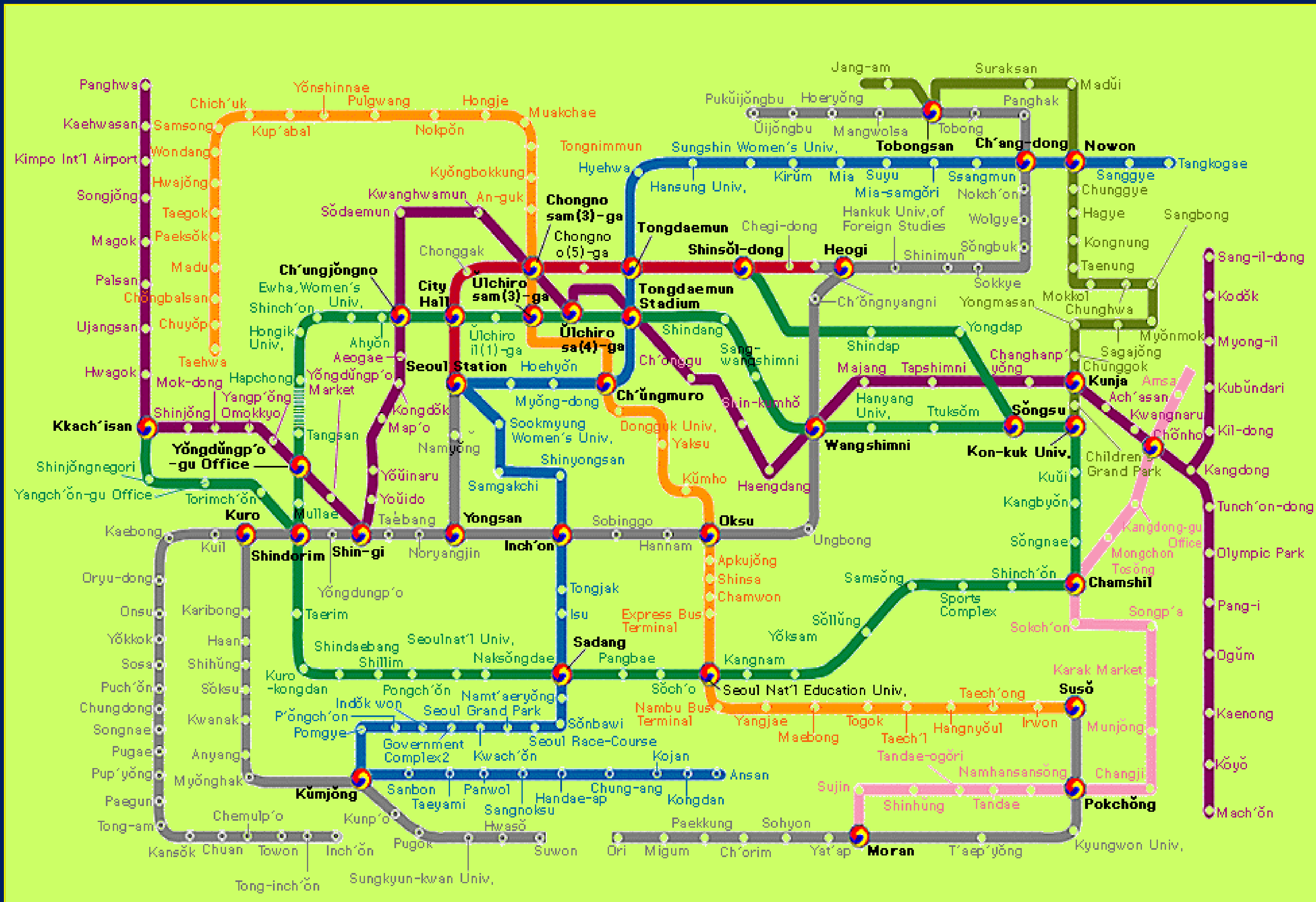

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OTTAVIANO
LEPANTO
P.ZA DI SPAGNA
BARBERINI
REPUBBLICA
C. PRETORIO
POLICLINICO
P.ZA BOLOGNA
TIBURTINA
QUINTILIANI
M B
REBIBBIA
P.MAMMOLO
S.M.SOCCORSO
PIETRALATA
M.TIBURTINI
FS
STAZ.TERMINI
CAVOUR
COLOSSEO
CIRCO MASSIMO
PIRAMIDE
OSTIENSE
VITTORIO E.
MANZONI
SAN GIOVANNI
RE DI ROMA
PONTE LUNGO
FURIO CAMILLO
COLLI ALBANI
ARCO DI TRAVERTINO
QUADRARO
NUMIDIO QUADRATO
LUCIO SESTIO
GIULIO AGRICOLA
SUBAUGUSTA
CINECITTA'
ANAGNINA
M A
MAGLIANA
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L.DA VINCI AEREOPORTO
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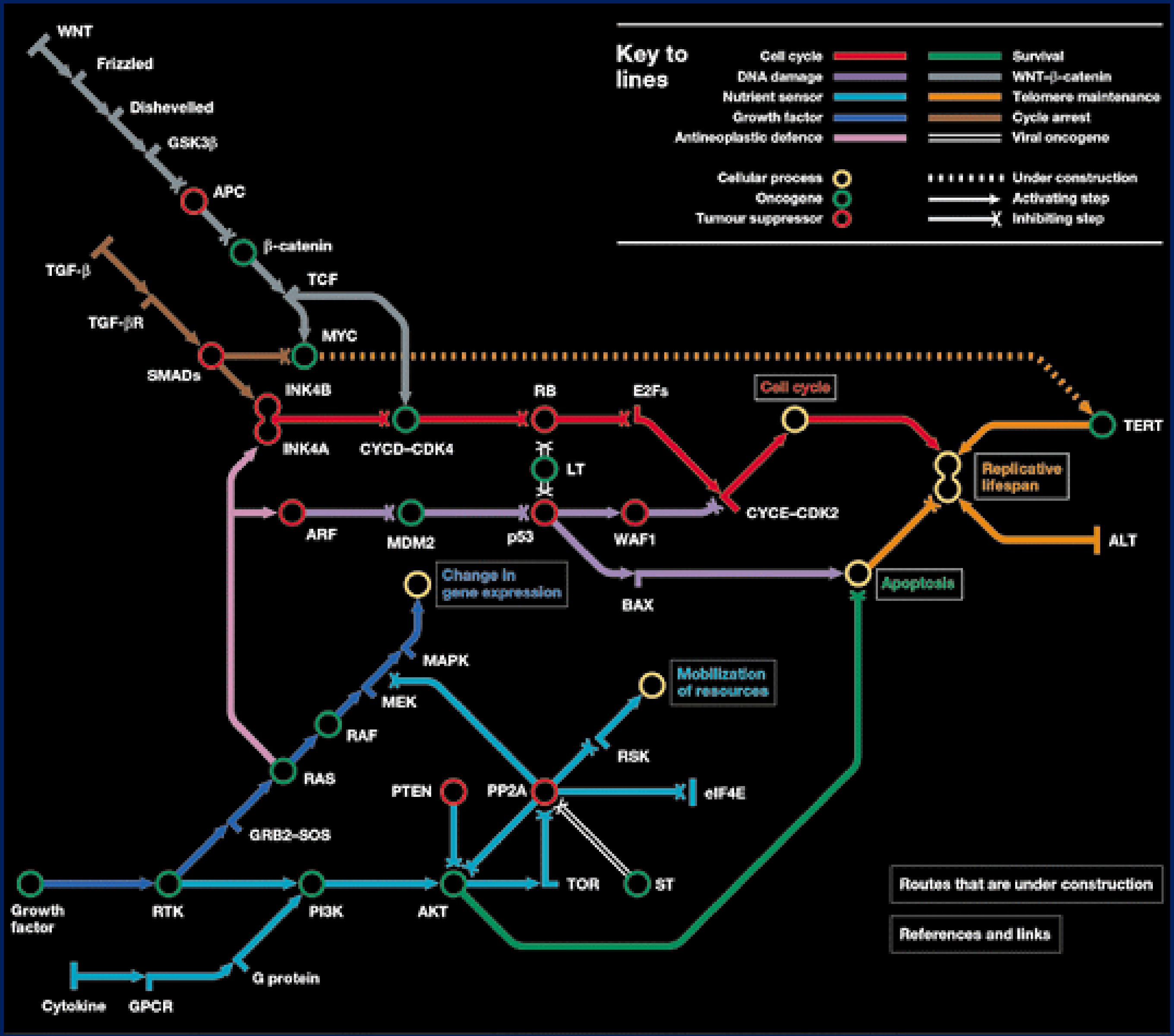
P.GALERIA
GARBATELLA
BASILICA DI S. PAOLO
MARCONI
MAGLIANA
M B
LAURENTINA
EUR FERMI
EUR PALASPORT
TOR DI VALLE
VITINIA
CASAL BERNOCCHI
ACILIA
OSTIA ANTICA
LIDO CENTRO
STELLA POLARE
CASTEL FUSANO
C.COLOMBO
M A METRO LINEA "A"
ANAGNINA - OTTAVIANO
M B METRO LINEA "B"
LAURENTINA - REBIBBIA
FIUMICINO - OSTIENSE
PIRAMIDE - C.COLOMBO (OSTIA)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Cell cycle
DNA damage
Nutrient sensor
Growth factor
Antineoplastic defence
Survival
WNT-β-catenin
Telomere maintenance
Cycle arrest
Viral oncogene
Cellular process
Oncogene
Tumour suppressor
Under construction
Activating step
Inhibiting step
WNT
Frizzled
Dishevelled
GSK3β
APC
β-catenin
TCF
TGF-β
TGF-βR
SMADs
MYC
INK4B
INK4A
CYCD–CDK4
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ARF
MDM2
p53
WAF1
CYCE–CDK2
ALT
BAX
Apoptosis
Change in gene expression
MAPK
MEK
RAF
RAS
Mobilization of resources
RSK
PTEN
PP2A
eIF4E
GRB2–SOS
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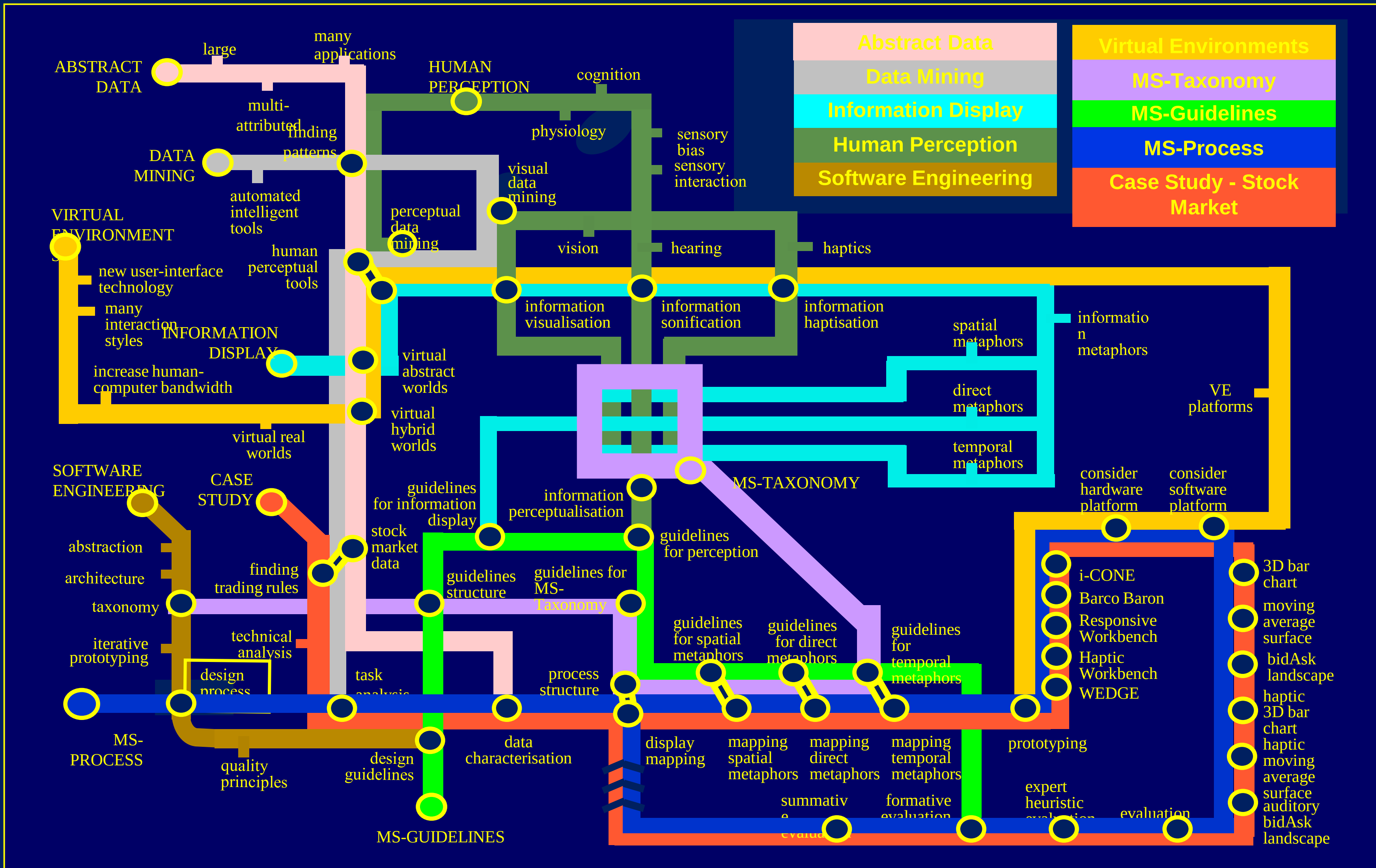
RTK
PI3K
AKT
TOR
ST
G protein
Cytokine
GPCR
Routes that are under construction
References and links

J. Hallinan



Keith Nesbitt

Scientific Question: Is there an ***E^3*** computer algorithm that can produce a layout of a metro map graph?

(E^3 = Effective, Efficient, Elegant)

Force directed method

1. Define forces that map good layout to low energy

2. Use continuous optimization methods to find a minimal energy state

Force directed method

Optimisation goals
- Routes straight
- Routes horizontal/ vertical/ 45º.

Set of forces:
- Stations→steel rings
- Interconnections → springs
- Vertical/horizontal/45º magnetic field
- Futher forces to preserve input topology

Find a layout with minimum energy

Elegance

Force directed method

Very intuitive mapping

Optimisation goals

- Routes straight
- Routes horizontal/ vertical/ 45^o.

Set of forces:

- Stations→steel rings
- Interconnections → springs
- Vertical/horizontal/45^o magnetic field

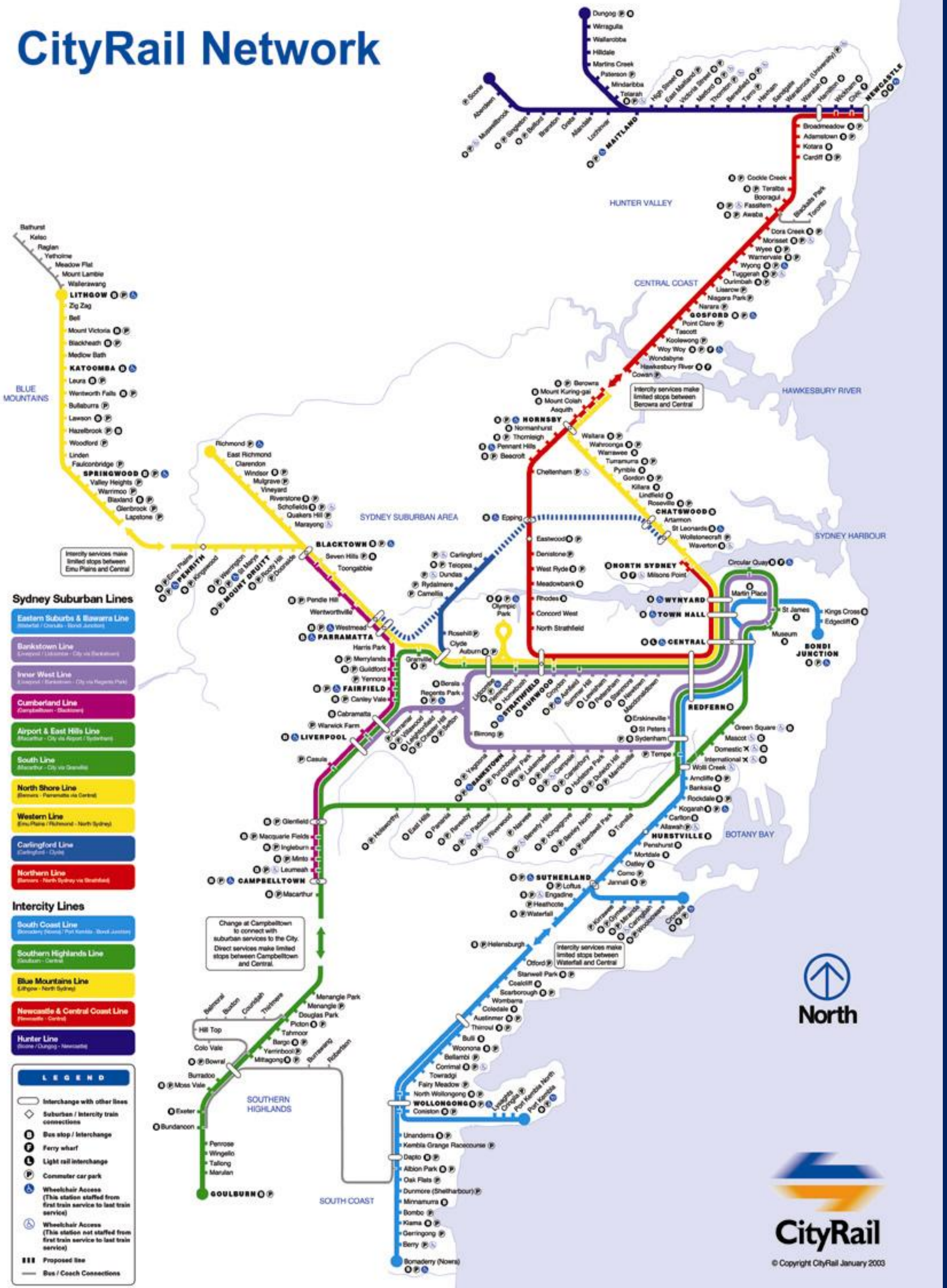
Very Elegant!

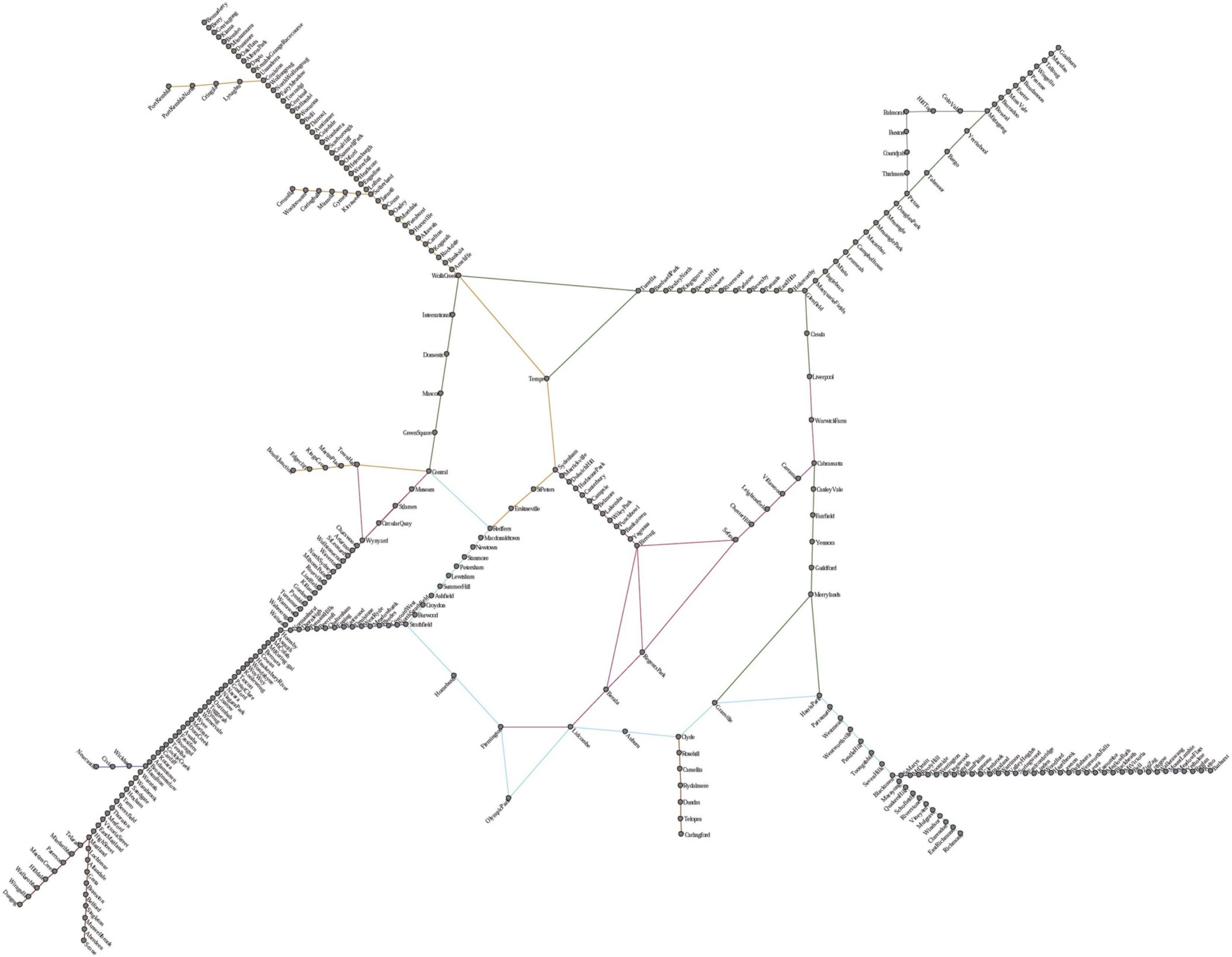
Find a layout with minimum energy

Classical numerical methods

Effectiveness

CityRail Network

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Newcastle
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Dungog
Scone
PortKembla
PortKemblaNorth
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Woolooware
Caringbah
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Gymea
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L5 Horta
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L4 Trinitat Nova
L3 Zona Universitària
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L1 Fondo
L2 Pep Ventura
L4 La Pau
L2 Paral·lel
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Tibidabo
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Av Tibidabo
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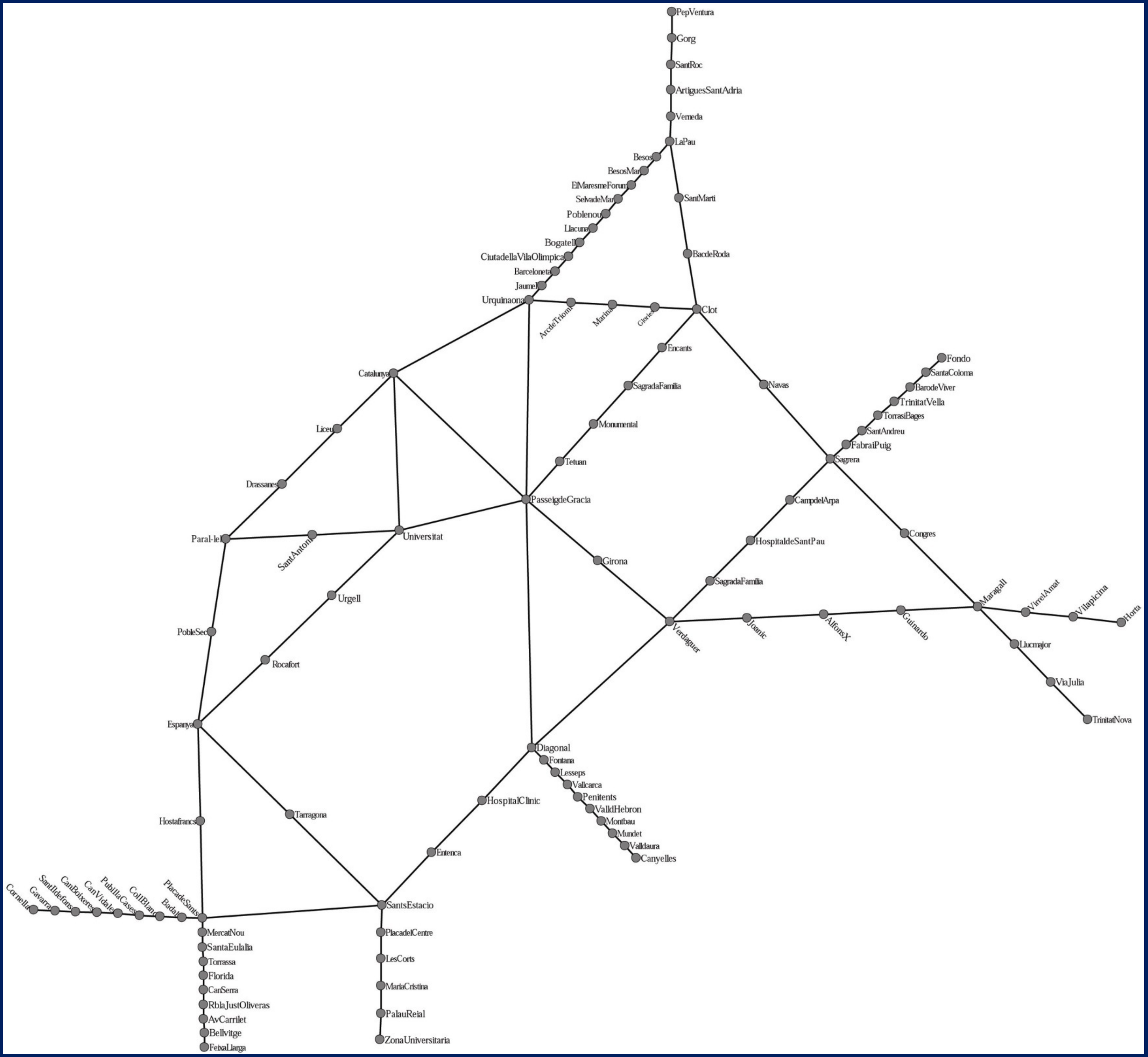
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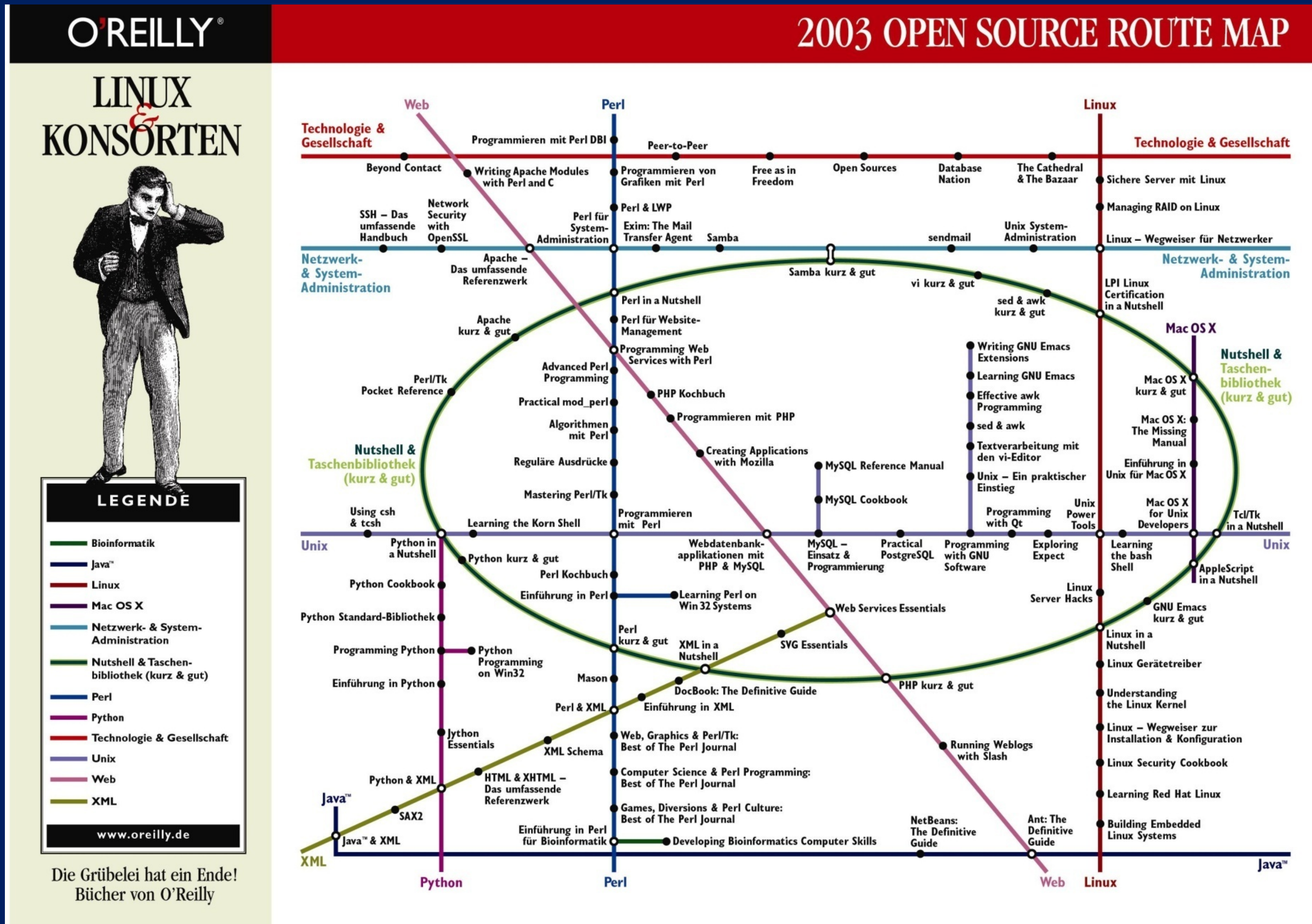

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XML in a Nutshell
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Java™ & XML
Einführung in Perl für Bioinformatik
Developing Bioinformatics Computer Skills
XML
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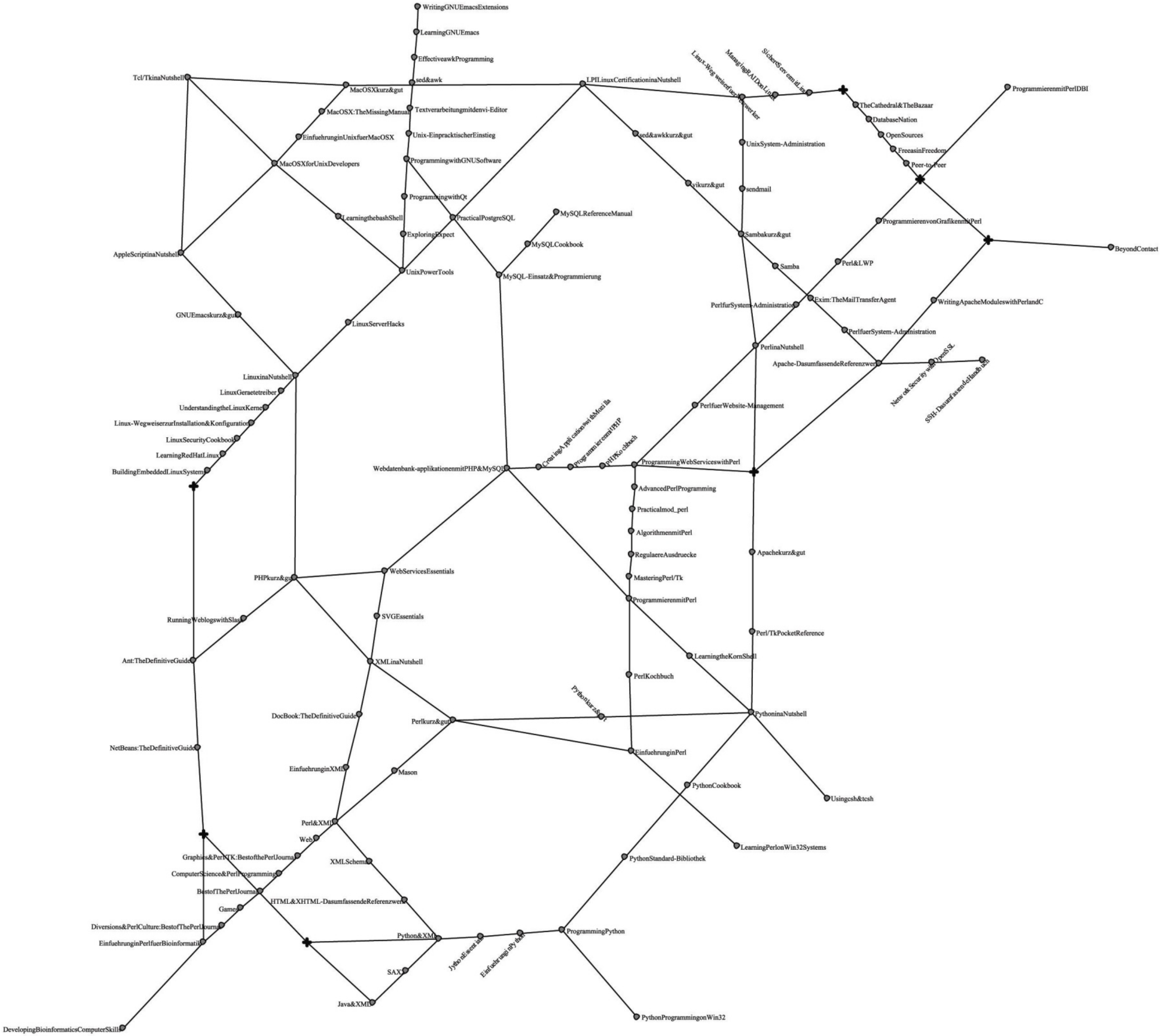

WritingGNUEmacsExtensions
LearningGNUEmacs
EffectiveawkProgramming
Tcl/TkinaNutshell
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Exim:TheMailTransferAgent
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Practicalmod_perl
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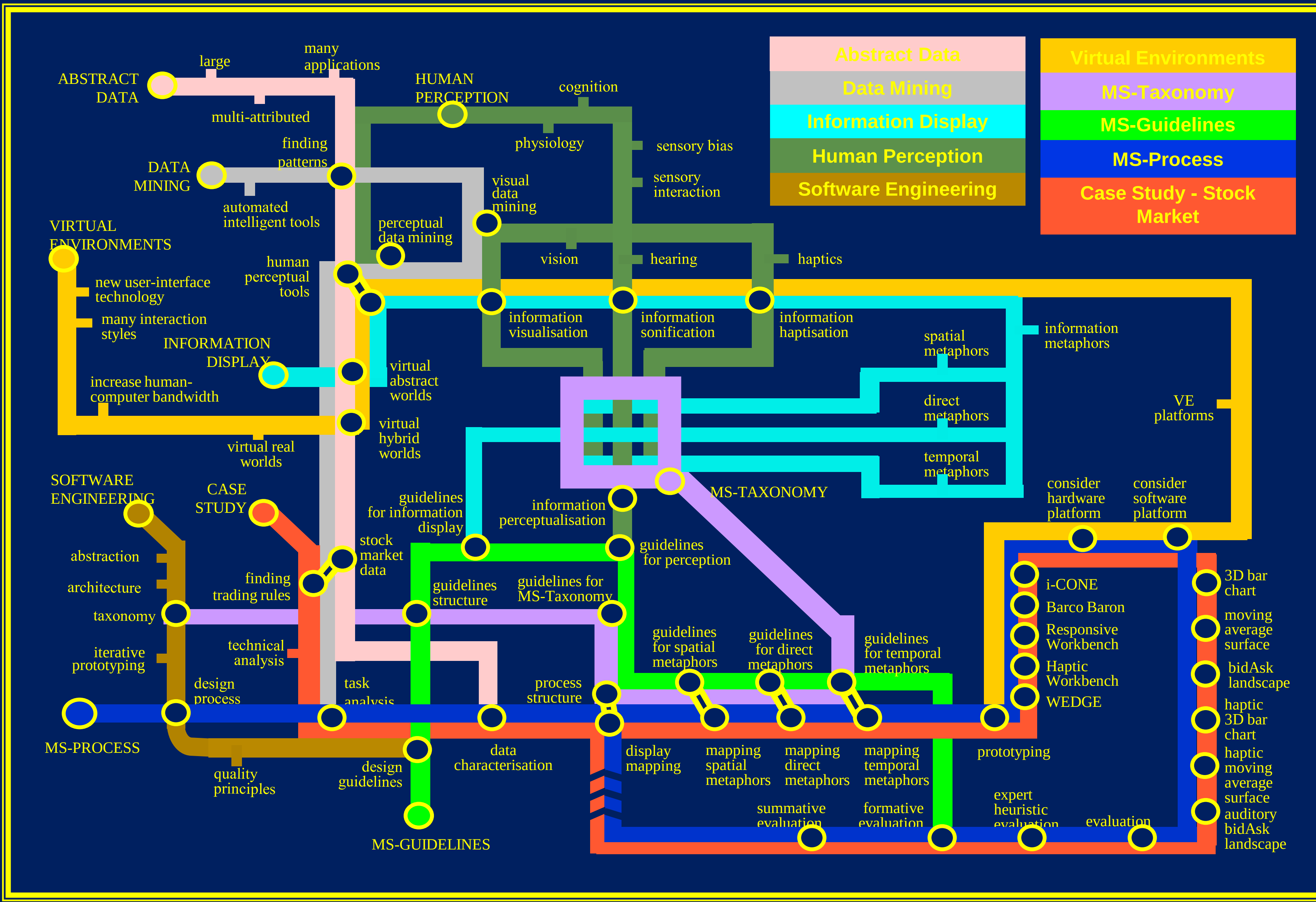

Abstract Data
Data Mining
Information Display
Human Perception
Software Engineering
Virtual Environments
MS-Taxonomy
MS-Guidelines
MS-Process
Case Study - Stock Market
ABSTRACT DATA
large
many applications
multi-attributed
HUMAN PERCEPTION
cognition
physiology
sensory bias
sensory interaction
DATA MINING
finding patterns
automated intelligent tools
visual data mining
perceptual data mining
VIRTUAL ENVIRONMENTS
new user-interface technology
many interaction styles
human perceptual tools
vision
hearing
haptics
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information sonification
information haptisation
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spatial metaphors
direct metaphors
temporal metaphors
increase human-computer bandwidth
virtual abstract worlds
virtual hybrid worlds
virtual real worlds
VE platforms
SOFTWARE ENGINEERING
CASE STUDY
MS-TAXONOMY
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information perceptualisation
consider hardware platform
consider software platform
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architecture
taxonomy
iterative prototyping
stock market data
finding trading rules
technical analysis
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guidelines for MS-Taxonomy
guidelines for perception
guidelines for spatial metaphors
guidelines for direct metaphors
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Haptic Workbench
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haptic moving average surface
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task analysis
process structure
MS-PROCESS
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design guidelines
data characterisation
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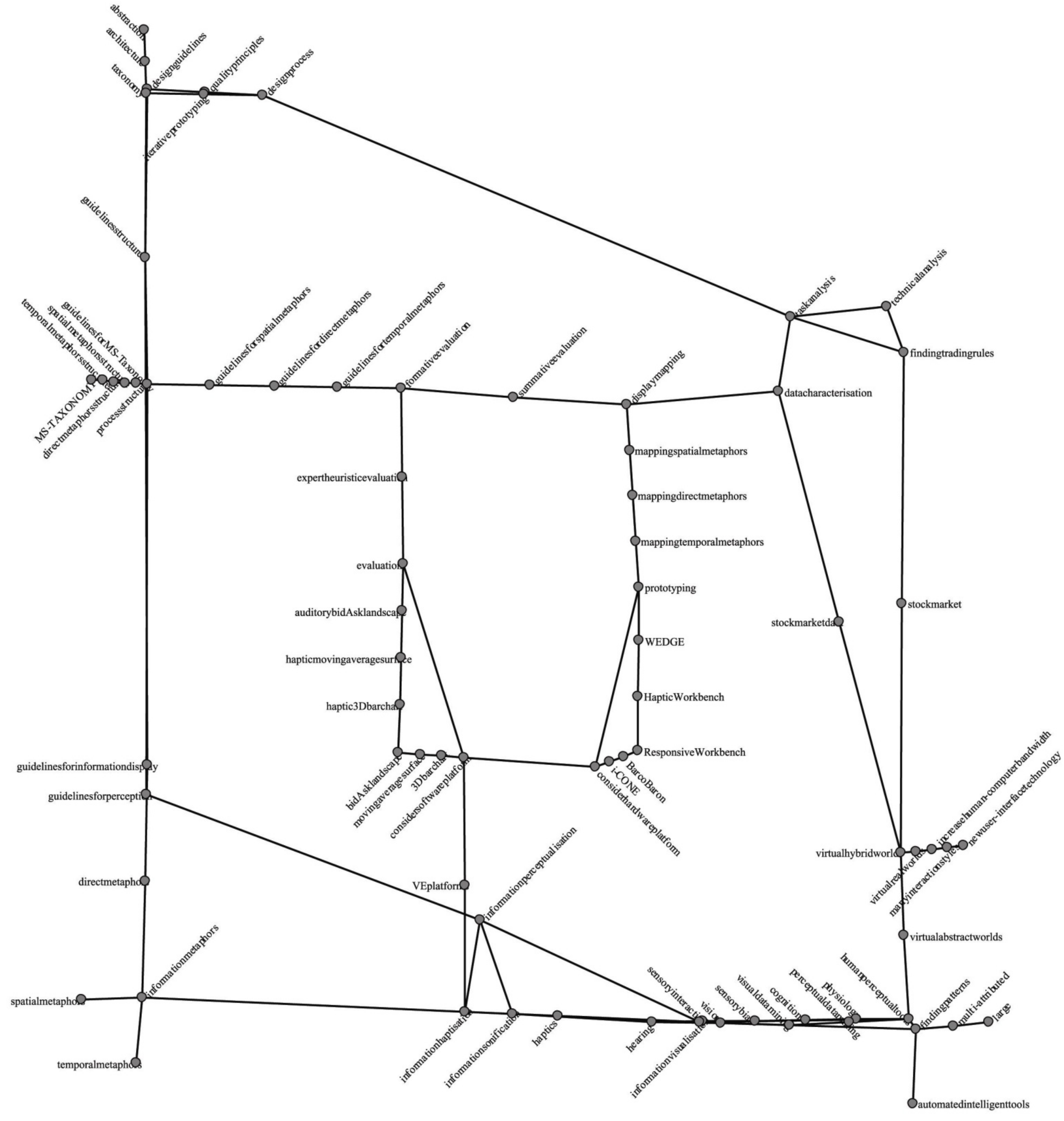
formative evaluation
expert heuristic evaluation
evaluation
MS-GUIDELINES



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ResponsiveWorkbench
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humanperceptualtools
findingpatterns
multi-attributed
large
automatedintelligenttools

Chesham
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Watford
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Chorleywood
Rickmansworth
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Northwood
Northwood Hills
Pinner
North Harrow
Harrow-on-the-Hill
West Harrow
West Ruislip
Ruislip
Hillingdon
Ruislip Manor
Eastcote
Uxbridge
Ickenham
Ruislip Gardens
Rayners Lane
South Ruislip
South Harrow
Northolt
Greenford
Sudbury Hill
Perivale
Sudbury Town
Hanger Lane
Alperton
Park Royal
North Ealing
North Acton
West Acton
Ealing Broadway
East Acton
Acton Central
Ealing Common
Acton Town
South Acton
South Ealing
Northfields
Boston Manor
Osterley
Hounslow East
Hounslow Central
Hounslow West
Hatton Cross
Heathrow Terminals 1, 2, 3
Heathrow Terminal 4
Chiswick Park
Turnham Green
Stamford Brook
Ravenscourt Park
Gunnersbury
Kew Gardens
Richmond
Harrow & Wealdstone
Kenton
Preston Road
Northwick Park
South Kenton
North Wembley
Wembley Central
Stonebridge Park
Harlesden
Willesden Junction
Wembley Park
Kensal Rise
Brondesbury
Brondesbury Park
Kensal Green
Queen's Park
Kilburn Park
Maida Vale
Warwick Avenue
Royal Oak
Westbourne Park
Ladbroke Grove
Latimer Road
White City
Shepherd's Bush
Holland Park
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Goldhawk Road
Hammersmith
Barons Court
West Kensington
Earl's Court
West Brompton
Fulham Broadway
Parsons Green
Putney Bridge
East Putney
Southfields
Wimbledon Park
Wimbledon
Stanmore
Canons Park
Queensbury
Kingsbury
Neasden
Dollis Hill
Willesden Green
Kilburn
West Hampstead
Finchley Road & Frognal
Finchley Road
Swiss Cottage
St. John's Wood
Edgware
Burnt Oak
Colindale
Hendon Central
Brent Cross
Golders Green
Hampstead
Belsize Park
Chalk Farm
Hampstead Heath
Gospel Oak
Kentish Town West
Camden Road
Camden Town
Mornington Crescent
Kentish Town
Mill Hill East
High Barnet
Totteridge & Whetstone
Woodside Park
West Finchley
Finchley Central
East Finchley
Highgate
Archway
Tufnell Park
Cockfosters
Oakwood
Southgate
Arnos Grove
Bounds Green
Wood Green
Turnpike Lane
Manor House
Arsenal
Holloway Road
Caledonian Road
Drayton Park
Finsbury Park
Caledonian Road & Barnsbury
Highbury & Islington
Silverlink Metro
Tottenham Hale
Seven Sisters
Blackhorse Road
Walthamstow Central
Paddington
Edgware Road
Marylebone
Baker Street
Great Portland Street
Euston
Euston Square
Warren Street
Regent's Park
Oxford Circus
Bayswater
Queensway
Marble Arch
Notting Hill Gate
Lancaster Gate
Bond Street
High Street Kensington
Hyde Park Corner
Knightsbridge
Gloucester Road
South Kensington
Sloane Square
Green Park
Piccadilly Circus
Victoria
St. James's Park
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Pimlico
Vauxhall
Stockwell
Brixton
Clapham North
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Clapham Common
Clapham South
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Tottenham Court Road
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Embankment
Blackfriars
Temple
Mansion House
Cannon Street
Monument
Tower Hill
Fenchurch Street
Tower Gateway
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Moorgate
Liverpool Street
St Paul's
Bank
Aldgate
Aldgate East
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Essex Road
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Dalston Kingsland
Hackney Central
Homerton
Hackney Wick
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Bow Church
Bromley-by-Bow
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New Cross
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Heron Quays
South Quay
Crossharbour & London Arena
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East Ham
Barking
Royal Victoria
Custom House
Prince Regent
Royal Albert
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Cyprus
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Beckton
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Epping
Theydon Bois
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Hainault
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Woodford
South Woodford
Snaresbrook
Redbridge
Wanstead
Gants Hill
Leytonstone
Leyton
Upminster
Upminster Bridge
Hornchurch
Elm Park
Dagenham East
Dagenham Heathway
Becontree
Upney
River Thames
Under construction
Mondays to Fridays until 2030 hours only
Key to lines
Bakerloo
Central
peak hours only
Circle
District
East London
peak hours and Sunday mornings
Hammersmith & City
Jubilee
under construction
Metropolitan
peak hours only
Northern
Piccadilly
Victoria
Waterloo & City
Docklands Light Railway
under construction
National Railways
restricted service
Interchange stations
Connections with National Railways
Connections with National Railways within walking distance
Airport interchange
Closed Sundays
Closed Saturdays and Sundays
Served by Piccadilly line trains early morning and late evening
For opening times see poster journey planners.
Certain stations are closed on public holidays.
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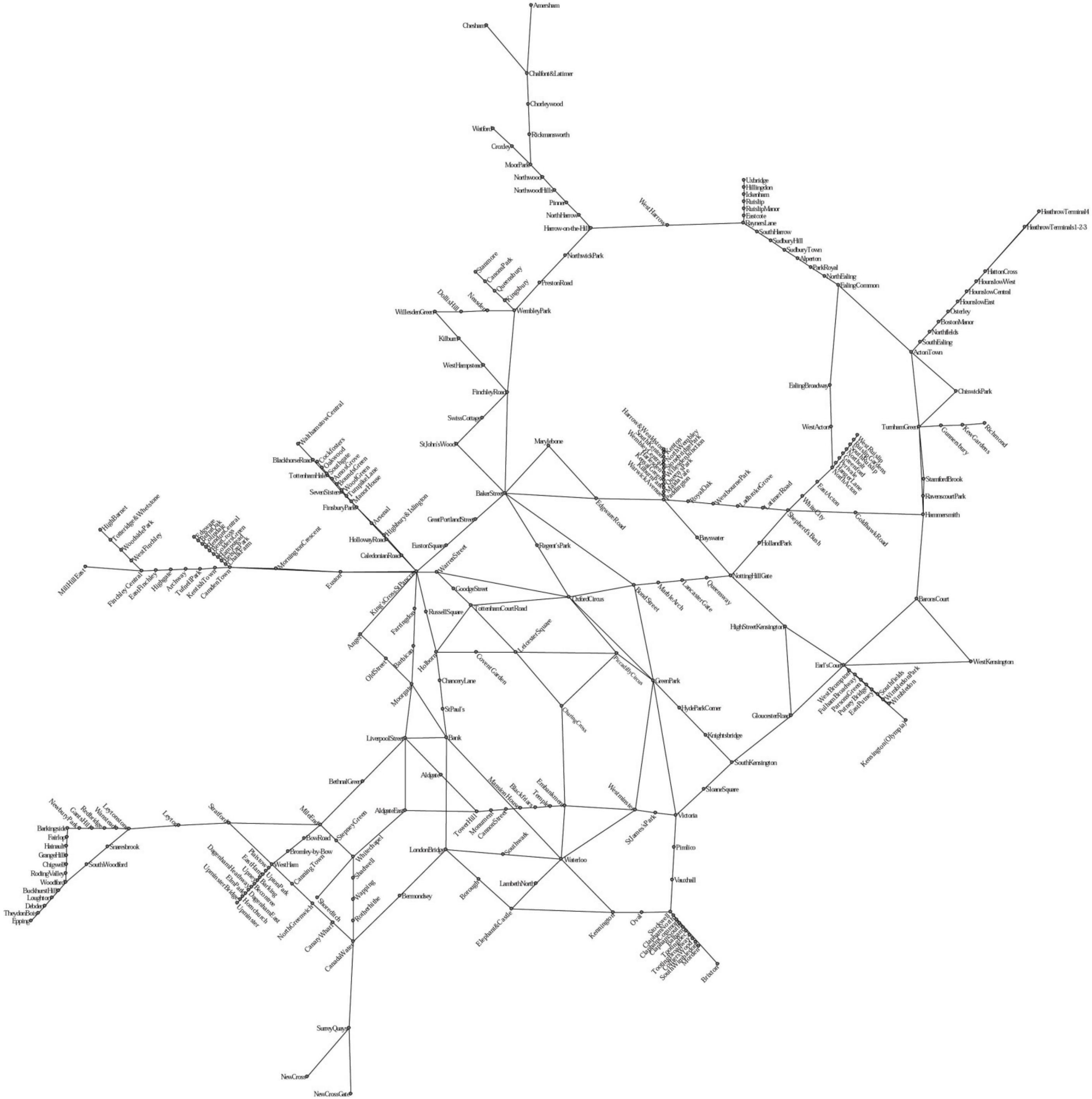

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RuislipManor
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HounslowCentral
HounslowEast
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EalingBroadway
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TurnhamGreen
Gunnersbury
KewGardens
Richmond
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Hammersmith
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NorthwickPark
PrestonRoad
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CanonsPark
Queensbury
Kingsbury
WembleyPark
Neasden
DollisHill
WillesdenGreen
Kilburn
WestHampstead
FinchleyRoad
SwissCottage
StJohn'sWood
BakerStreet
Marylebone
EdgwareRoad
Regent'sPark
GreatPortlandStreet
EustonSquare
WarrenStreet
Euston
MorningtonCrescent
CamdenTown
KentishTown
TufnellPark
Archway
Highgate
EastFinchley
FinchleyCentral
MillHillEast
WestFinchley
WoodsidePark
Totteridge&Whetstone
HighBarnet
WalthamstowCentral
BlackhorseRoad
TottenhamHale
SevenSisters
FinsburyPark
Arsenal
Highbury&Islington
HollowayRoad
CaledonianRoad
King'sCross StPancras
GoodgeStreet
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TottenhamCourtRoad
OxfordCircus
BondStreet
MarbleArch
LancasterGate
Queensway
NottingHillGate
Bayswater
RoyalOak
WestbournePark
LadbrokeGrove
LatimerRoad
WhiteCity
EastActon
Shepherd'sBush
HollandPark
HighStreetKensington
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NewCross
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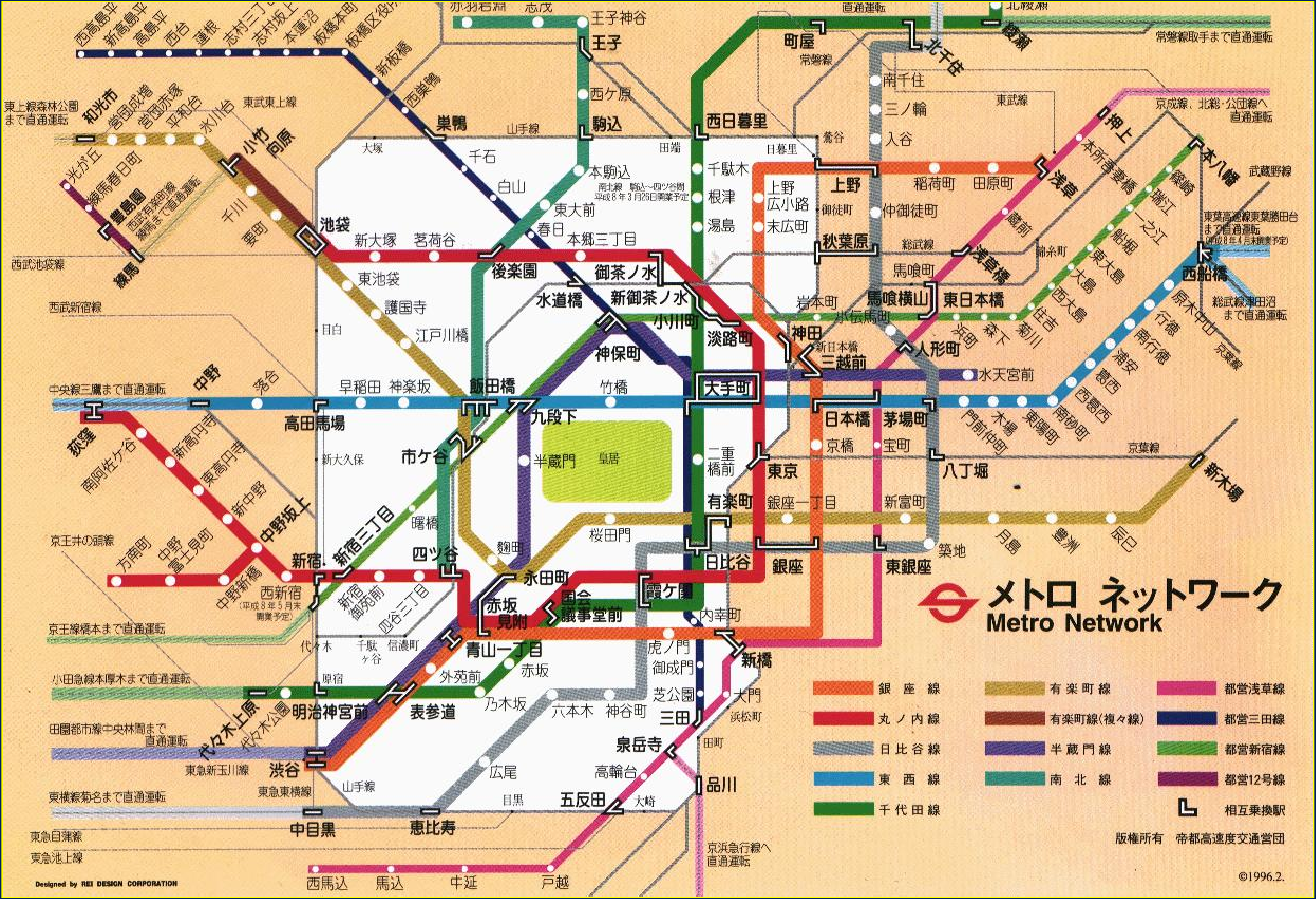
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都営浅草線
都営三田線
都営新宿線
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後楽園
本郷三丁目
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池袋
新大塚
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辰巳
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内幸町
新橋
虎ノ門
御成門
芝公園
大門
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三田
田町
泉岳寺
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赤坂
青山一丁目
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代々木公園
高田馬場
早稲田
神楽坂
落合
中野
中野坂上
新中野
東高円寺
新高円寺
南阿佐ケ谷
荻窪
中野新橋
中野富士見町
方南町
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根津
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小伝馬町
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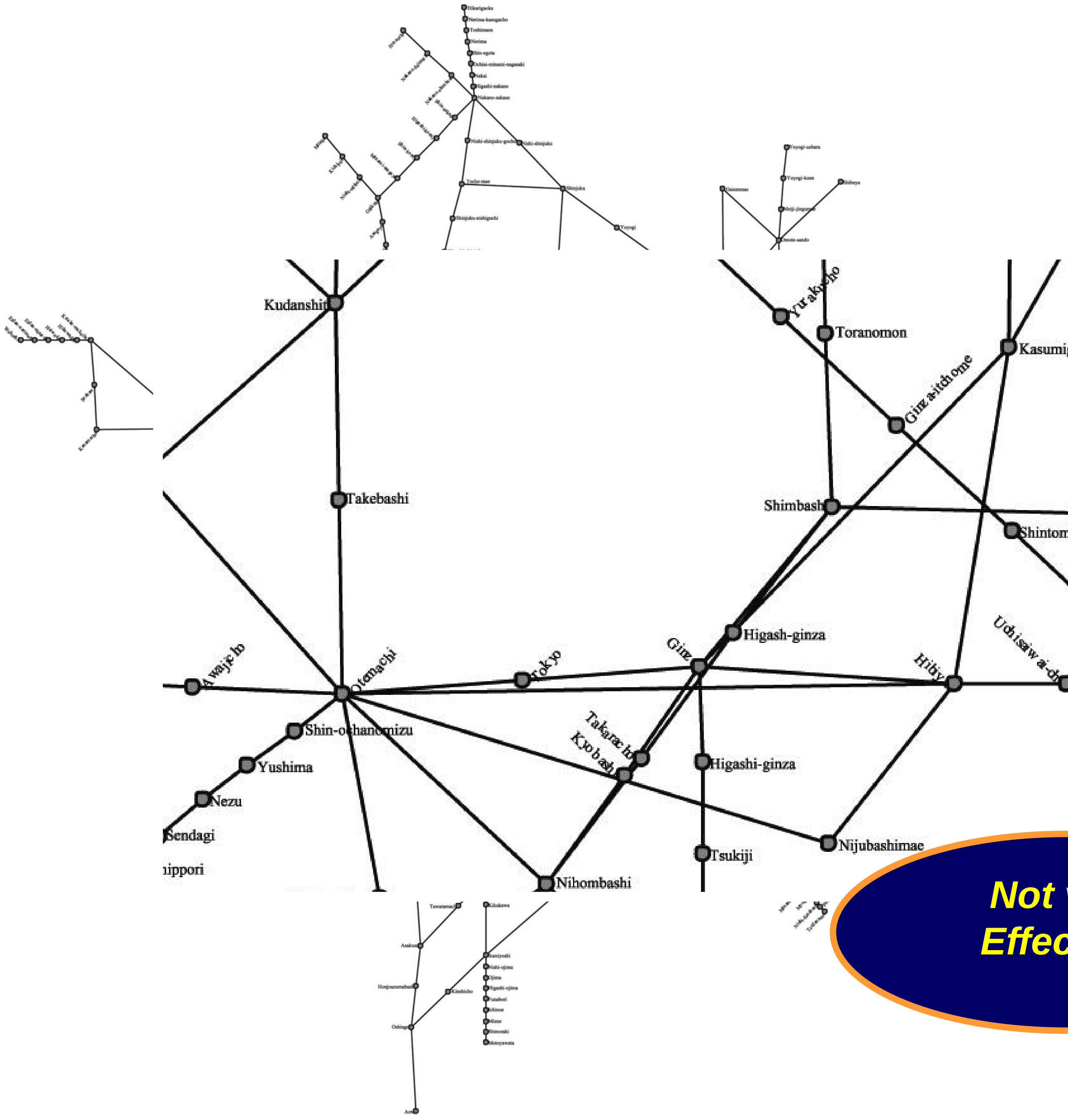
千川
要町
西馬込
馬込
中延
戸越
山手線
常磐線
総武線
京葉線
東武東上線
東武線
西武池袋線
西武新宿線
京王井の頭線
東急新玉川線
東急東横線
東急目蒲線
東急池上線
中央線三鷹まで直通運転
常磐線取手まで直通運転
京王線橋本まで直通運転
小田急線本厚木まで直通運転
田園都市線中央林間まで直通運転
東横線菊名まで直通運転
総武線津田沼まで直通運転
京浜急行線へ直通運転
東上線森林公園まで直通運転
京成線、北総・公団線へ直通運転
西武有楽町線練馬まで直通運転



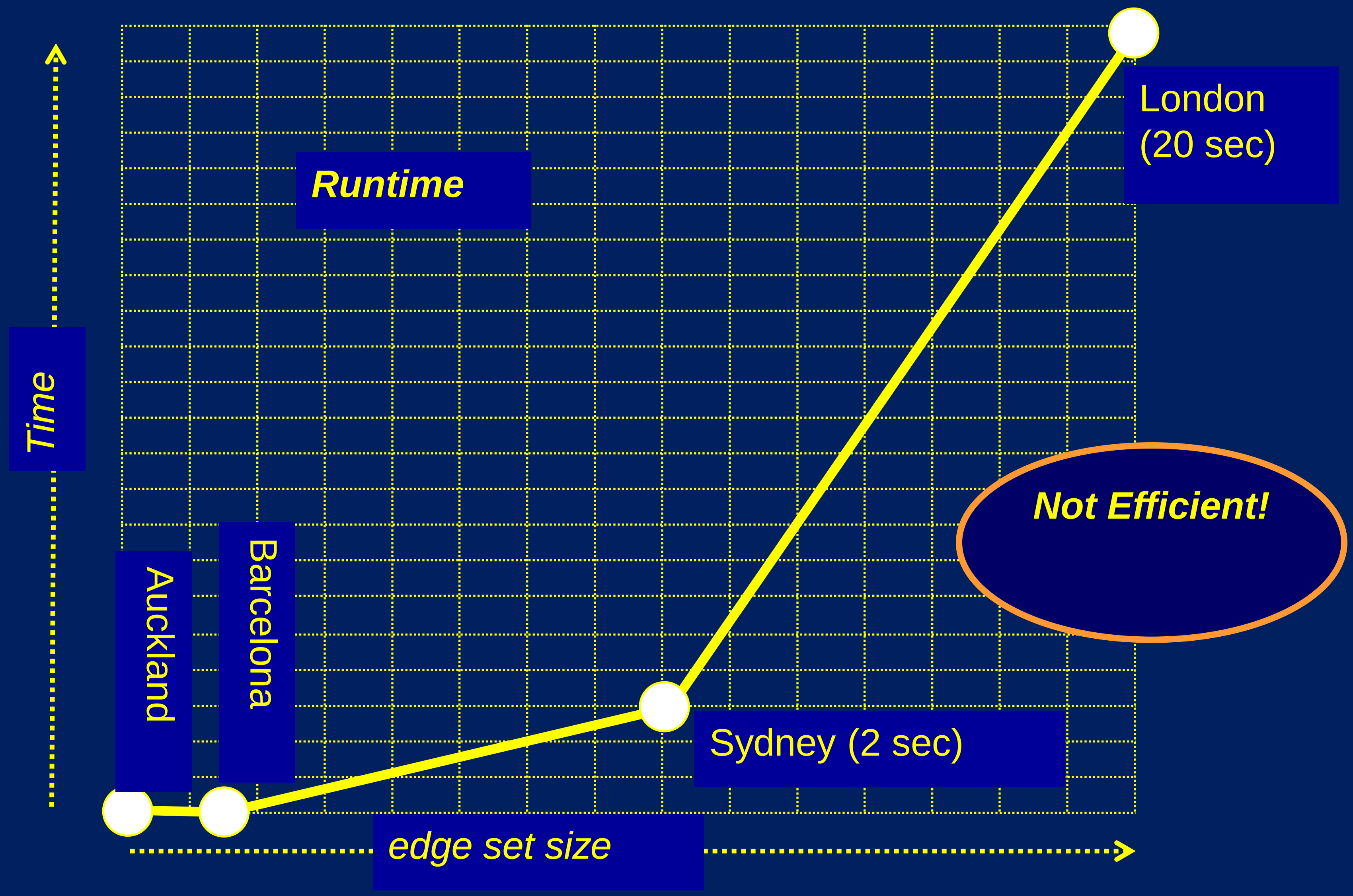
Not very Effective!

The force directed method is a little bit ***effective***, but not very effective.

It needs manual post-processing:

- This uses the time of a professional graphic artist
- Increases cost
- Increases time-to-market

Efficiency

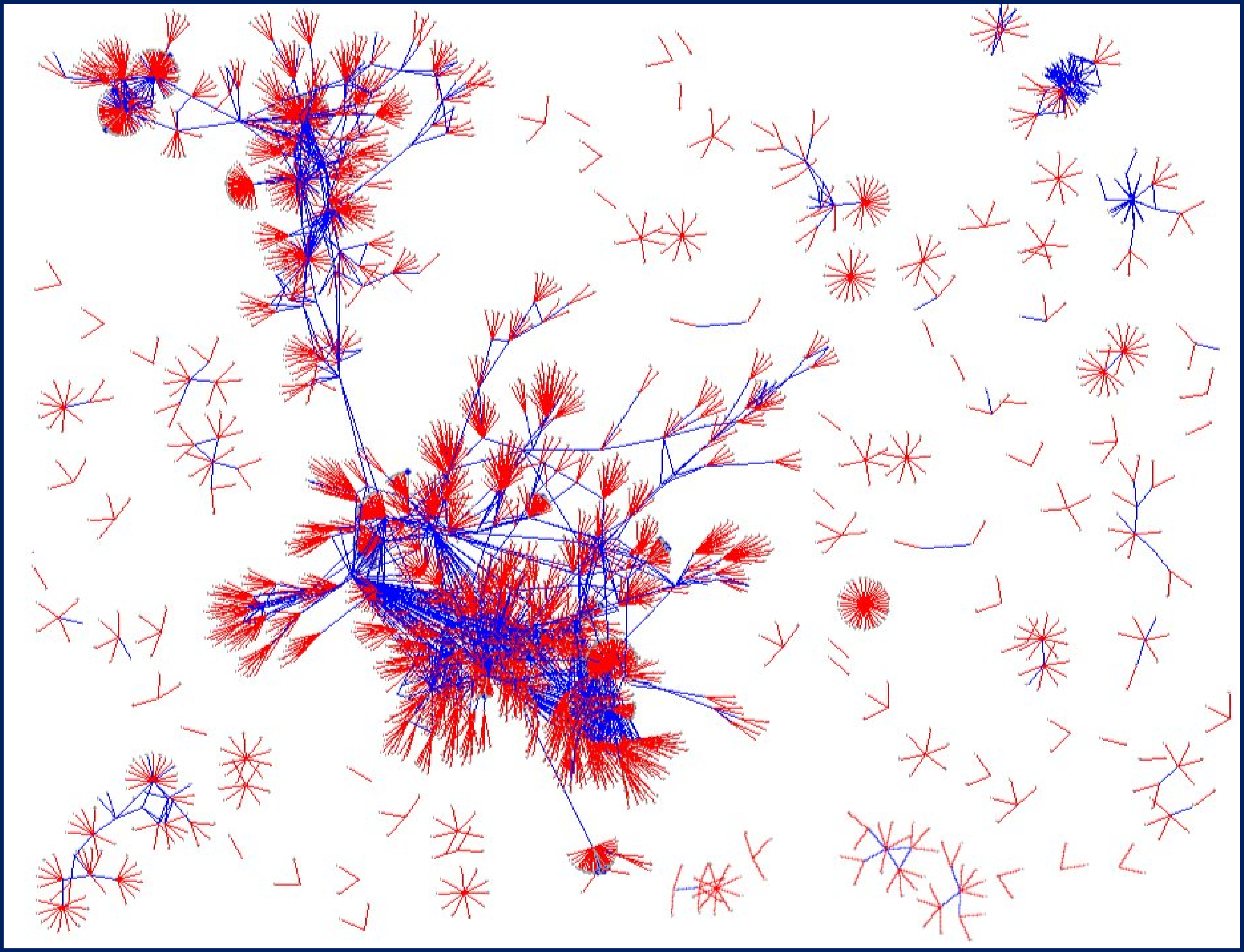

Runtime
Time
edge set size
Auckland
Barcelona
Sydney (2 sec)
London
(20 sec)
Not Efficient!

The force directed method for metro maps is *not* computationally *efficient*.

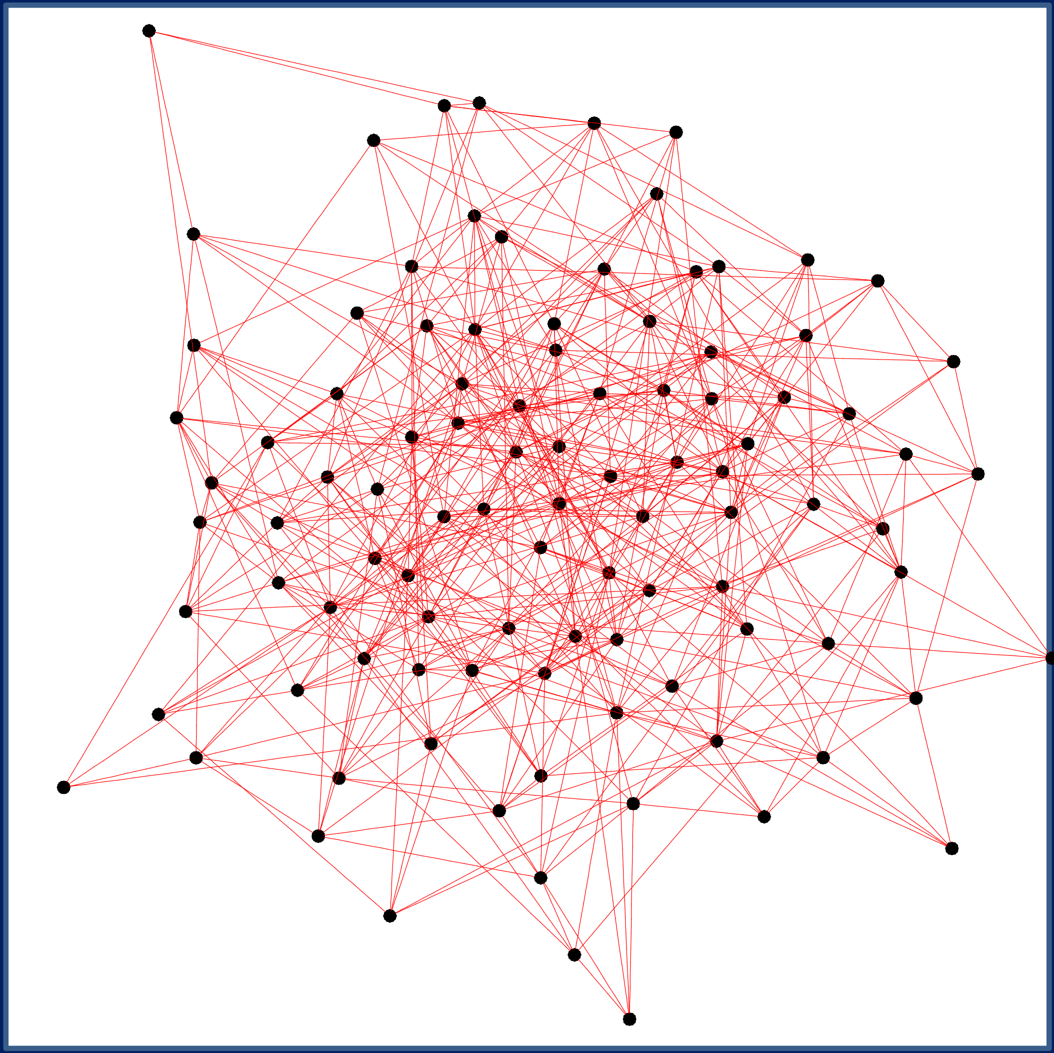
We need better ways to solve the equations.

The performance of force directed methods on metro maps is typical .

For some data sets, force directed methods give reasonable drawings.



For some data sets, force directed methods do not give reasonable drawings.



How good are current force directed methods?

Efficiency:

- ➢ OK for small graphs
- ➢ Sometimes OK for larger graphs

Elegance:

- ➢ Many simple methods, easy to implement
- ➢ Numerical software often available

Effectiveness:

- ➢ Very flexible ☑
- ➢ Straight-line edges ☑
- ➢ Planar graphs are *not* drawn planar ☒
- ➢ Very poor untangling for large graphs ☒

The state-of-the-art for force directed methods in practice:

- Many commercial force-directed tools graph drawing methods are available
  - IBM (ILOG)
  - TomSawyer Software
  - yWorks
- Much free software available
  - GEOMI
  - GraphVis
- Force-directed methods account for 60 – 80% of commercial and free graph drawing software

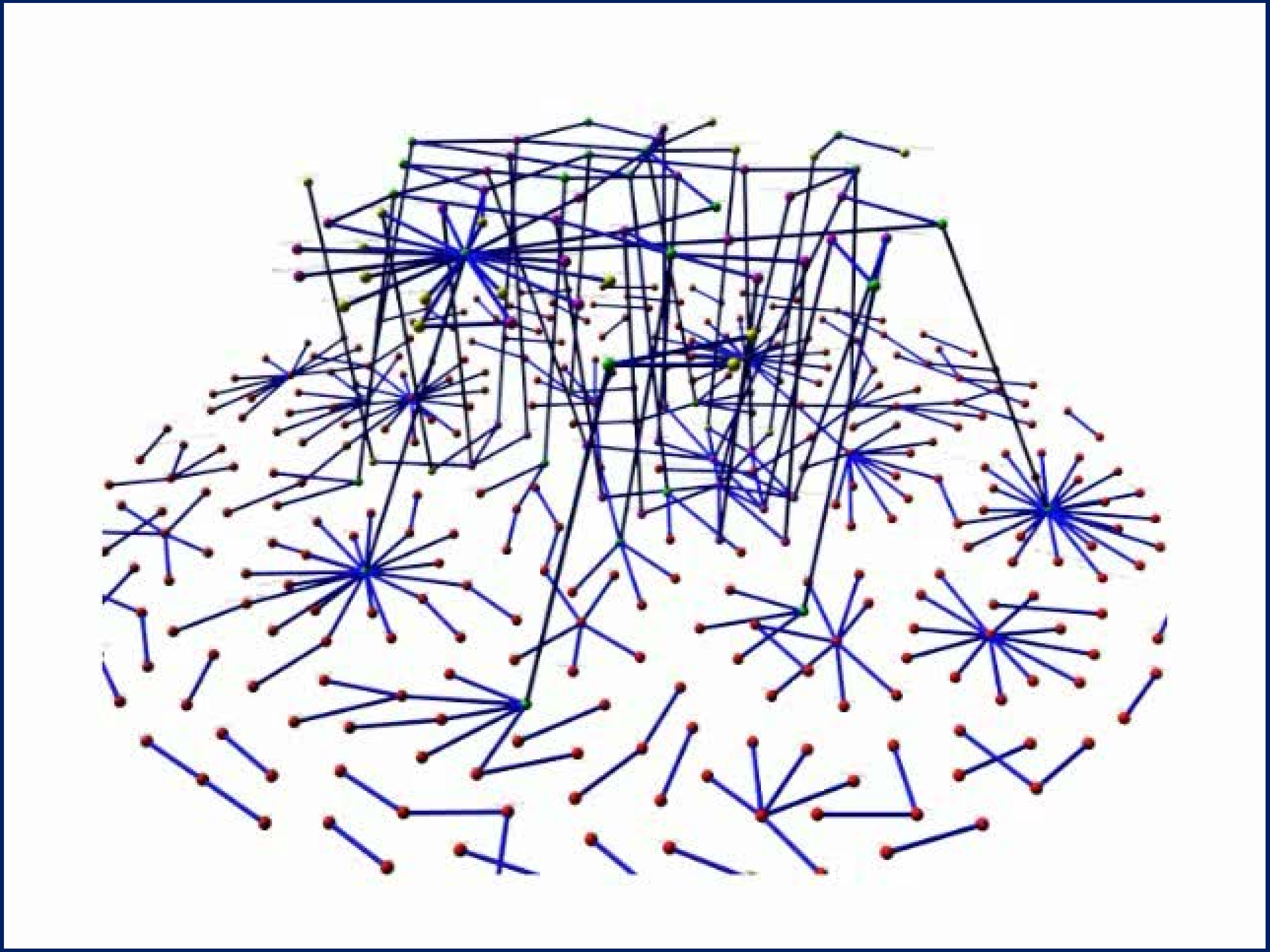
2. How to draw a planar graph?

   a) Before Tutte: 1920s – 1950s
   b) Tutte: 1960s
   c) After Tutte: 1970s – 1990s
   d) Recent work: since 2000

Recent work: since 2000

- Faster force-directed algorithms
- New metaphors in 2.5D
- New edge-crossing criteria: slightly non-planar graphs

New metaphors in 2.5D



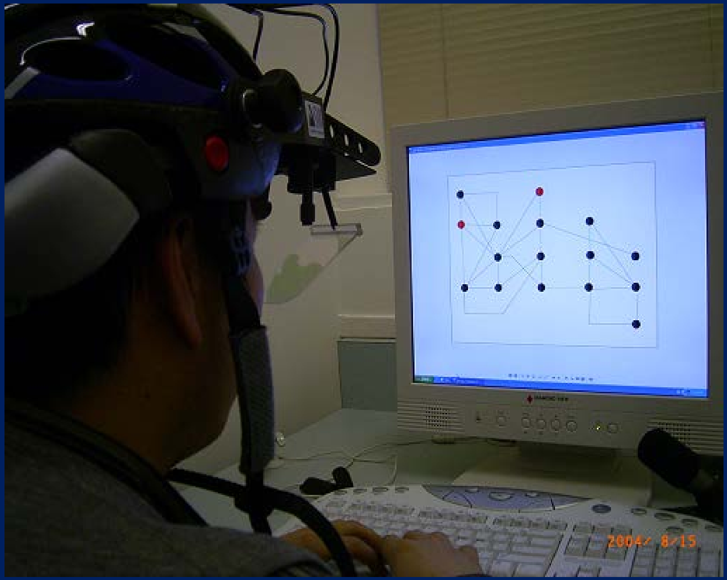
New edge-crossing criteria:

Slightly non-planar graphs

Motivation

Tony Huang 2003^{+}: Series of human experiments

- Eye tracking experiments to suggest and refine theories
- Controlled lab experiments to prove theories.



Eye-tracking: <u>suggestions:</u>

Large angle crossings are OK:

no effect on eye movements

Lab experiments: <u>proof</u>

- Suggestion was confirmed with traditional controlled human lab experiments

Huang's thesis

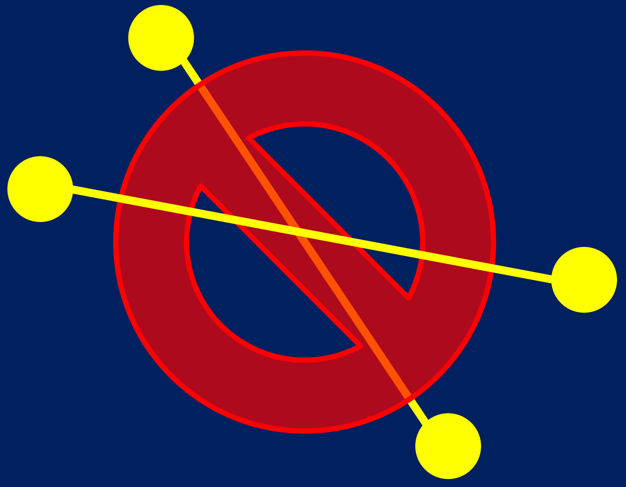
If the crossing angles are large, then non-planar drawings are OK.

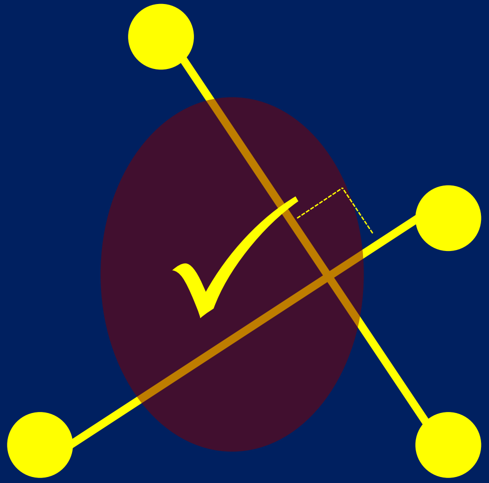
How can we draw graphs with large crossing angles?

Right Angle Crossing (RAC) Graphs

Right-Angle Crossing (RAC) graphs:

- Straight-line edges
- If two edges cross, then the crossing makes a right angle





Questions for slightly non-planar graphs:

- How dense can a RAC graph be?

Theorem (Liotta, Didimo, Eades, 2009)

Suppose that ***G*** is a RAC graph with ***n*** vertices and ***m*** edges.
Then $m \leq 4n-10$.

Questions for slightly non-planar graphs:

- How dense can a RAC graph be?
- How can you compute a drawing of a RAC graph?

*Theorem (Liotta, Eades)**

The following problem is NP-hard:

*Input: A graph **G***

*Question: Is there a straight-line RAC drawing of **G**?*

*Independently proved by Argyriou, Bekos and Symvonis

*Theorem (Liotta, Eades)**

The following problem is NP-hard:

*Input: A graph **G***

*Question: Is there a straight-line RAC drawing of **G**?*

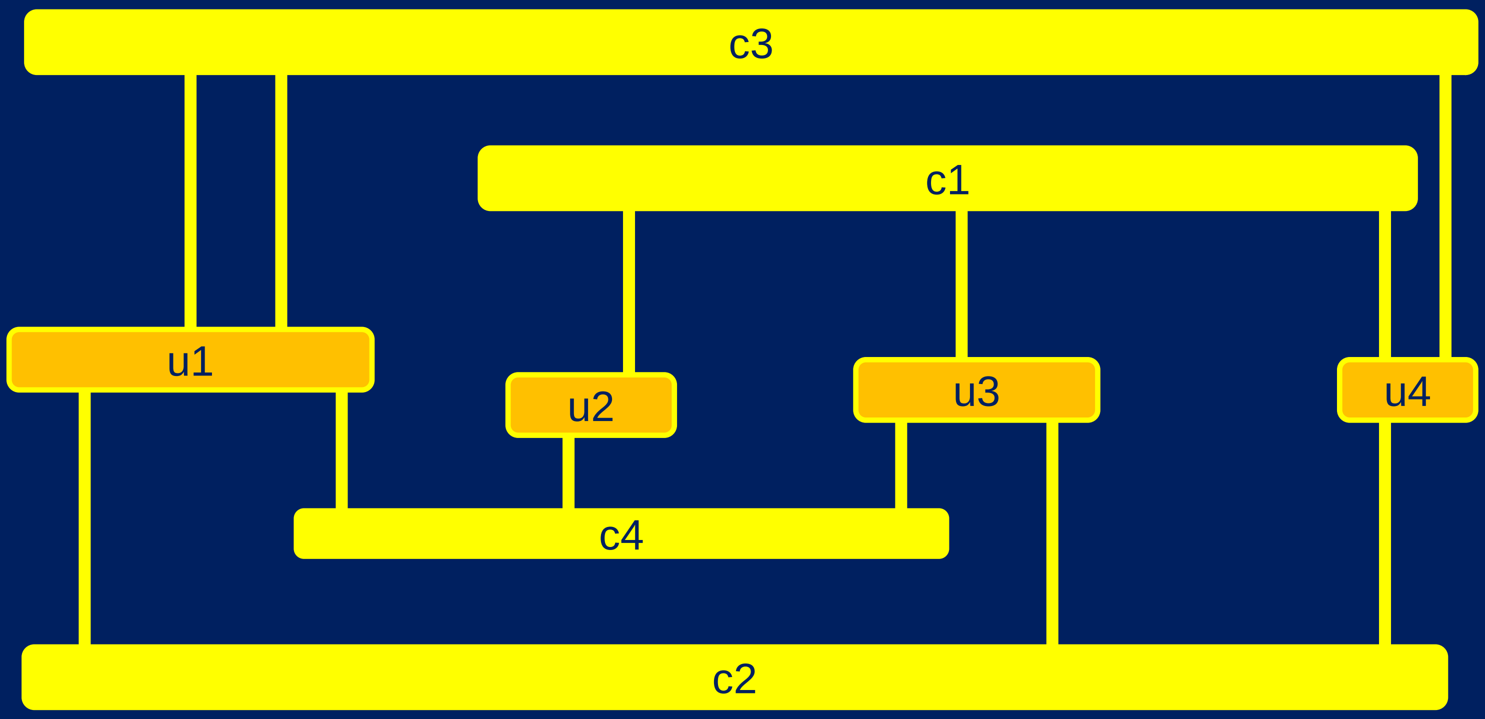
Proof

- Reduction from *planar-3-sat.*
- Draw the instance ***H*** of planar-3-sat as a template.
- Fill in details of the template ***H*** to form a graph ***G*** that has a RAC drawing if and only if ***H*** is satisfiable.

Proof

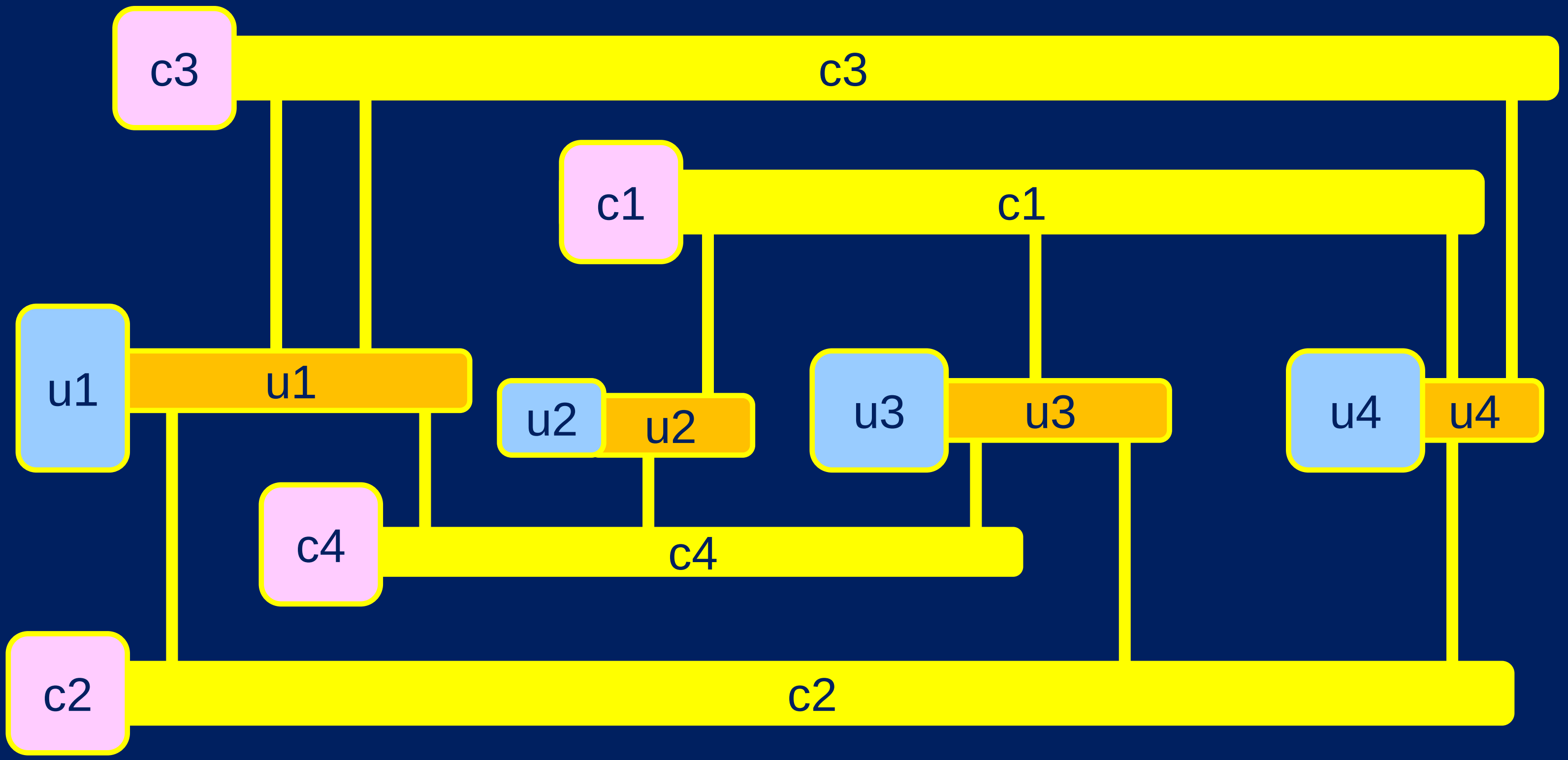
- Reduction from planar-3-sat
- Draw the instance ***H*** of planar-3-sat as a template
- Fill in details of the template ***H*** to form a graph ***G*** that has a RAC drawing if and only if ***H*** is satisfiable.

- Fairly generic proof strategy for NP-hardness for layout problems.

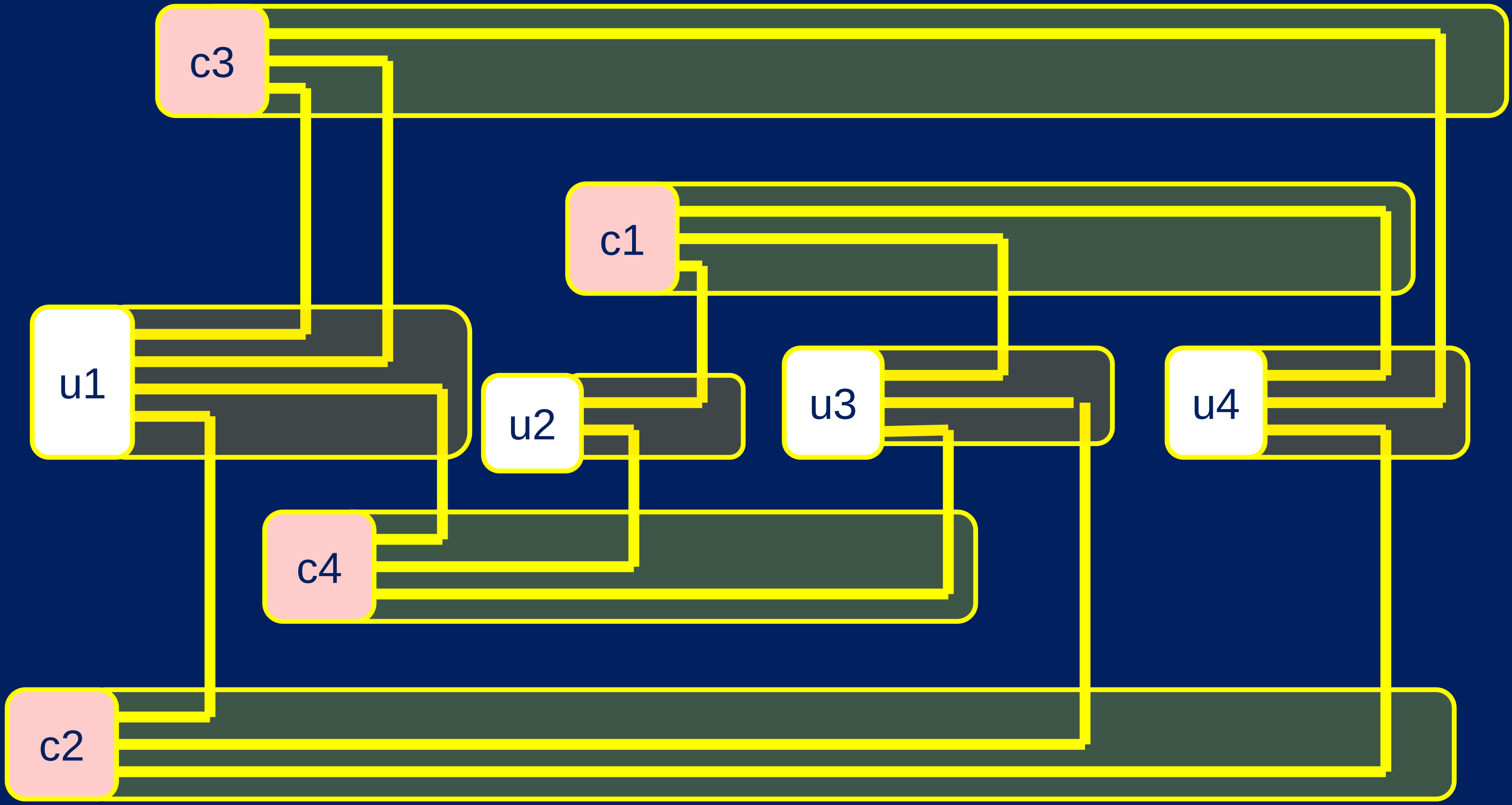


Instance ***H*** of planar 3-sat graph

1. Draw ***H*** as a visibility drawing

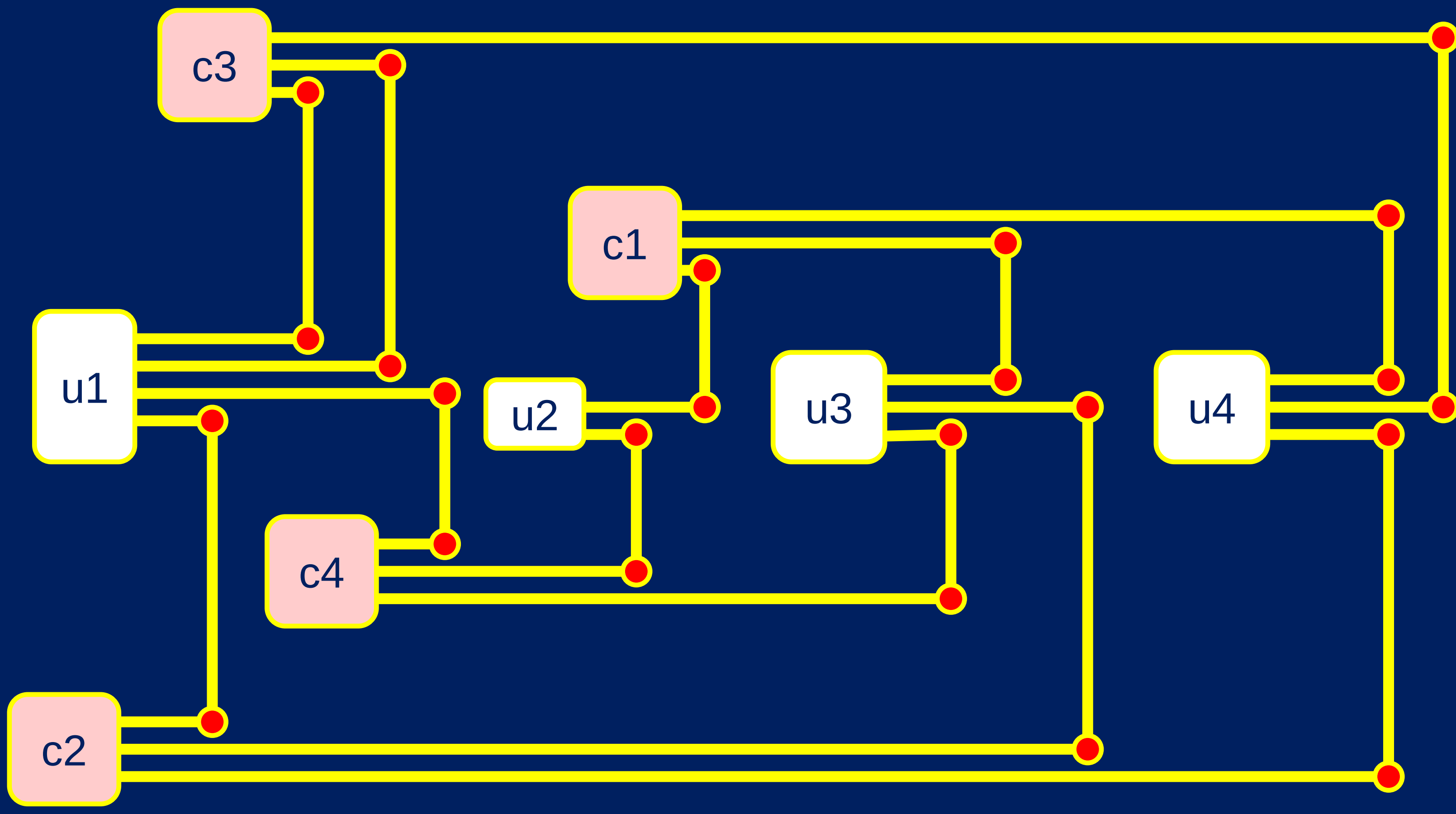


2. Enhance the drawing:
- “node boxes” for
  - ➢ clauses c1, c2, …
  - ➢ variables u1, u2, …



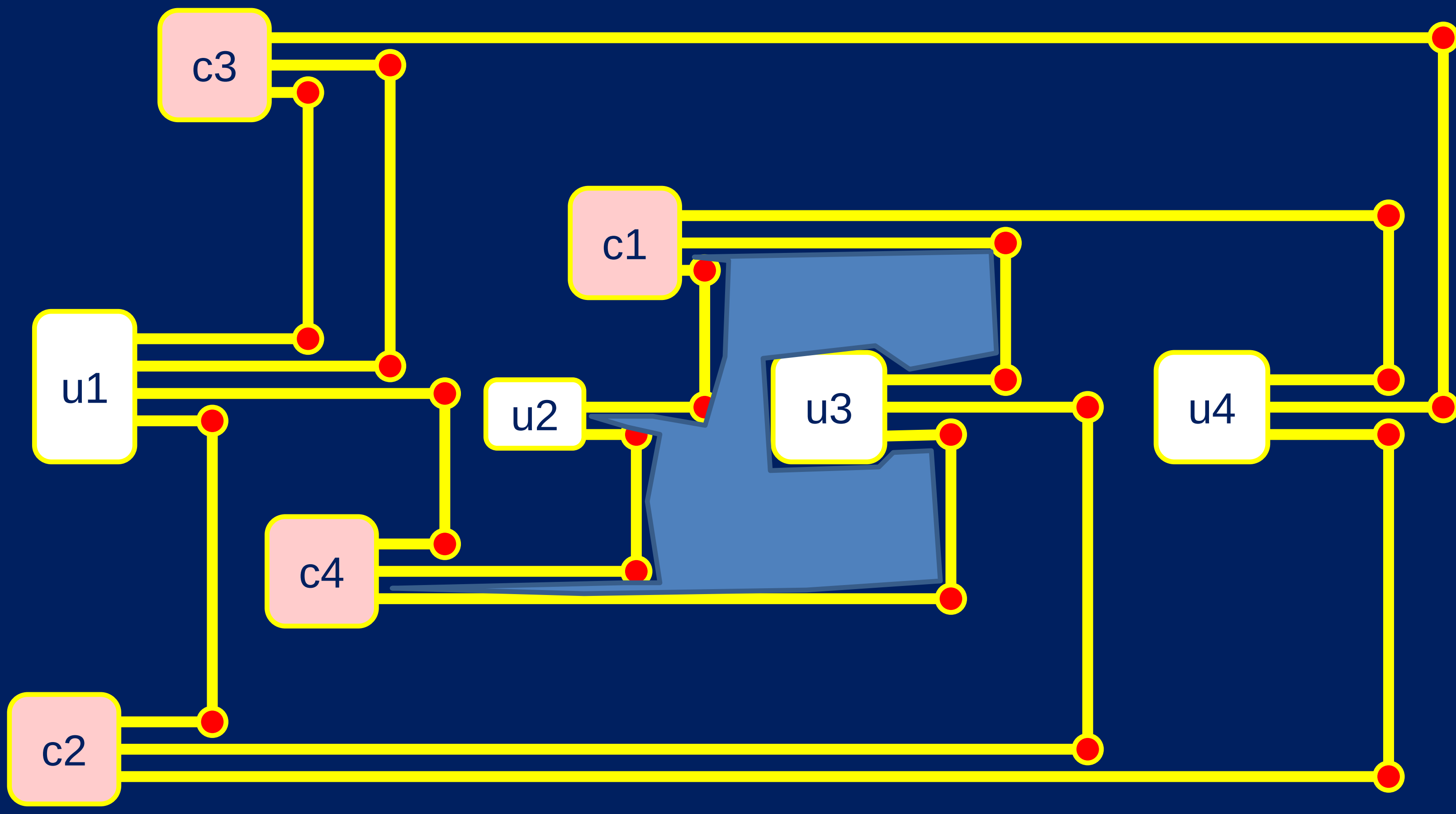
3. Transform to a 2-bend drawing

- ➢ “pipes” to communicate between variables and clauses

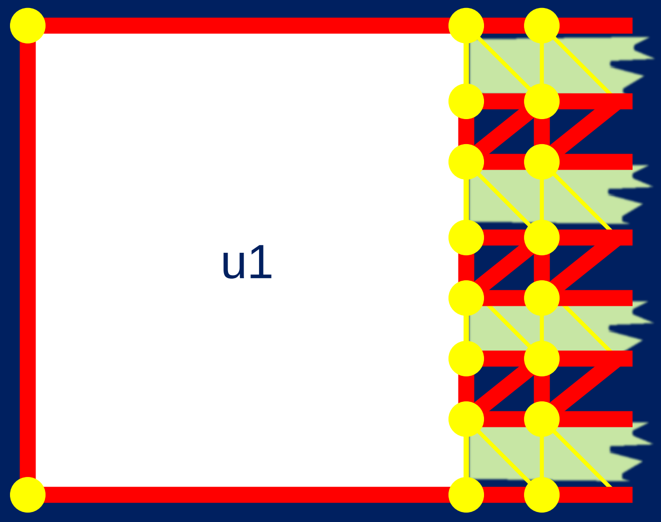


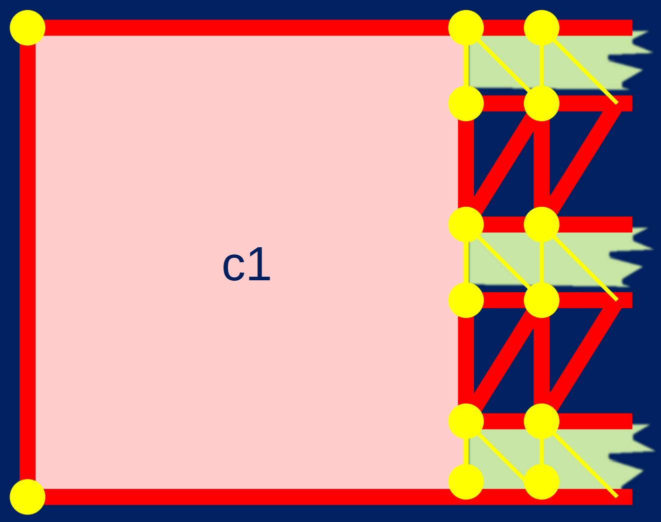
4. Transform to a no-bend drawing

➢ extra nodes at bend points

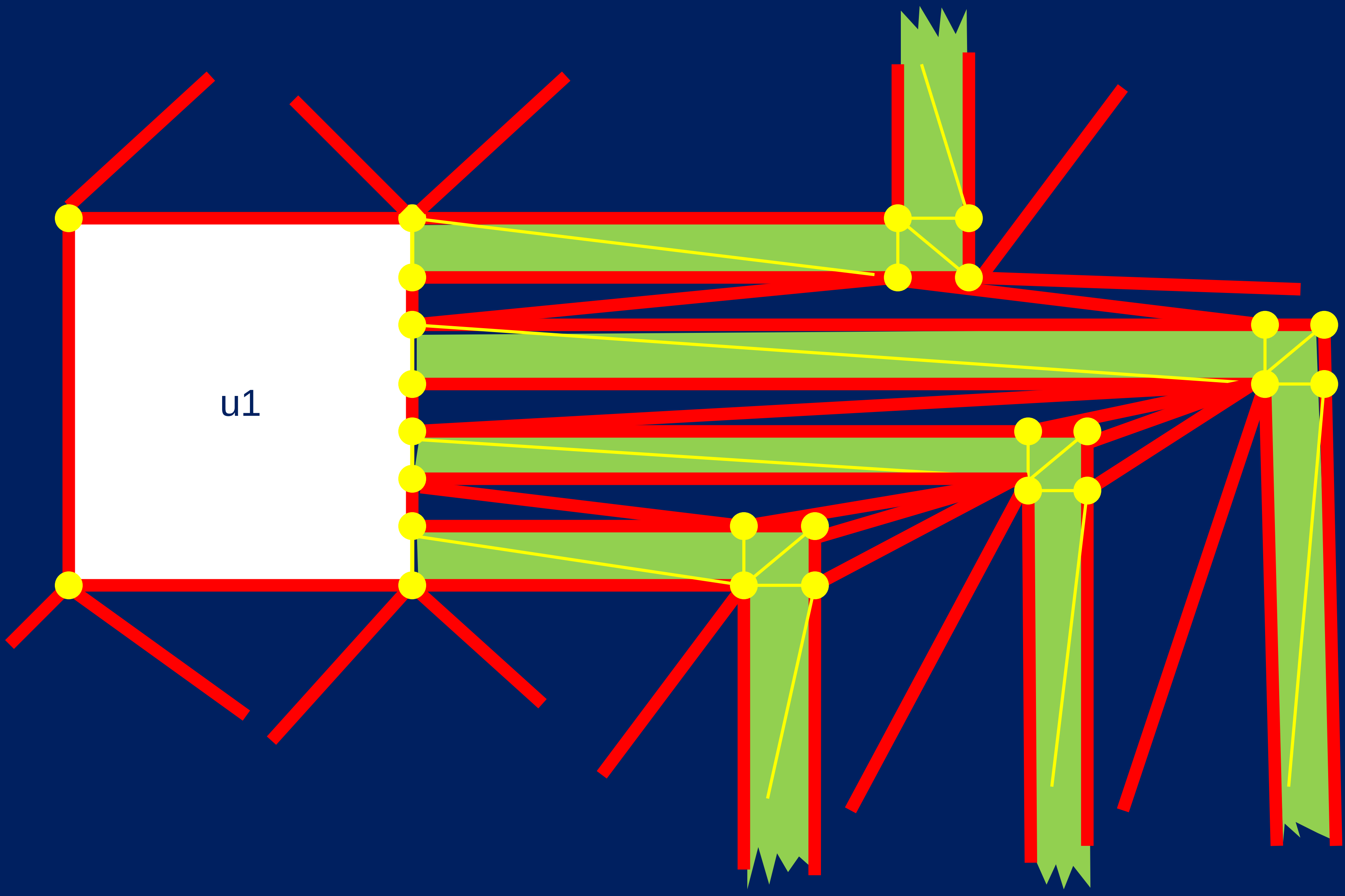


5. Triangulate every face to make it impassable

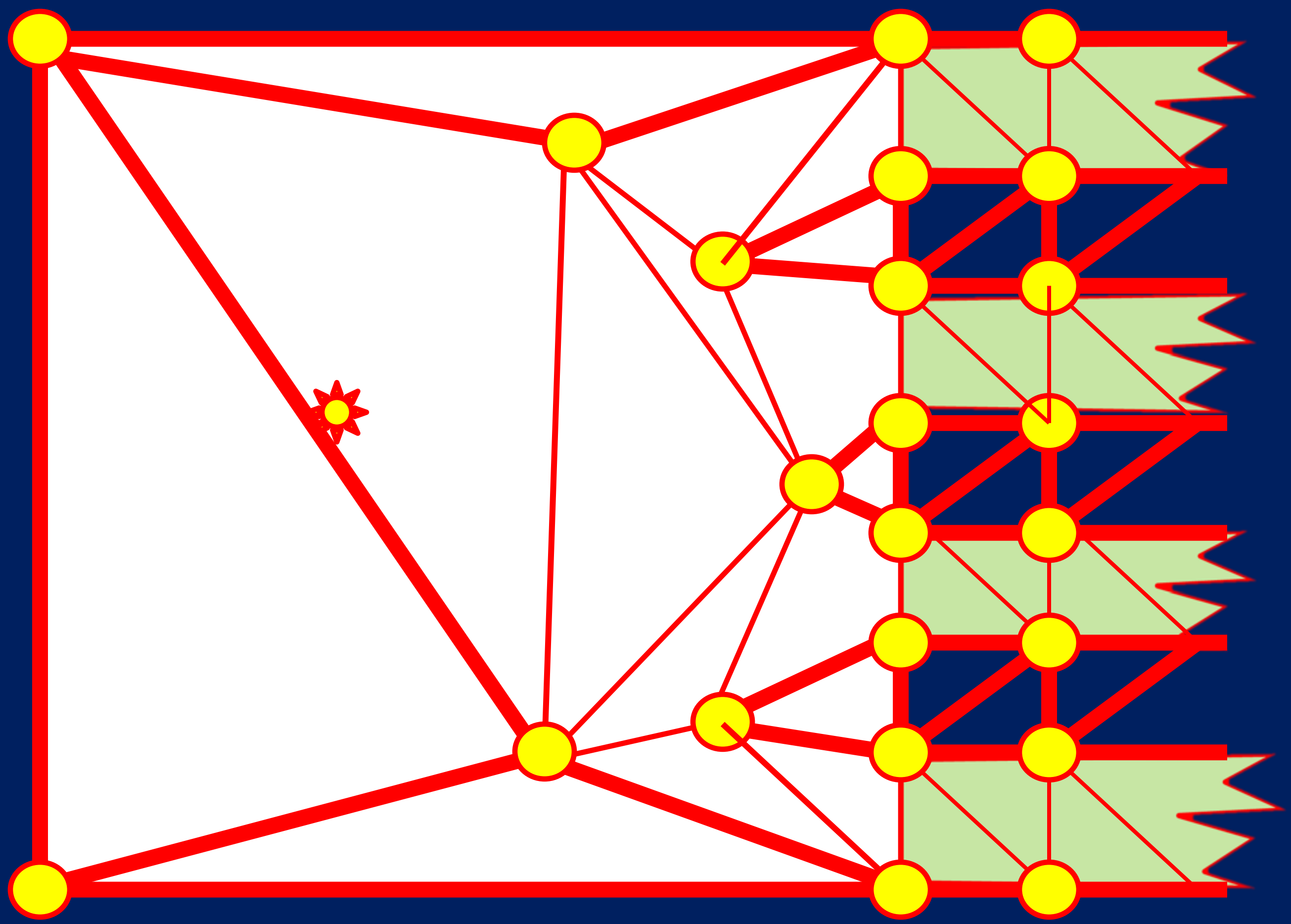




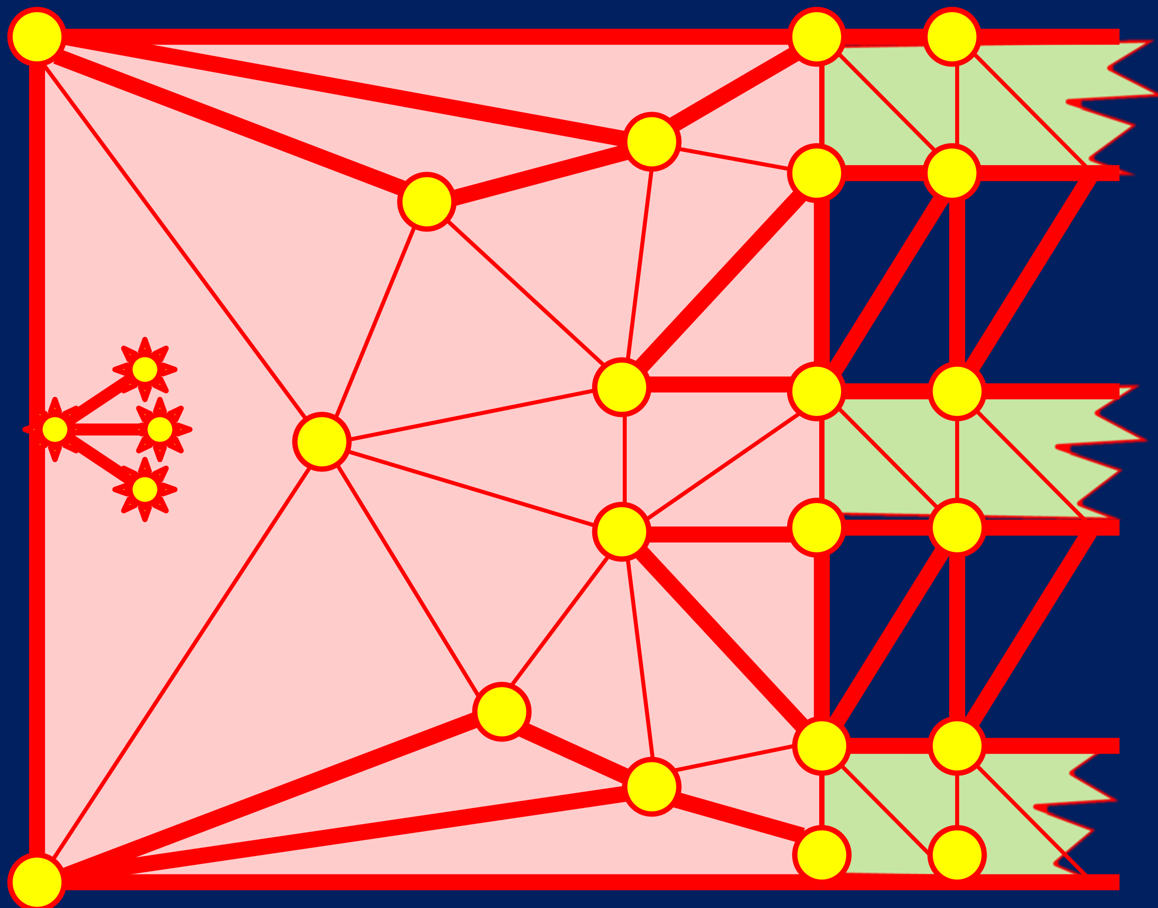
External appearance of “node boxes”, with “pipes” attached



External appearance of “node box”, with pipes attached, showing some of the external triangulation

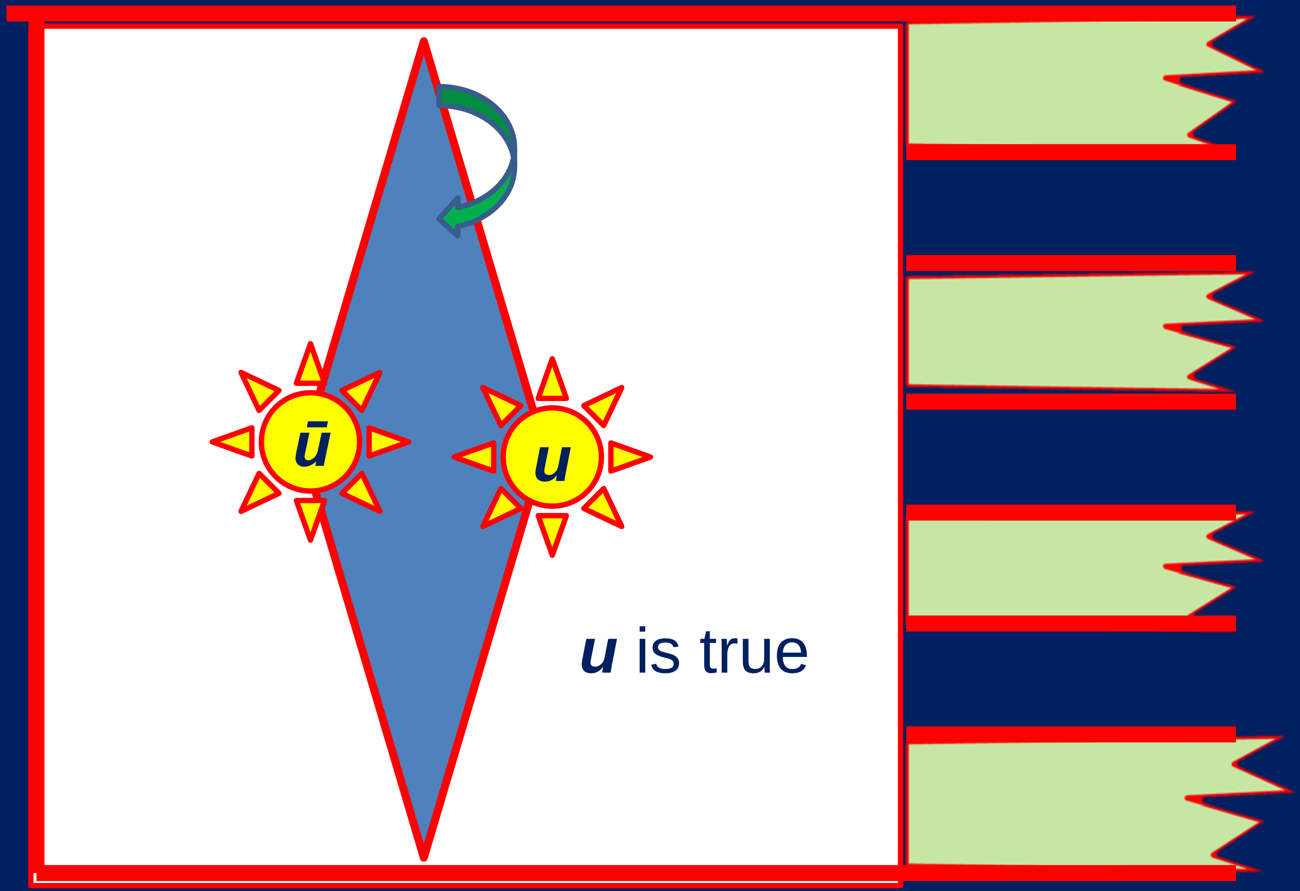


Variable gadget with pipes attached

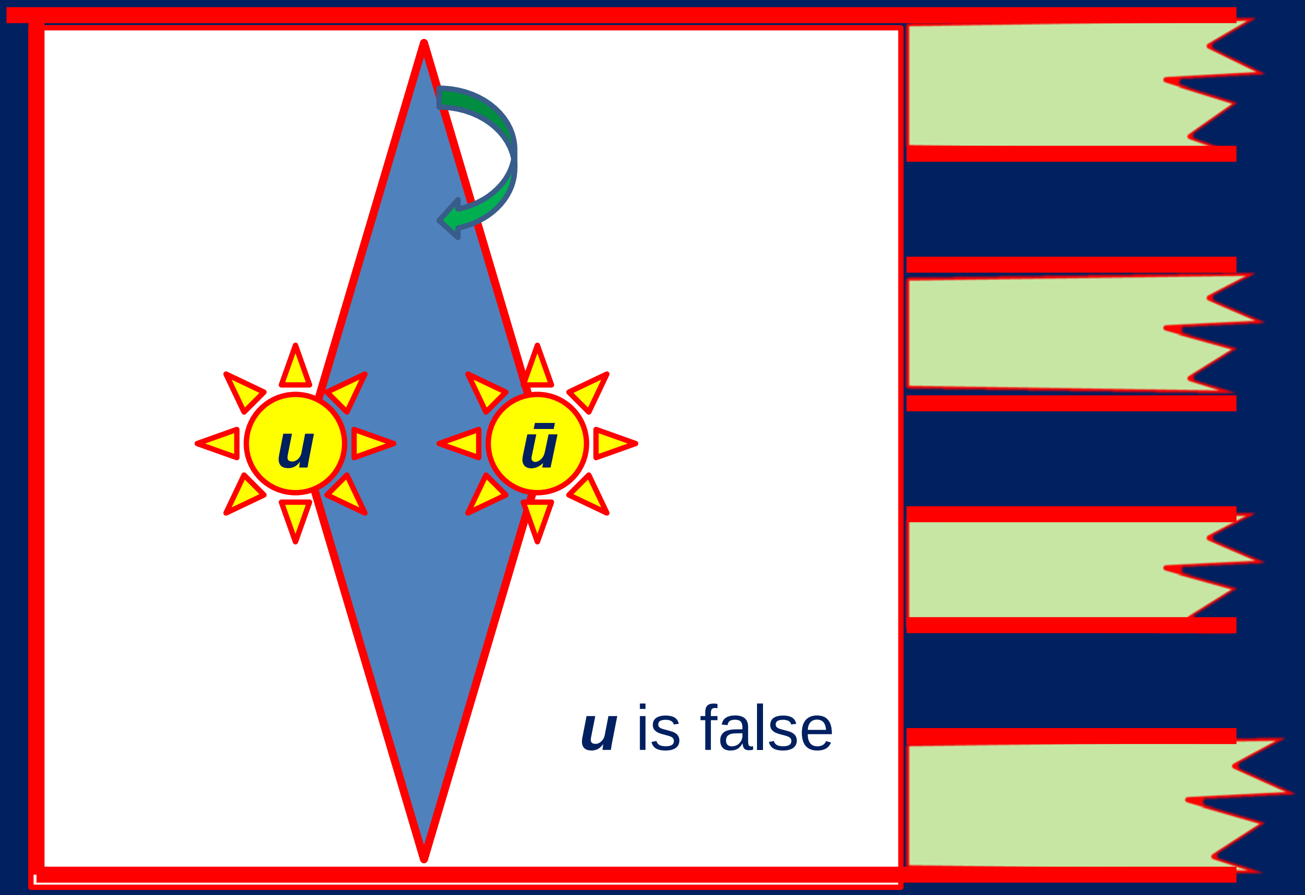


Clause gadget with pipes attached

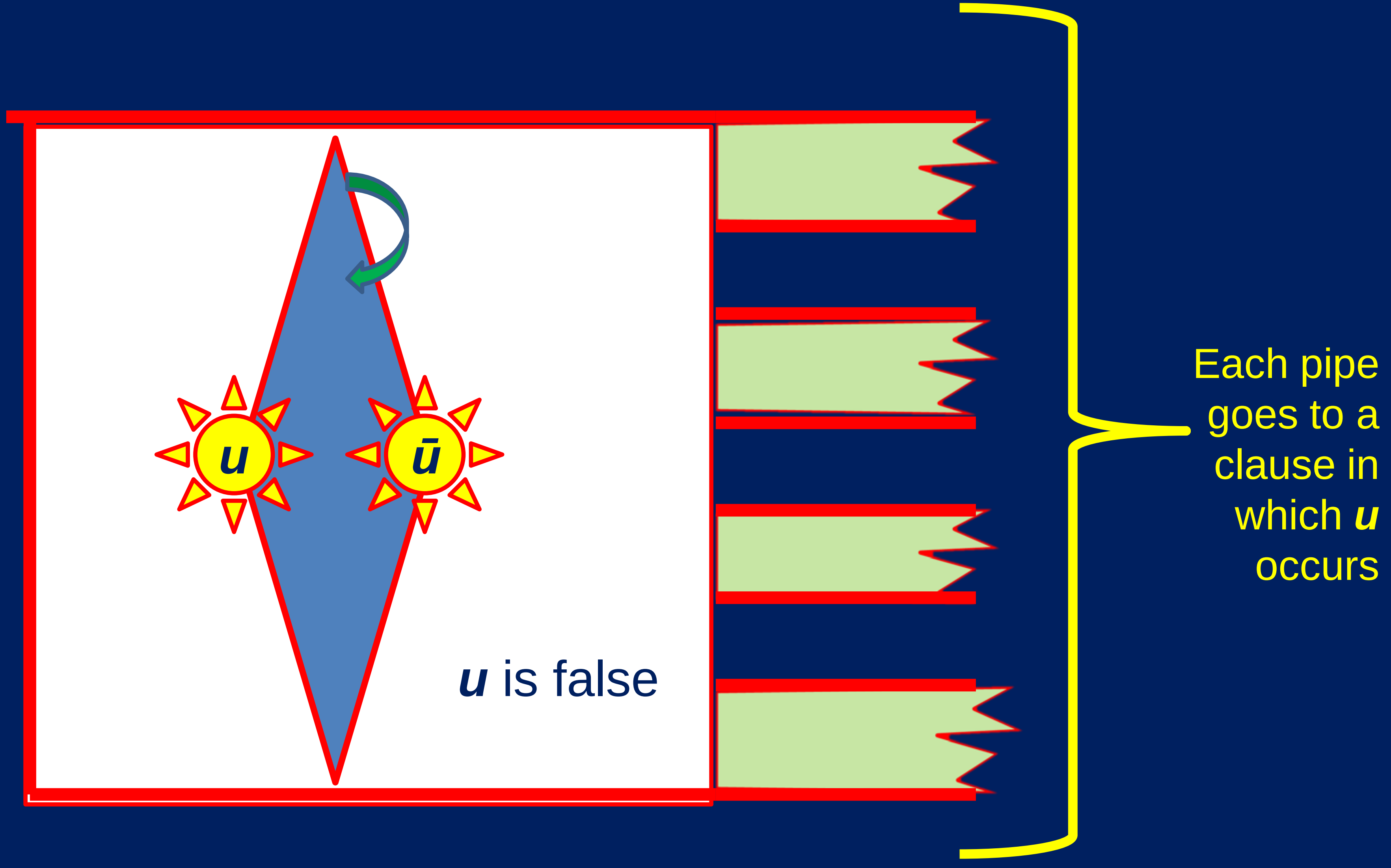
Logical view of variable gadget



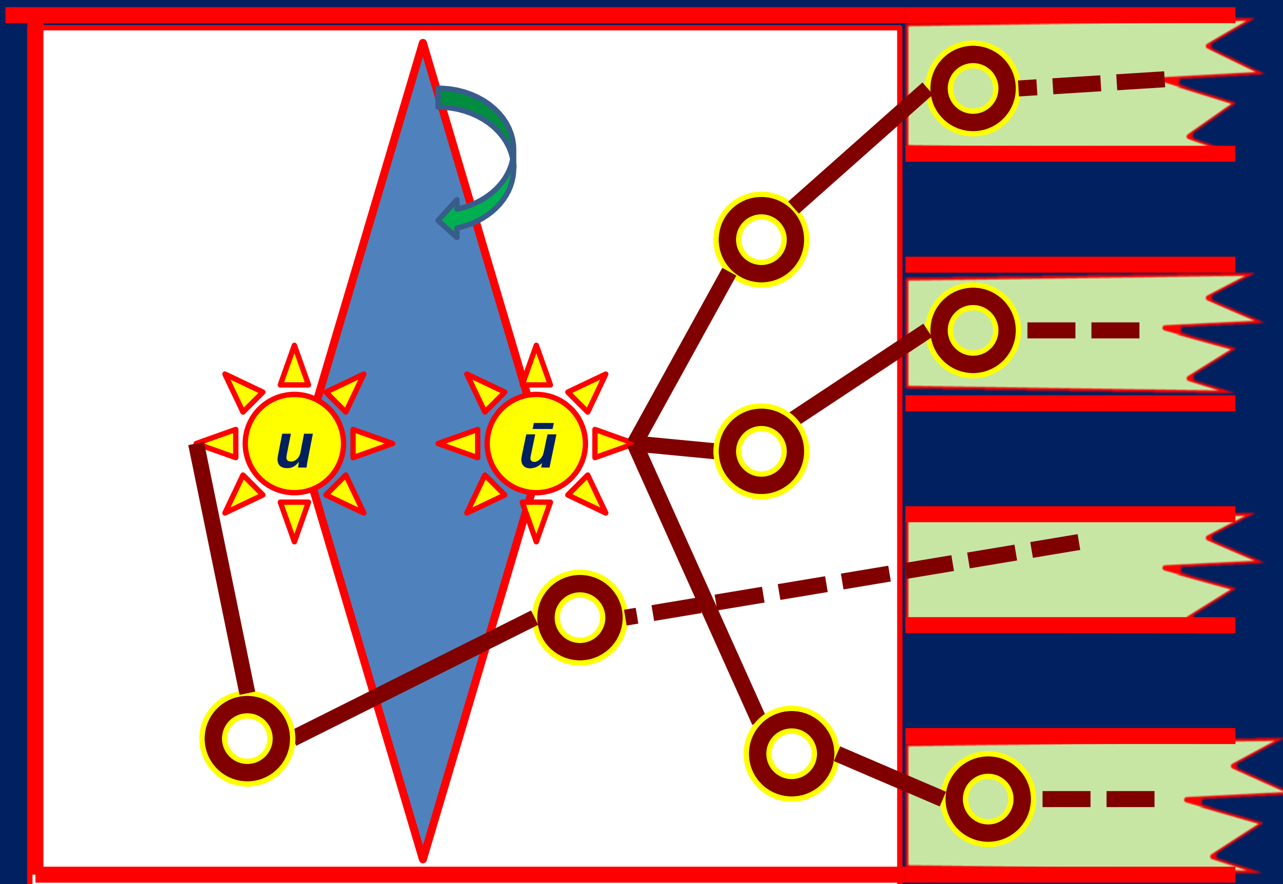
Logical view of variable gadget



Logical view of variable gadget

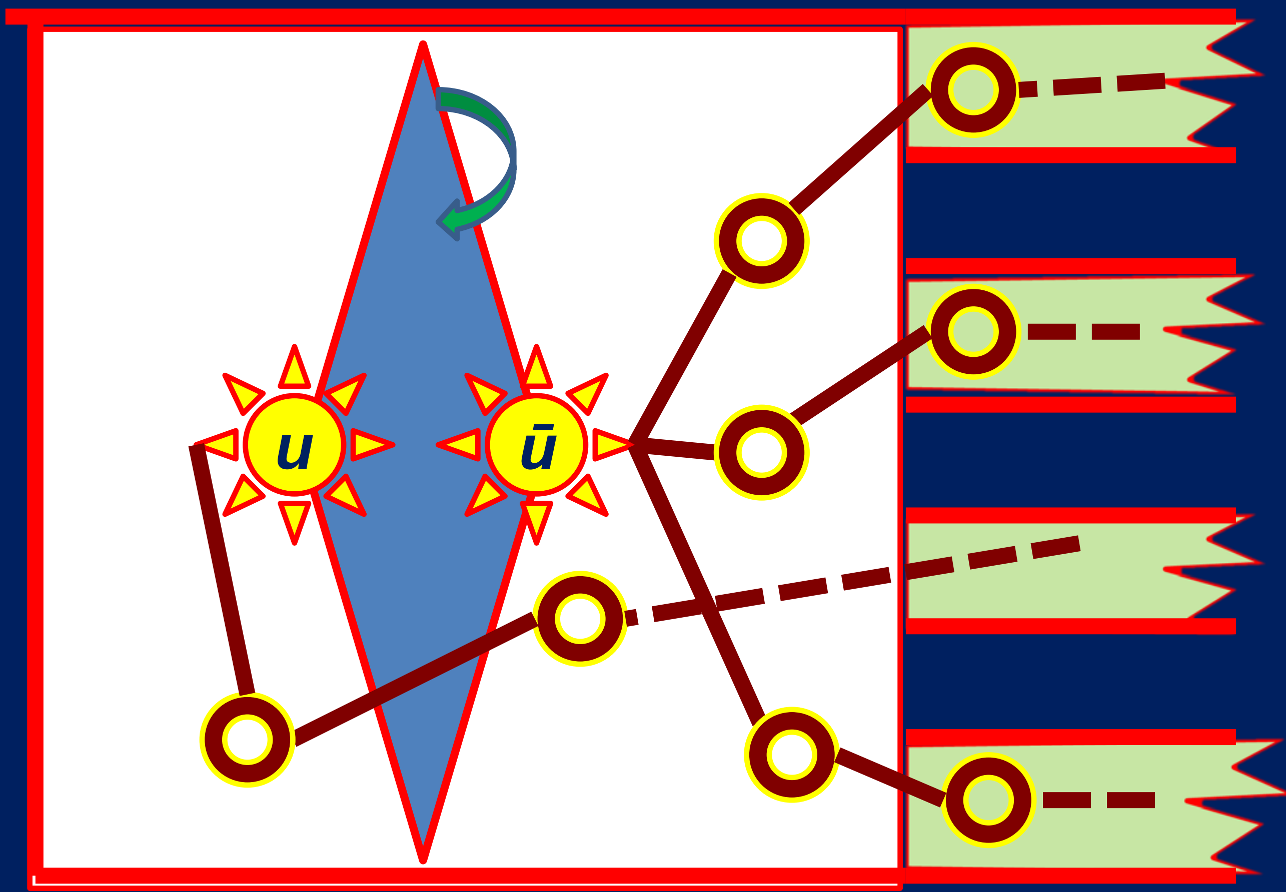


Logical view of variable gadget



Literals are attached to the clauses in which they occur, using chains threaded through the pipes

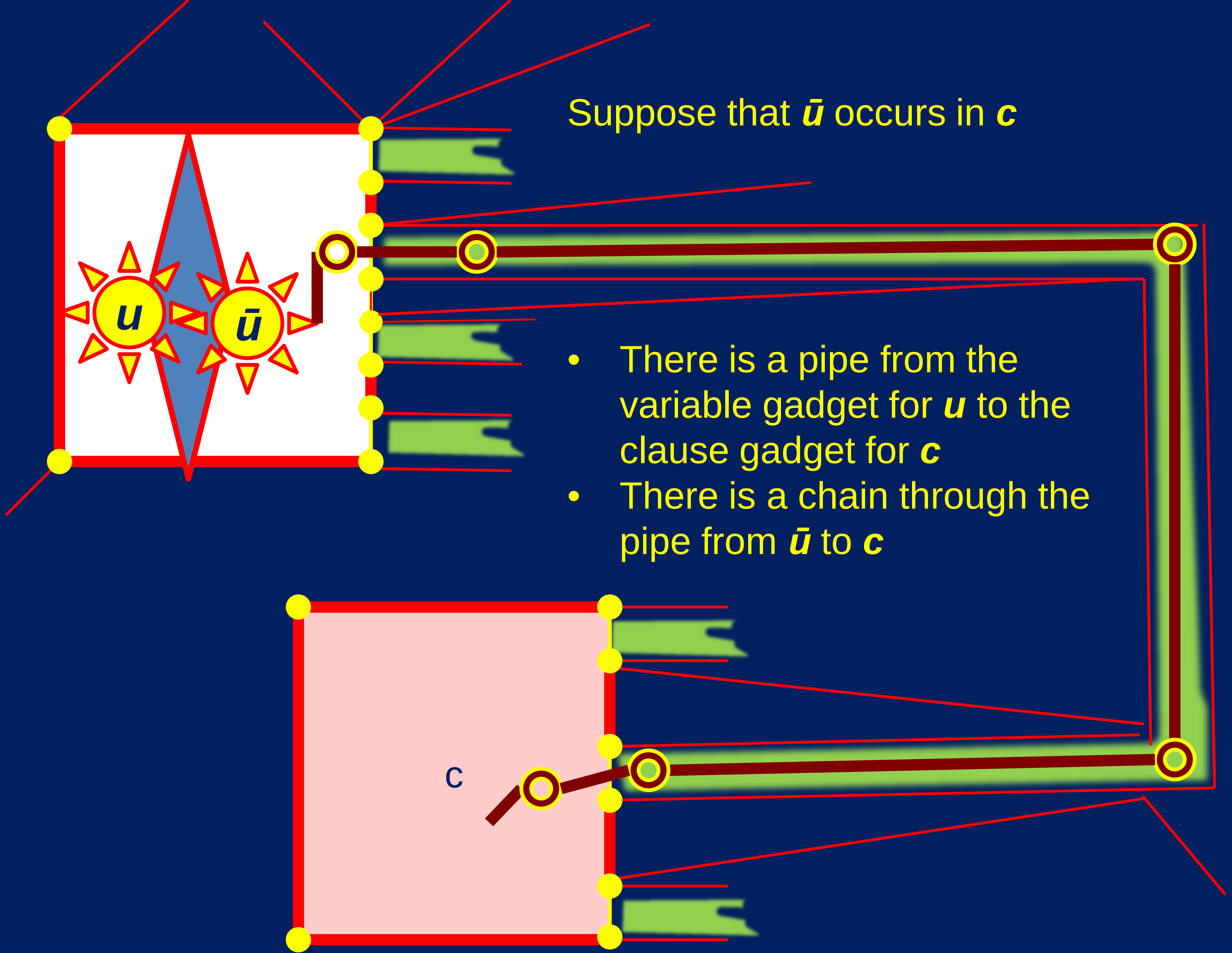
Logical view of variable gadget



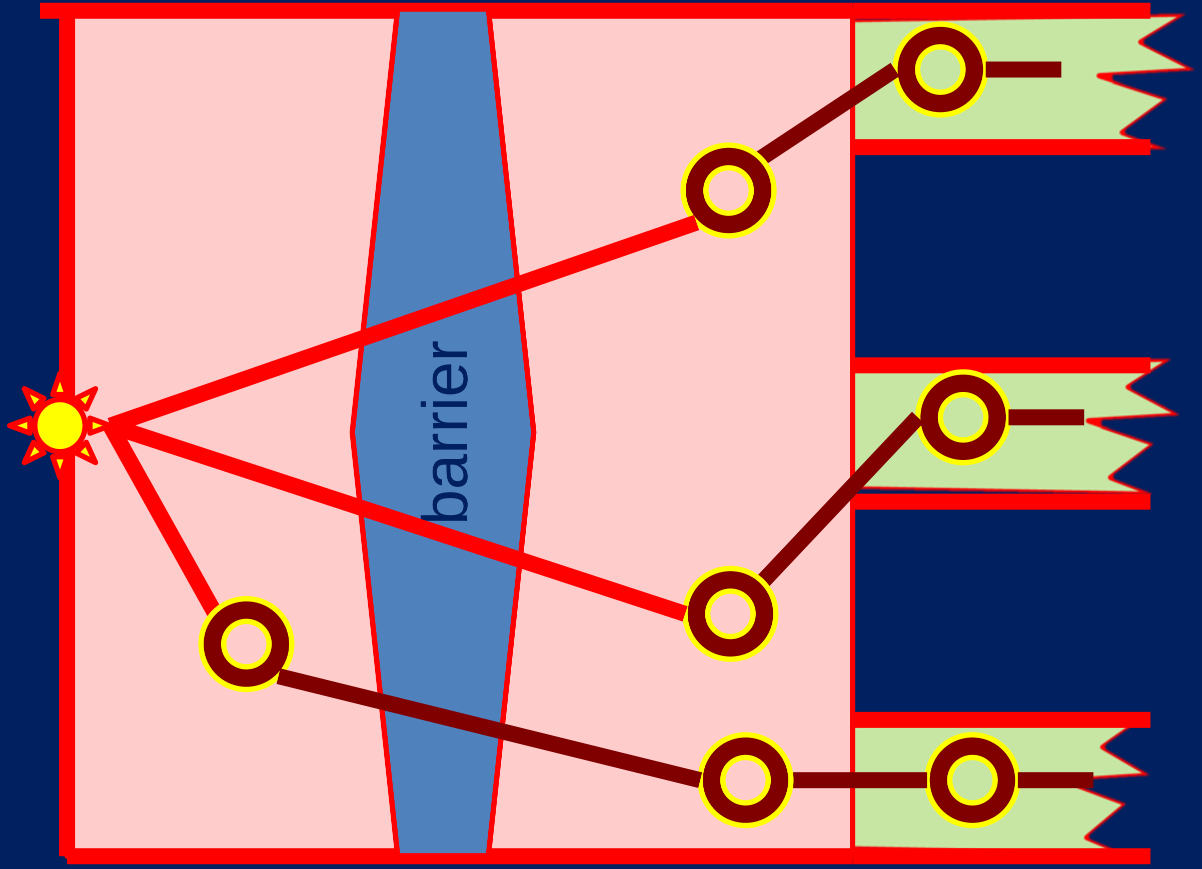
Chains attached to the rear literal spend an extra link before getting into the pipe.

Suppose that ***ū*** occurs in ***c***

- There is a pipe from the variable gadget for ***u*** to the clause gadget for ***c***
- There is a chain through the pipe from ***ū*** to ***c***



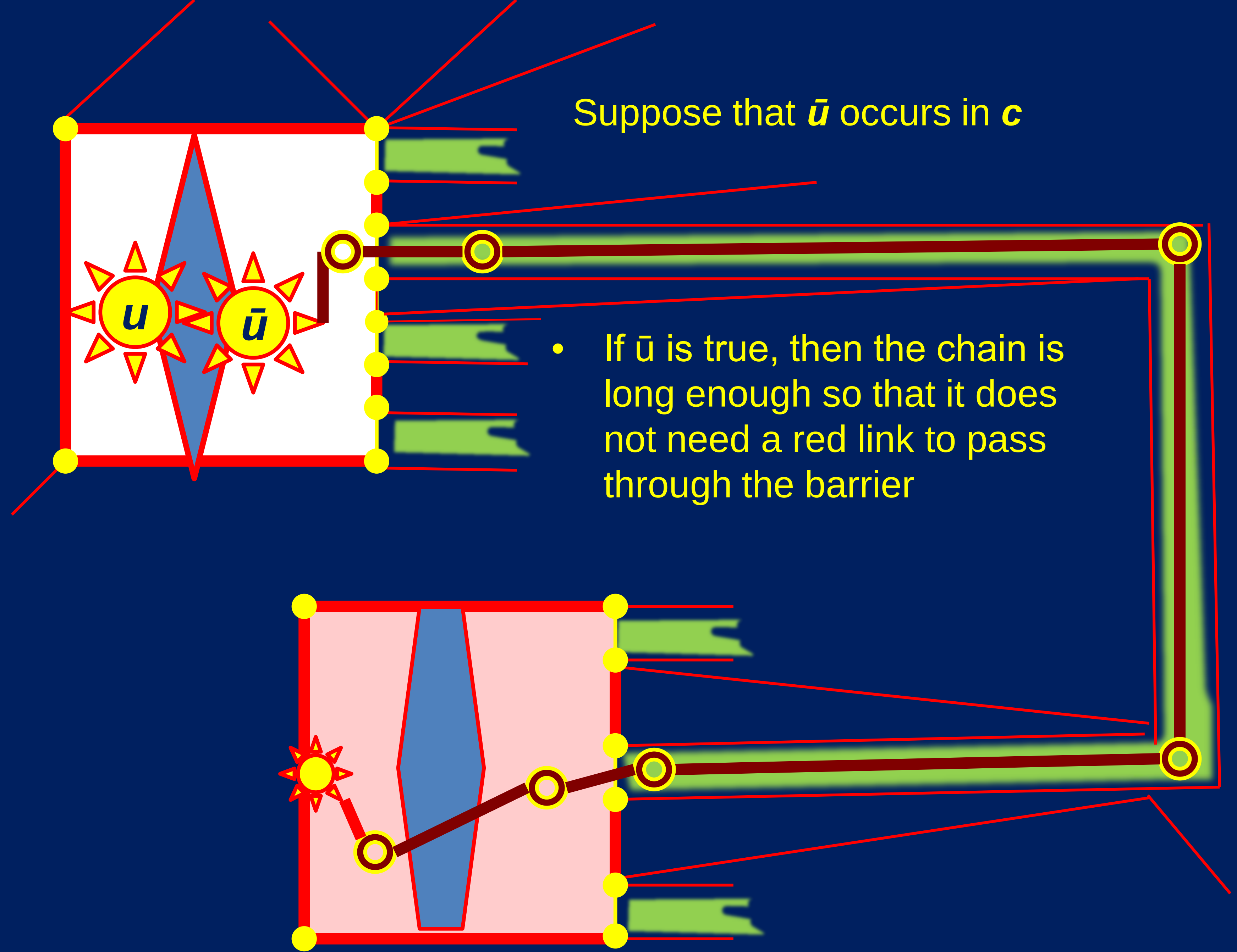
Logical view of clause gadget



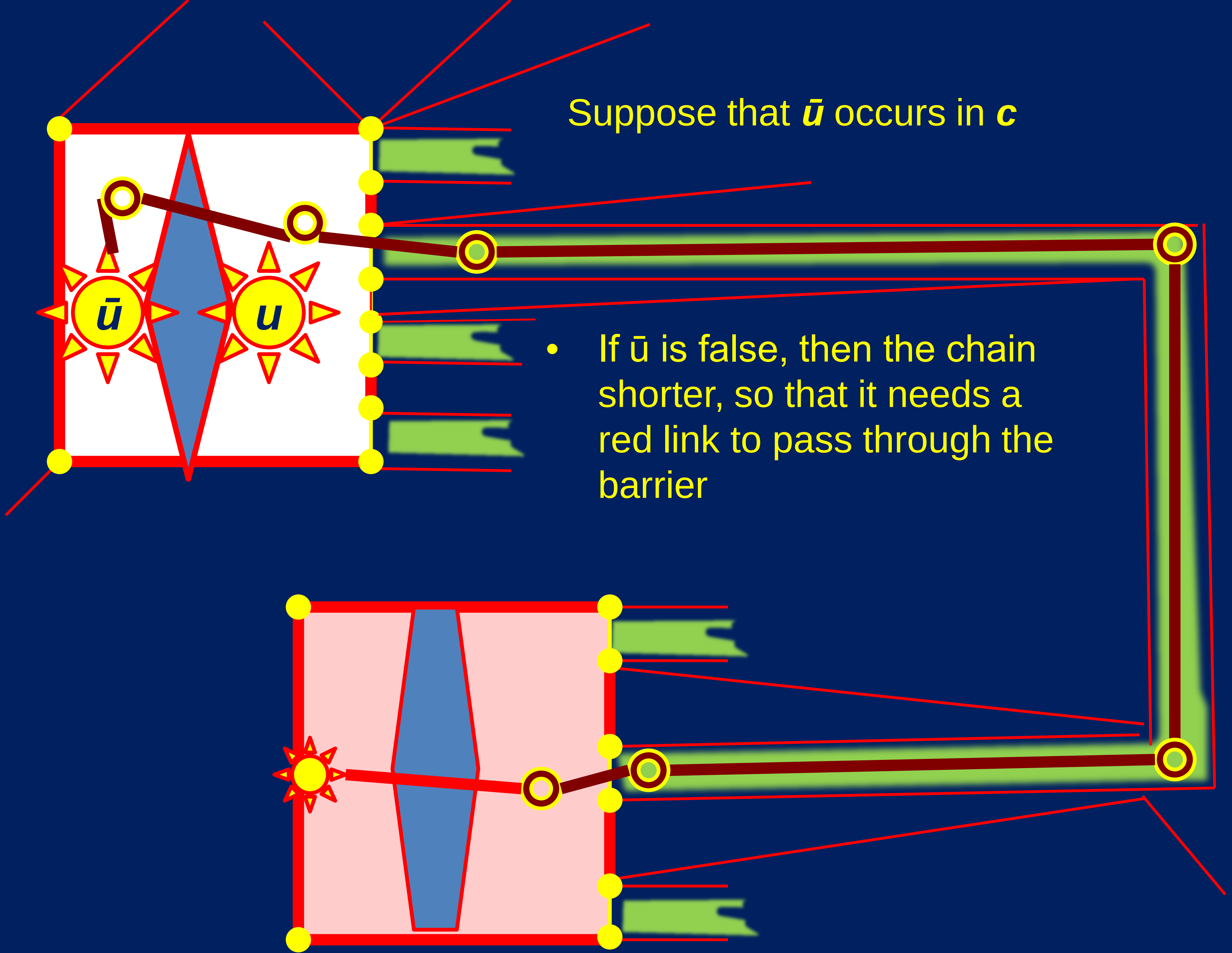
The barrier allows

- Any number of brown links to pass through
- At most two red links to pass through

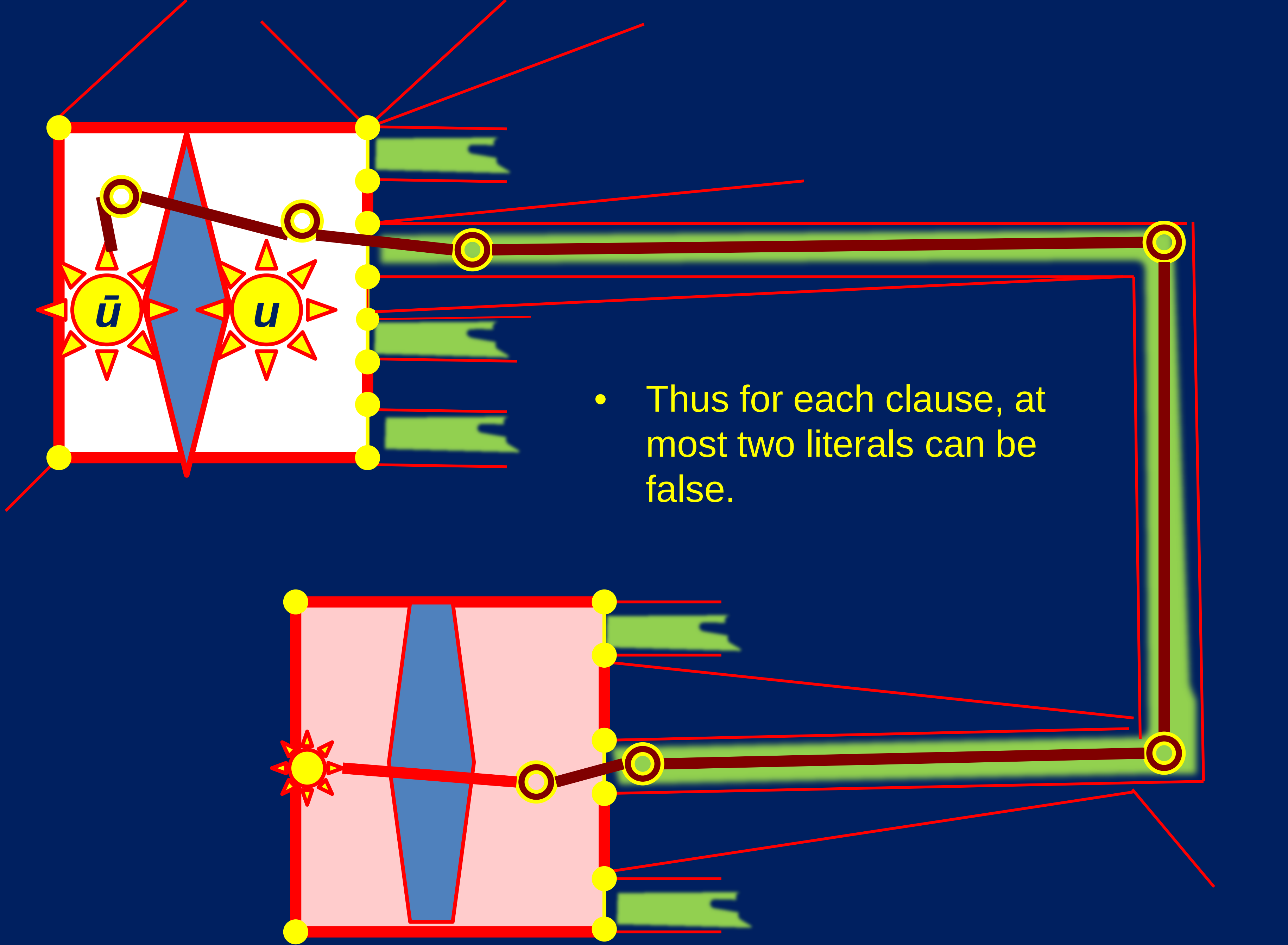
Thus at least one chain needs to be long enough to reach past the barrier


Suppose that ***ū*** occurs in ***c***
u
ū
• If ū is true, then the chain is long enough so that it does not need a red link to pass through the barrier

Suppose that ***ū*** occurs in ***c***



- If ū is false, then the chain shorter, so that it needs a red link to pass through the barrier



- Thus for each clause, at most two literals can be false.

Notes

- This is a fairly generic proof strategy for NP-hardness for layout problems.
- Details of clause and variable gadgets are straightforward but tedious
- The same proof works for 1-planar graphs: just choose different gadgets for clauses and variables.

Questions for slightly non-planar graphs:

- How can you compute a drawing of a RAC graph?

More generally,

- How can we draw a graph with large crossing angles?

Answers

- There are some force directed heuristics that use forces to enlarge angles
- There are some special algorithms for some special classes of graphs

However, other than the NP-hardness result, the problem remains mostly open from both practical and theoretical points of view.

open problem

Given a graph drawing, what is the smallest crossing angle?